A. A. Shiyan Cand. Sc(Phys-Math)

MATHEMATICAL MODELING OF JOINT ECONOMIC ACTIVITY OF PEOPLE

Mathematical apparatus intended for modeling of joint economic behaviour and classification of relations between the types of economic behaviour of people has been elaborated.

Keywords: *informative space, types of activity, joint management, operator.*

Introduction and analysis of literature

Economic behaviour of an individual is the basis of modern economic theory [1]. Activity of people is carried out collectively, mainly in collectives consisting only of a few persons. However, such problems definitions are not considered in modern economic theory: attention of researchers is concentrated on the problems of collective choice and formation of coalitions [2,3].

The purpose of the paper is construction of mathematical apparatus intended for modeling of joint economic activity of people aimed at enhancement of efficiency of managerial decisions. The criteria of results achievement of result are correspondence of theoretical forecast to real results of joint activity of the set of people.

Problem set-up

Using the results [4, 5], where it is proved that random activity of a man can be described as an operator (two-component abstract information automaton – 2AIA) which operates in *the* specially built *informative space*, elaborate mathematical apparatus for modeling of *joint* activity of several 2AIA.

Description of algebra of relations between the types of 2AIA

As it was shown in [4, 5], the activity of a man corresponds to one of 16 types of 2AIA, each of which can be presented as a 4-component vector, having the form *{a,b,c,d}*. Thefirst two components describe the programming function of this 2AIA (and correspond to information space, built prior *to* realization of the activity), and two components – his creative function (and belong, accordingly, to information space after realization of human activity). The first component of this vector determines the pole of dichotomy «generalizing – detailing» of programming function, and the *second* is a given class of information, it belongs to. The *third* component of information describes the precise class of information, the creative function belongs to, and the *fourth* sets, whether the creative component describes the state or the process.

We will note that due to such determination the second and the third components of 2AIA type of vector differ by the pole of dichotomy «generalizing – detailing» into a detail, and in case of the change *of this pole* of 2AIA the second and the third components in the type record must change places.

As follows from the definition given above for 2AIA type record, each vector component of the type can take two values: *0* or *1*. The choice of fixing of concrete correspondences fixing for the values of variables – poles of corresponding dichotomies – for further consideration is of minor importance: in fact it means *the randomness* of choice of «type 2AIA, from which counting begins». Thus, type 2AIA as a vector can be written as *{a,b,c,d}*, where *a,b,c,d=0;1*. We will denote the set of all 16 types of 2AIA as $\{T_i\}$.

We will introduce the class of operators which are determined at the set $\{T_i\}$ and which transform one type of 2AIA into another *certain* type of 2AIA. We note that this class of operators can be presented as component-wise addition with the vector of certain 4-component vector type which is the presentation of the corresponding operator. Addition must be performed by *mod 2*. Thus, the components of all these vectors form in algebraic sense *the field* of two elements: *0* and *1* [6].

The basis of this presentation of operators forms *four* vectors, which can be written as $e_1=\{1,0,0,0\}$, $e_2=\{0,1,0,0\}$, $e_3=\{0,0,1,0\}$, $e_4=\{0,0,0,1\}$.

Note that there exists only 16 different operators which translate one type of 2AIA into another one : besides the basic vectors described above and zero vector *of* $e_0=\{0,0,0,0\}$ (identical transformation) *t:* $e_5= e_1+ e_2$, $e_6=e_1+ e_3$, $e_7=e_1+ e_4$, $e_8=e_2+ e_3$, $e_9=e_2+ e_4$, $e_{10}=e_3+ e_4$, $e_{11}=e_1+ e_2+ e_3$, $e_{12}=e_1+ e_2+ e_4$, $e_{13}=e_1+ e_3+ e_4$, $e_{14}=e_2+ e_3+ e_4$, $e_{15}=e_1+ e_2+ e_3+ e_4$.

After the action of the operator e_1 the pole of dichotomy «generalizing – detailing» in the record of 2AIA type changes , and we must change the places of generalizing and detailing classes of information in the record of vector type (i.e. change the places of the second and third numbers). Due to this reason for the operators $e_5 – e_{15}$ operation of «translation of type in type» – that is , «law of addition» for information components – will be defined as follows.

The operator e_1 acts *the first (*changing the pole of dichotomy «generalizing – detail ing» in the record of 2AIA type: «0» into «1» or vice versa, accordingly), as a result *the second and third components of vector type change places*.

And only after that the action of other basic operators must be performed (i.e. the addition with other operators e_i if $i>1$ *for* vector type).

