# A. D. Azarov Dr. Sc. (Eng.), O. O. Reshetnik, S. V. Bogomolov SYSTEM OF CALCULATION WITH WEIGHT REDUNDANSY FOR PROCESSING SPEED AD CONVERTORS OF SEQUENTIAL APPROXIMATION AND DIGITAL ANALOUGE CONVERTER ABLE FOR SELF-CALIBRATION 

There had been given the review of calculation system with weight redundancy. There had been analyzed the peculiarities of theoretic and numeral calculation systems with weight redundancy. There had been analyzed the possibility of realization of AD converter and digital-analog converter with weight redundancy on the basis of nonredundant digital-analog converter. There had been shown the possibility of usage of calculation redundant systems with weight redundancy and artificial base on the basis of dual series witch allows to build redundant digital-analog converter on the basis of nonredundant dual digital-analog converter without the creation of specialized elementary base.

Key words: AD converter, redundant positional calculation systems (RPCS), calculation system with weight redundancy(CSWR).

## Introduction

The majority of modern information form converters (IFC) in the kind of AD converter and digital-analog converter are realized with the usage of classical dual calculation system [1-5]. At the same time the creation of AD converter and digital-analog converter on the basis of calculation system with weight redundancy [9] allows to solve the problems of improvement of processing speed and accuracy of AD converter as well as digital-analog converter, built on low - precised elementary base in complex.

Principal disadvantage of calculation dual system usage in IFC is that the availability of instrumental static failures causes the appearance of zones in the characteristics of AD converter conversion, in which the output analogue value cannot be composed by any code combination. Such zones are called the breakage of transformation characteristic [1, 2, 5, 7, 8]. This, in turn, cases the appearance of the so called missing codes in the AD converter created on such digitalanalog converter [9]. For the realization of AD converter without the missing code it is necessary to use digital-analog converter with the weight redundancy with no breakage of the conversion characteristic. Such an approach simplifies at the same time the calibration of the weight digits and allows formulating the conversion results in digital-analog converter together with the compensation of dynamic failures. This helps reduce the conversion time and improve the processing speed.

## Urgency of the problem

Breakage of the conversion characteristic in AD converter of digit approximation causes both, the availability of instrumental static failures (weight digits deviation) in digital-analog converter as well as dynamic failures, when the durability of stroke is no sufficient for completion of transient processes during the digit balancing.

The usage of weight redundancy during the creation of AD converter and digital-analog converter allows forming the indissoluble conversion characteristic with the availability of both, the static and dynamic failures, which appear in digital-analog converter during the digits balancing.

It should be noted that the given approach allows to improve the accuracy of multidigit AD converter and digital-analog converter, created on inaccurate elements (failures of $5 \%-10 \%$ ) by self calibration of digit weights [9]. Apart from that, there is the possibility of substantial (by one or two orders) increasing the processing speed of AD converter of digit balancing. Hence the level of calculation system research with weight redundancy (CSWR) as well as the peculiarity of its usage in AD converter and digital-analog converter techniques is inefficient. That is why the analysis of
the theoretic and numerical peculiarities of CSWR, directed to the improvement of processing speed and accuracy of analog digital and digital-analog conversion on the basis of usage of different types of CSWR is urgent.

## AIM

The aim of the paper is the systematization of the calculation systems with weight redundancy used in IFC for the increasing of the processing speed and accuracy.

## Tasks

According to the objective set, the following tasks are formed:

1) Examination of calculation system with the weight redundancy and the estimation of the level of this redundancy according to their types.
2) The analysis of theoretical and numerical peculiarities of CSWR.
3) The analysis of the realization possibilities of $A D$ converter and digital-analog converter with their weight redundancy on the basis of non redundancy dual digital-analog converter.

## Tasks solution

The calculation system or numeration [11] is the aggregate of methods and rules for denomination and designation of numbers. The aggregate of digits weights of calculation system is the basis $\left\{Q_{0}, Q_{1}, Q_{2}, Q_{3}, \ldots, Q_{n-1}\right\}$. Alphabet - is the amount of values that each of calculation system digit can except $a \in\{0,1\}, a \in\{-1,1\}$ or $a \in\{-1,0,1\}$. Thus the calculation system can be set by the basis of calculation system, base and alphabet. Classic dual calculation system has the basis 2 , base $1 ; 2 ; 4 ; \ldots 2^{n-1}$ and alphabet $\{0,1\}$. Depending on the weight setting law of the $i$-th digit, the calculation systems can be divided into systems with natural basis and systems with artificial basis. In natural basis there is a steady correlation between digit weights $-\alpha=\frac{Q_{i}}{Q_{i-1}}$, besides, the correlation is the basis for calculation system. In calculation systems with artificial basis the weight of each of digit is formed by some specific law from the smaller digit (for example, as the sum of two previous digits in Fibonachi's system of calculation).

