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MATHEMATICAL MODELS OF FORCES, ACTING ON WIND WHEEL WITH VERTICAL AXIS OF ROTATION AT NON – ZERO ANGLES OF BLADES TURN

Vector diagrams of the air fluxes velocities, acting on the blades of the wind wheel with vertical rotation axis, and forces, created by this flux at non – zero angles of blade turn are constructed. Mathematical models, interconnecting these velocities, forces and angles in the process of wind wheel rotation have been suggested.

Key words: *wind electric station, wind motor with vertical rotation axis, wind velocity, angle of blade rotation, angle of attack, aerodynamic force*

Problem set – up

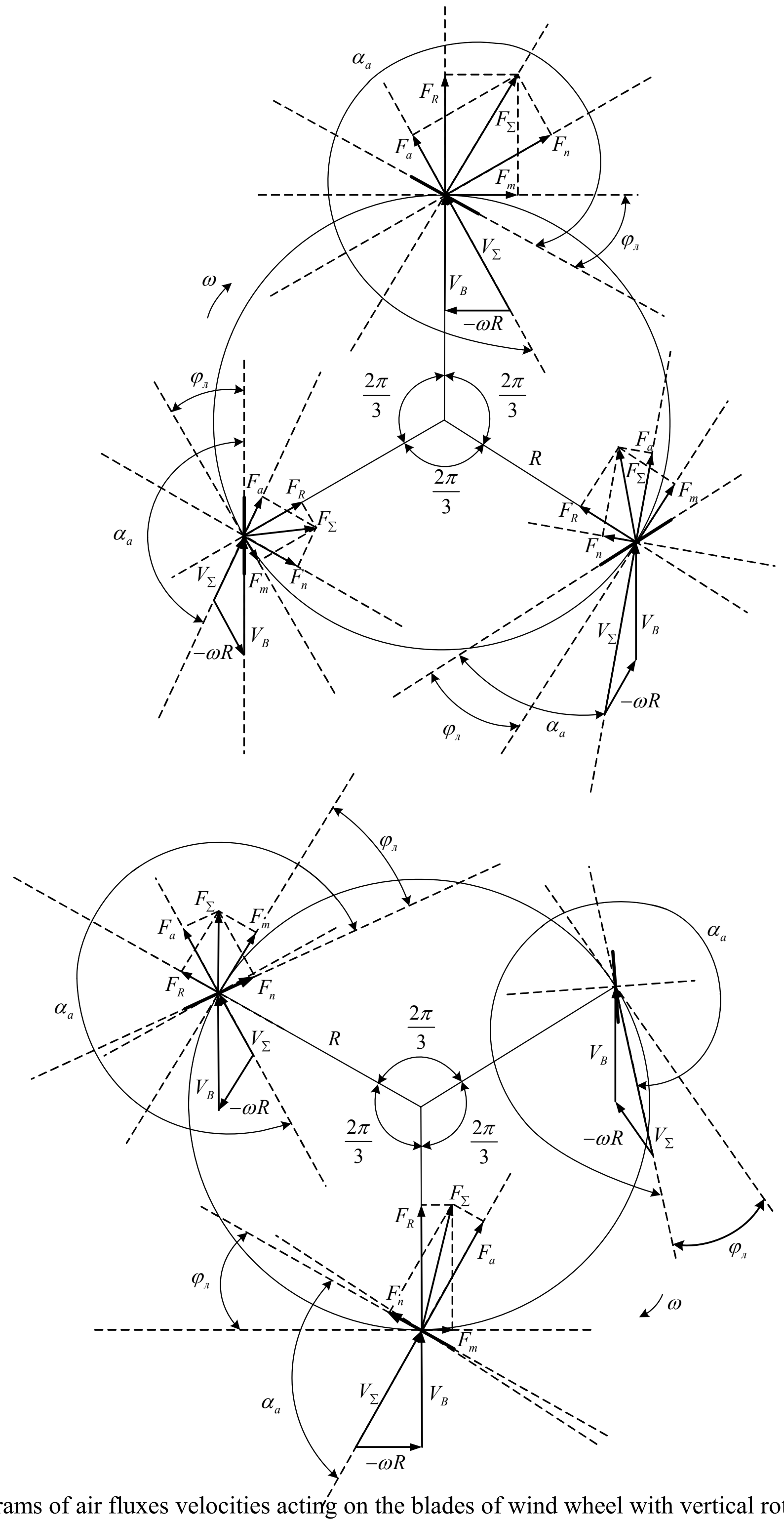
In [1] it is shown how simple and efficient mathematical models enabling to carry out analysis of forces, emerging in the process of rotation of the wind wheel with vertical axis can be constructed. However, these models were obtained on condition of zero turn angles of wind wheel blades. For practical application mathematical models which take into consideration not zero value of blade turn angle are more useful.