Due to this condition the aggregate of operators *of* $e_0 – e_{15}$ is examined further as the aggregate of arranged operators, according to V. Maslov [7].

Thus, the system *of operators* $\{e_i\}$ the action of which is performed on the set $\{T_i\}$ *is obtained.* . The structure of this set is assigned by the following theorems.

Theorem 1. The aggregate of operators forms $\{e_i\}$ noncommutative group.

Proof is obvious: for example, $e_7 \bullet e_{13} \neq e_{13} \bullet e_7$.

Theorem 2. Group $\{e_i\}$ has *11* cyclic sub-groups of the order 2.

Theorem 3. Group $\{e_i\}$ is decomposed into *3* types of complexes the elements of which possess the followings properties: $e_0 \bullet e_0 = e_0$ (1 complex), $e_i \bullet e_i = e_i^2 = e_0$ (11 sets of complexes – cyclic sub-groups of the order 2,we will call such operators «symmetric»), and $e_i^4 = e_0$ (4 sets of complexes – cyclic sub-group of the order4, such operators will be called «asymmetric»).

Theorem 4. Group $\{e_i\}$ is vector space of dimensionality *4*.

Corollary. If there is a description of action of random four linearly-independent operators from $\{e_i\}$, then the action of the rest *11* operators can be expressed in terms of action of these operators (action of *identical* operator *of* $e_0$is trivial).

Asymmetric operators from the set $\{e_i\}$ structure the set of 2AIA type $\{T_i\}$ as follows.

Theorem 5. The set of 2AIA type $\{T_i\}$ is divided by each of asymmetric operator into *4* equal power non- intersecting sets (4 orbits, which consist of *four* 2AIA of different types).

Corollary. The set $\{T_i\}$ is the sum of four sets, each of which is formed by the operator, possessing the property of $e_i^4 = e_0$.

Definition 1. Operator e_i from $\{e_i\}$, which translates one type of 2AIA into another, will be called *the relation* between the given types of 2AIA.

Thus, on the set of 2AIA of types $\{T_i\}$ due to the theorems given above there exists only *16* different relations: *1 identical* relation, *11 symmetric* relations (when repeated application of operators of transition from type to type does not lead outside of this pair of types) and *4* asymmetric relations (when by successive application of this relation *4* different types of 2AIA form a ring).

Asymmetric relation *of* e_{13} is allocated, because it provides the highest level of self-programming between the pair of 2AIA types. Indeed, as it is seen , only at such correlation between these types of 2AIA the creative function of the first type of 2AIA *coincides* with programmatic function of the other type of 2AIA. In other words, the activity of the first type of 2AIA is perceived by the second type of 2AIA as completely equivalent description of the whole surrounding world (in fact this, second type of 2AIA, «sees» only one component of information- and exactly the information , which is creative for the first type of 2AIA).

The set of operators $\{e_i\}$ can be presented in the form of graphs – segments which connect two

points (two 2AIA with different types). Then it will be seen that asymmetric relations can be presented as the *oriented* graphs.

As it follows from determination of operators $\{e_i\}$, if we apply the random *asymmetric* operator e_i twice, then we will get a symmetric operator: $e_i^2=e_8$. The following relations also take place: $e1$ $e_{12}\bullet e_{13}=e_{13}\bullet e_{12}=e_5\bullet e_6=e_6\bullet e_5=e_0$. *Presence of «intersecting» relations* $e_{14}\bullet e_5=e_{13}$ *and* $e_{14}\bullet e_6=e_{12}$ *and similar to them allows to select the relation* e_{14} *among all symmetric relations. Thus relation of* $e_{14}\bullet e_5=e_{13}$ *due to orientation graph* e_{13} *results that the graph* e_5 *also appears to be oriented (since the graph* e_{14} *– is not oriented).Hence ,we come to the theorem.*

Theorem 6. The system of graphs $\{e_i\}$ is structured as follows: e_0 is a ring (point), $e_1 – e_4$, $e_7 – e_{11}$, e_{14} , e_{15} are non-*oriented* graphs (symmetric relations, and the graph e_{14} is allocated concerning the connection of orbits, formed by the action of symmetric operators), e_5 , e_6, e_{12} and e_{13} are the *oriented* graphs (information spreads only by graphs e_{13} and e_5, and the graphs e_{12} and e_6 *are oriented oppositely* to the direction of distribution of information, and can be considered as «informative bottle necks»). Graph e_1 is allocated, because its application results in radical alteration of vector of type presentation.