In positional calculation systems with fractional digit weights ("golden" p and s proportions) any real number can be shown as:

$$
A^{*}=\sum_{i=-\infty}^{n-1} a_{i} \alpha^{i},
$$

where $a_{i}$ - the number in the $i$-th digit, $\alpha$ - the basis of calculation system. But in the techniques of information conversion the given expression is unacceptable as it suggests the availability of endless long digital net. In practice the length of digital net is restricted [9]. That is why in this case it is convenient to use the expression for the real numbers:

$$
N^{*}=\sum_{i=0}^{n-1} a_{i} \alpha^{i} .
$$

In CSWR with the natural basis there appear the problems with accurate image of the integers. This peculiarity must be taken into consideration during the creation of IFC within the realization of the digital part. In the same time the inaccuracy in presentation of integers does not influence the accuracy of convention of the analogue-code or code-analogue. This is a forcible argument to CSWR, as in AD converter or digital-analog converter irrespective the fact what calculation systems is used, there are always the instrumental failures on the level of half of low-order digit.

The essence of weight redundancy is, that the sum of low-order digit weights is bigger or equal to the weight of high-order digit (weight redundancy is displayed as in the systems with natural basisas well as in the systems with an artificial basis):

$$
\sum_{j=0}^{j=i-1} Q_{j} \geq Q_{i} .
$$

Thus the absolute weight redundancy is calculated as follows:

$$
\Delta Q_{i}=\sum_{j=0}^{j=i-1} Q_{j}-Q_{i},
$$

The notion of relative weight redundancy determines the level of weight redundancy for statics, the level of weight redundancy in statics shows the possible total level of deviation of digits weight digits when the characteristic of remains indissoluble:

$$
\delta Q_{i}=\frac{\sum_{j=0}^{j=i-1} Q_{j}-Q_{i}}{\sum_{j=0}^{j=i} Q_{j}} .
$$

On condition of constancy of $\alpha$ for the systems of calculation with natural basisthe following correlation comes true:

$$
\delta Q_{i} \approx \frac{2-\alpha}{\alpha} .
$$

For dynamics, when the temporary changes of digit weight in the process of balancing take place, the notion of the resulted relative weight redundancy is used [9]:

$$
\widetilde{\delta} Q_{i}=\frac{\sum_{j=0}^{j=i-1} Q_{j}-Q_{i}}{Q_{i}}
$$

Thus dynamic failure is formed as the part of one digit. Dynamic failure of the I type exists during the short interval (within one or a few steps).

It should be differentlevels of weight redundancy participate in correction of static and dynamic failures. For correction of static failures the values of is being fixed and does not change during some time, there shall be used the relative weight redundancy which is proportionally reflected from all the digits of AD converter or digital-analogue converter. So the static failure is formed as the sum, contributed by all the digits.

CSWR can be formed on the basis of natural as well as on the basis of artificial basis. The systems of calculation of «gold» p and s that are proportions or RPCS belong to CSWR with natural basis [9]. Such systems of calculation have the basis and stable level of weight redundancy.

Fibonacci system of calculation, some dual-coded codes (DCC) as well as CSWR on the base of dual digit described bellow belong to the CSWR with artificial basis. It should be noted that DCC are widely used in measuring equipment for convenient presentation of measuring results in decimal system of calculation. For artificial basis it is not possible to use the notion of the base of the calculation system, since the correlation between weights of neighbor digits is not constant.

Level of weight redundancy in CSWR with natural basis depends on the value of the base $\alpha$ (fig. 1a). Table 1 shows the value of relative weight redundancy for different $\alpha$ in case of natural basis.

| Table 1 |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\alpha$ | 2,00 | 1,90 | 1,80 | 1,70 | 1,618 | 1,60 | 1,50 | 1,40 |
| $\delta Q_{i}(\%)$ | 0 | 5,26 | 11,11 | 17,65 | 23,62 | 25,00 | 33,33 | 42,86 |
| $\widetilde{\delta} Q_{i}(\%)$ | 0 | 11,11 | 24,99 | 42,86 | 61.81 | 66.66 | 99.99 | 149.94 |

Table 1 shows that $\delta Q_{i}<\widetilde{\delta} Q_{i}$, that is why for the compensation of dynamic failures there are much more possibilities than for the correction of static ones.