The given research is aimed at construction of the models of wind wheel with vertical rotation axis.

Problem solution

Applying the theory of aerodynamics [2] let us construct vector diagrams of velocities and forces for wind wheel with vertical rotation axis and three blades, spaced at the angle of $2\pi/3$, horizontal section of which is shown in Fig.1, along with vector diagrams for two, shifted relatively each other by 180°, positions of wind wheel.

In these vector diagrams ω – angular velocity of wind wheel rotation around the axis, V_w – velocity of wind flux, R – wind wheel radius, ωR – circumferential velocity of the blade, V_Σ – velocity of resulting wind flux, carrying over the blade, φ_b – angle of blade turn relatively the plane, perpendicular to the radius of wind wheel at the point of blade attachment, α_a – angle of attack of the blade by resulting wind flux at non – zero angle φ_B of its turn, F_a – force of aerodynamic pressure of resulting wind flux on the blade, F_n – aerodynamic lifting force, acting on the blade F_Σ – resulting aerodynamic lifting force, acting on the blade F_R – radial force of ram wind pressure on the blade, F_t – traction force, creating the torque.



a)

b)

Fig 1. Vector diagrams of air fluxes velocities acting on the blades of wind wheel with vertical rotation axis and forces, created by these fluxes at non – zero values of blade turn angle, constructed for two , shifted by 180°, positions of wind wheel.

Note, that on these vector diagrams the count of angles of attack and blades turn angles is carried out from blade plane counter-clockwise, that allows to write down the value of these angles in mathematical models with the sign “+”.

While constructing mathematical models of forces, acting on the blade of wind wheel with vertical axis of rotation at non-zero values of its turn angles relatively the plane, perpendicular to the radius of wind wheel, we will use the analogy, suggested in [1].

We have the right for the force $F_{\sum}$, created by aerodynamic pressure of the sum of air fluxes, to write the mathematical model in the following form:

$$\begin{aligned} F_{\Sigma 1} &= F_{\max} \sin(\omega t + \alpha_{an} + \varphi_{л}), \\ F_{\Sigma 2} &= F_{\max} \sin(\omega t + \alpha_{an} + \varphi_{л} - 2\pi/3), \\ F_{\Sigma 3} &= F_{\max} \sin(\omega t + \alpha_{an} + \varphi_{л} - 4\pi/3), \end{aligned} \tag{1}$$

where α_{an} – the value of the first blade angle of attack at the moment of the beginning of time count if its angle were zero; $F_{\sum 1}$, $F_{\sum 2}$, $F_{\sum 3}$ – total force of aerodynamic pressure correspondingly on the first, second and third blades, F_{max} – its amplitude value which can be found, for instance, from the first equation of the system (1), on condition, that

$$\omega t + \alpha_{an} + \varphi_{л} = \pi/2. \tag{2}$$

We should remember that in velocity system of coordinates –

$$F_{\Sigma}^2 = F_a^2 + F_n^2, \tag{3}$$

and in connected system of coordinates

$$F_{\Sigma}^2 = F_R^2 + F_m^2. \tag{4}$$

We should also remember that of all these forces for designers and operating staff the most important is the force F_m, creating the torque, and the force F_R, tending to bend the axis of wind wheel and destroy the support bearing.

From vector diagram in Fig 1 it is obvious that for numerical determination of radial force F_{R1}, acting on the first blade, it is necessary to use the formula, slightly different from the formula, suggested in [1], namely:

$$F_{R1} = k_F^R S_b \rho_w^2 \cos\varphi_b, \tag{5}$$

where, as in [1], S_b – blade section area (m^2), ρ – specific air density (kg/m^2), V_w – wind velocity (m/sec), K_F^R – factor less than one, charactering the difference of the “corridor” of air flux motion, directed at the blade, from the tube with rectangular section, equal to the section of this blade.