In CSWR, in comparison with dual system of calculation, the digit net is continued [9]. The degree of extension is determined by the factor of digit net extension (fig. 1 b ):

$$
\gamma_{n}=\frac{\ln 2}{\ln \alpha} .
$$

Table 2 shows the value of factors $\gamma_{n}$ for some $\alpha$ from the natural basis. It should be noted, that for $\alpha=1$ we have $\gamma_{n}=\frac{2^{n}}{n}$. The given formula $\gamma_{\mathrm{n}}$ describes the case when dual and redundancy range strictly coincides. If there are exactly number of digits in system of calculation, such formula gives only an approximate result. In practice it should be taken into account that real factor of extension can be much higher (or lesser) on $1 / \mathrm{n}$. ( n - quantity of dual digit), but CSWR digit in this case is going to increase or reduce. It stipulates that the CSWR digit of present numerous can be higher or lesser than the digit of present numerous in dual system of calculation, but there is infrequent situation when digit are coincided. While the chosen of CSWR it should taken into account the level of weight redundancy as well as real factor of digit net extension. At the same time the factor of digit net extension for CSWR with artificial basis is determined by number of digit.

Table 2

| $\alpha$ | 1,20 | 1,30 | 1,414 | 1,50 | 1,60 | 1,70 | 1,80 | 1,90 | 2,00 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\gamma_{n}$ | 3,80 | 2,64 | 2,00 | 1,71 | 1,48 | 1,31 | 1,18 | 1,08 | 1,00 |




Fig. 1. Dependence of additional parameters of CSWR on $\alpha$ : a) level of relative weight redundancy, b) extension factor of digital net

It should be noted that the construction of IFC on the basis of CSWR requires creation of specific element base for every separate CSWR. At the same time, without regarding to the Наукові праці ВНТУ, 2008, № 3
possibility of usage of simplified technology for making the analog knots of AD converter or digital-analog, it can become a hindrance on the way of technical realization.

For overcoming of this engineering and economic barrier during practical realization of IFC authors offer a few approaches in relation to a construction AD converter with gravimetric surplus on the basis of classic not surplus of dual system of calculation [12]. The last enables to design surplus AD converter or digital-analog converter with improving static and dynamic descriptions on the base of traditional dual AD converter and does not require creation of original element base. The examples of forming of surplus rows on the base of dual rows and their description are resulted in a table 3:

Table 3

| № | Row | Relative weight <br> redundancy, $\partial Q$ |  | Coefficient of word <br> length prolongation, <br> $\gamma_{n}$ | Specific weight <br> redundancy, $\gamma_{E}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\max$ | 2 | $0,14 / 0,25$ |  |  |
| 1 | $1 ; 1 ; 2 ; 2 ; 4 ; 4 ; \ldots$ <br> $2^{n-1} ; 2^{n-1} ;$ | 0,27 | 0,49 | 2 | $0,19 / 0,22$ |
| 2 | $1 ; 1,5 ; 2 ; 3 ; 4 ; 6 ; \ldots$ <br> $2^{n-1} ; 1.5 \cdot 2^{n-1} ;$ | 0,39 | 0,43 | 2 | 0,207 |
| 3 | $1 ; \sqrt{2} ; 2 ; 2 \sqrt{2} ; 4 ;$ <br> $4 \sqrt{2} ; \ldots 2^{n-1} ;$ <br> $2^{n-1} \sqrt{2} ;$ | 0,414 |  | 2 | $0,13 / 0,17 / 0,22$ |
| 4 | $1 ; 2 ; 3 ; 4 ; 8 ; 12 ; \ldots$ <br> $2^{n-2} ; 2^{n-1} ;$ <br> $2^{n-2}+2^{n-1}$. | 0,19 | 0,33 | 1,5 |  |

If the correlations between the neighboring members of base are not constant, the extension factor of digit net can be determined as the relation of the proper number of redundant digits to the number of dual digits on condition of sameness of ranges of representation of numbers [9].

Choice of the type of CSWR is expedient to make using the - specific weight redundancy $\gamma_{\text {IBH }}$ witch is determined as the relation of the level of relative weight redundancy to the extension factor of digital net.

$$
\gamma_{\text {IBBH }}=\frac{\delta Q}{\gamma_{n}}=\frac{\left(\frac{\alpha^{i}-1}{\alpha-1}-\alpha^{i}\right) \ln (\alpha)}{\left(\frac{\alpha^{i}-1}{\alpha-1}+\alpha^{i}\right) \ln (2)},
$$

and the given specific weight redundancy:

$$
\widetilde{\gamma}_{\text {IBH }}=\frac{\widetilde{\delta} Q}{\gamma_{n}}=\frac{\left(\frac{\alpha^{i}-1}{\alpha-1}-\alpha^{i}\right) \ln (\alpha)}{\alpha^{i} \ln (2)} .
$$

Let us analyze the given functions as for extremums in the section from 1 to 2. Having differentiated the functions we get the following equations:

$$
\begin{aligned}
& \gamma_{\text {IBH }}^{\prime}=\frac{1}{\ln (2)}\left(\left(\alpha^{i} \frac{i}{\alpha}-2 \alpha+1\right) \frac{\ln (\alpha)}{\alpha^{i+1}-1}+\frac{\alpha^{i}-\alpha^{2}+\alpha-1}{\alpha\left(\alpha^{i+1}+1\right)}-\right. \\
& \left.-\left(\frac{\left(\alpha^{i}-\alpha^{2}+\alpha-1\right) \ln (\alpha)}{\left(\alpha^{i+1}+1\right)^{2}} \alpha^{i+1} \frac{i+1}{\alpha}\right)\right) \\
& \widetilde{\gamma}_{\text {IBH }}^{\prime}=\left(\alpha^{i} \frac{i}{\alpha(\alpha-1)}-\frac{\alpha^{i}-1}{(\alpha-1)^{2}}-\alpha^{i} \frac{i}{\alpha}\right) \frac{\ln (\alpha)}{\alpha^{i} \ln (2)}+ \\
& +\frac{\frac{\alpha^{i}-1}{\alpha-1}-\alpha^{i}}{\alpha^{i+1} \ln (2)}-\left(\frac{\alpha^{i}-1}{\alpha-1}-\alpha^{i}\right) \frac{\ln (\alpha) i}{\alpha^{i+1} \ln (2)}
\end{aligned}
$$

The roots of the equation are the optimal values of $\alpha$ for the static $\alpha=1,375$ and the dynamic of $\alpha=1,1575$. Normalizing each function on its own extremum allows to find the crossing point in witch the dynamic and static characteristics are balanced $\alpha=1,2553$.


Fig. 2. Dependence the relative weight extension from
Analyzing the given graphs (fig. 1-2) it is easy to make the conclusion, that the zone of optimum for CSWR on the base of natural basis is the value of the base from 1,2 to 1,6 . But within the limits of the value $1.2 \leq \alpha \leq 1.4$ the extension factor of digital net increases sharply with the reduction of $\alpha$. And within the limits from 1.4 to 1.6 the factor of extension remains lower that two.

The considered CSWR with the artificial basis on the base of dual system of calculation have the factor of extension of digital net 2 . In this situation the level of weight redundancy of such systems can reach $\delta Q=0.4-0.5$. The advantage of the given systems of calculation is the possibility of creation IFC with the weight redundancy on their base without the creation of the specialized elementary base.

The usage of CSWR on the basis of dual series allows to create the redundant AD converter on the base of no redundant dual AD converter. Lets consider the structure of realization of such AD
converter (fig. 3) on the example of basis $1 ; \sqrt{2} ; 2 ; 2 \sqrt{2} ; 4 ; 4 \sqrt{2} ; \ldots 2^{n-1} ; 2^{n-1} \sqrt{2}$. AD converter contains two dual AD converters, the summator of analog signals and the scale block. Outputs of AD converters are connected to the summator of analog signals, one of the AD converters is connected through the special scale block $M$ (in this case the factor of transferring $\sqrt{2}$ ). The outputs of dual AD converters are connected to the external output bus of the redundant AD converter, in turn, according to the increasing of weight digits.


Fig. 3. Redundant AD converter on the basis of two dual no redundant AD converters
Thus the usage of the CSWR on the basis of dual digits allows us to built redundant AD converter on the basis of dual no redundant AD converter, which don't have brake in the transmitting characteristic even with the availability of the deviation of weight digits.

## Conclusions

1. There had been made the survey of calculation systems with weight redundancy (CSWR) on natural as well as artificial basis. There had been shown that for the natural basis the values of the calculation systems basis $\alpha \approx 1,37$ gives the biggest efficiency.
2. There had been shown the possibility of the creation of the redundant AD converter without the creation of the specialized elementary base.

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