But it is quite obvious, that the structure of mathematical models for forces F_{R1}, F_{R2}, F_{R3} in dynamics will be analogous to the structure, suggested in [1], the only difference is that in each structure we will have the increment of the angle of attack by the value of blade turn angle, i.e.

$$\begin{aligned} F_{R1} &= F_{\max}^R \sin(\omega t + \varphi_n + \varphi_b), \\ F_{R2} &= F_{\max}^R \sin(\omega t + \varphi_n + \varphi_b - 2\pi/3), \\ F_{R3} &= F_{\max}^R \sin(\omega t + \varphi_n + \varphi_b - 4\pi/3), \end{aligned} \tag{6}$$

where, as in [1], φ_b – initial turn angle of F_{Ri} force vector at the moment of time t=0, matching with the value of initial angle of attack $\alpha_{\alpha\varphi}$, and F_{max}^R – amplitude value of this force, which can be determined from the first equation of the system (6) on condition that t=0, i.e., from the expression

$$F_{\max}^R = \frac{F_{R1}}{\sin(\varphi_n + \varphi_b)}. \tag{7}$$

The formula for definition of the force of aerodynamic pressure at the first blade F_{al} will also differ from the formula obtained in [1]. Transforming the formula in accordance with the vector diagram, given in Fig 1,it is seen that the formula takes the following form.

$$F_{a1} = S_b \cos(\angle(F_m, F_n) + \varphi_b) V_\Sigma \rho V_\Sigma k_F^a = \\ k_F^a S_b \rho V_w^2 \frac{\cos(\angle(F_m, F_n) + \varphi_b)}{\cos^2(F_m, F_n)}, \tag{8}$$

where, as in [1], K_F^a - less than one coefficient, characterizing the difference of the "corridor" of the air flux motion, directed at blade, from the tube with rectangular section, equal to the section of the blade, and $\angle(F_m, F_n)$ - angle between corresponding axes of rate and connected coordinate systems, but additionally angle ($\angle(F_m, F_n) + \varphi_л$) appears between the plane of the blade and the plane, perpendicular to vector V_Σ.

It follows from vector diagram, shown in Fig 1, that at non – zero turn angle of the blade 1, the following system of two equations, obtained in [1] is valid, this system connects two known F_{al},

F_{Ri} and two unknown F_{ml}, F_{nl} forces:

$$\begin{aligned} F_{m1} &= F_{n1} \cos\angle(F_m, F_n) - F_{a1} \sin\angle(F_m, F_n), \\ F_{R1} &= F_{n1} \sin\angle(F_m, F_n) + F_{a1} \cos\angle(F_m, F_n), \end{aligned} \tag{9}$$

solving this system of equations, we could easily find these unknown forces.

But we need to known only one of them – $F_{ml.}$ In order to find it, multiply the first equation of the system (9) by $\sin\angle(F_m, F_n)$, and the second equation is to be multiplied by $\cos\angle(F_{m,} F_n)$, then from the second multiplied equation we subtract the first one.

As a result we obtain

$$F_{R1} \cos\angle(F_m, F_n) - F_{m1} \sin\angle(F_m, F_n) = F_{a1}, \tag{10}$$

hence

$$F_{m1} = \frac{F_{R1} \cos\angle(F_m, F_n) - F_{a1}}{\sin\angle(F_m, F_n)}. \tag{11}$$

Further, taking as a basis the system of equations (6) and taking into account that the vector of force F_{ml} lags behind the vector of force F_{R1} by angle $\pi/2$ (see Fig 1), we can write that in dynamics

$$\begin{aligned} F_{m1} &= F_{\max}^m \sin(\omega t + \varphi_n + \varphi_b - \pi/2), \\ F_{m2} &= F_{\max}^m \sin(\omega t + \varphi_n + \varphi_b - \pi/2 - 2\pi/3), \\ F_{m3} &= F_{\max}^m \sin(\omega t + \varphi_n + \varphi_b - \pi/2 - 4\pi/3). \end{aligned} \tag{12}$$

From the first equation of this system on condition that t =0, we find that.

$$F_{\max}^m = \frac{F_{m1}}{\sin(\varphi_n + \varphi_b - \pi/2)}, \tag{13}$$

where F_{ml} - it is the value of the pull force at the first blade, that can be determined from the expression (11).

Proceeding from the same arguments, presented in [1], acting value $M^{д}_{tr}$ of the torque can be found from the relation.

$$M_{tr}^{д} = F_m^{д} R = \frac{F_{\max}^m}{\sqrt{2}} R, \tag{14}$$

in which wind wheel radius R is the arm to which force $F^{\partial}{}_{p}$ is applied.

Conclusions

1. Vector diagrams of forces, produced by wind fluxes, acting on the blades of the wind wheel with vertical rotation axis at non – zero turn angle of the blade relatively the tangent at the point of blade connection with the rim of wind wheel are constructed

2. Mathematical models of forces, which emerge in dynamics on the blades of the wind wheel with vertical rotation axis under the impact of the air fluxes at non zero turn angle of the blades based on the analogy with three phase AC electric system of sinusoidal character are suggested.

3. The technique of the identification of the suggested mathematical models for forces, emerging at the blades of the wind wheel with vertical rotation axis, at non zero value of blades turn angle is elaborated.

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