Words from $\{e_i\}$ as chains of decisions making

Definition 2. Random sequence of operators from $\{e_i\}$ we will call *a word* (sequence of application of operators – from the right to the left).

Each such *word* sets the chain of decisions making. In other words, every word sets a definite chain of information distribution.

Note. We will see that only types, possessing the same pole of dichotomy «generalizing-detailing", can «communicate» with each other «on equal. In fact, *generalizing* type performs control «from general to private», where detailing 2AIA type – vice versa, «from private to general» (see Theorem 6).

It is important to note, that one and the same *word* can unite different sets of 2AIA (it is seen at presentation of operators in the form of graphs).

Definition 3. Words on the set of assigned types of 2AIA will be called *equivalent* in the sense of control realization, if they are based by their start and end on the fixed types of 2AIA (which can be both different and coincident; in the latter case we obtain *a cycle (ring)* of 2AIA).

We may say, that words – they are topologically invariant constructions on the set $\{T_i\}$.

The general algorithm of solution of problems dealing with management of socio-economic system of arbitrary nature with the help of assigned set 2AIA has the following form .

1)All the types of 2AIA are determined, which are in the set aggregate of people (that is, types are determined for all the people in this collective).

2)All of types of operators e_i are determined, which link pairs of different types of 2AIA, existing in the assigned set of 2AIA.

3)Words which are optimum for solution of the purpose of management and administrative decisions taking (that is, both the types of 2AIA for concrete people and relations between them are fixed).

Note, that management aims in general case can differ from those which are listed above: they are determined by the concrete problem to be solved. Thus, instead of studying chains of information transfer (chains of making new mode of management) between the concrete types of 2AIA, now it is possible to study *words* which are invariant and do not depend on the choice of concrete types.

Information classification of constructions for joint activity

We will consider information characteristics of construction which arises on the set $\{T_i\}$ at the requirement of maximally complete making of new joint management. In other words, it is necessary to find such construction in which all 16 types of 2AIA and all 16 types of relations between them are involved and which is maximally adjusted for making new management. Such

construction must have a maximal amount of the complete ways (cycles) which consist of asymmetric operators. We will build such a construction.

This type of 2AIA (that which «sets the problem» before other types of 2AIA) forms the ring (cycle) of *individual self-programming*) by successive application of e_{13} operator.

The action of the same operator e_{13} divides $\{T_i\}$ set into *3* rings (cycle) of c *self- self-programming.* Only one of the obtained rings of the individual self-programming can be jointed with this type of 2AIA so that to create a single unit, that is, a ring (cycle) of the *doubled («diadic»)* self-programming. Such ring will be obtained, if corresponding type of 2AIA with that ring(cycle) of individual self-programming in which this type is included is added by means of operator e_{14} *to* the given type of 2AIA. In this case every pair of types, which is in one link of such «doubled» ring (rings of dyadic self-programming) will be connected with the same operator e_{14}.

Thus, the set $\{T_i\}$ is divided into *two* rings of diadic self-programming , one of which contains the set type of 2AIA, and another – not.

Two rings of the individual self-programming which compose the *second* ring of dyadic self-programming (that is, which remained) is possible, in its turn, to add to the set type of 2AIA only with the help of four different operators which do not change in the type of selected by us 2AIA the pole of dichotomy «generalizing – detailing». In such case association takes place with such types of 2AIA, in which either programming or creative function *coincide* with corresponding functions of 2AIA of this type or the type which is obtained from it by means of the operator e_{14} (such type is called «*diadic»).*

In case of any other methods of joining the rings of individual self-programming to this type of 2AIA optimum transmission will not be achieved (as the information will be distorted while communication of the set type of 2AIA with other types of 2AIA, with whom he is *in the same link*, that is, operates jointly).

Now we will pass to the next theorem (mathematical details see, for example, in [8]).

Theorem 7. Construction on the set of types of 2AIA $\{T_i\}$, which is able to transform optimally new information, in topological sense is homotopically equivalent to a bouquet of 6 circumferences.

Corollary. Construction described in Theorem 7 is differomorphic to 2-D sphere with 7 tapes of Mebius. Number of Eclair for this construction $\chi=-5$.

Definition 4. Entered in Theorem 7 will name a construction *соционом*.

Note that in socion for random type of 2AIA all possible on the set $\{T_i\}$ operators are present (that is, all relations between types). Thus, socion is the object, which includes *the longest word* in which all the types from $\{T_i\}$ *are* present only once (that is, the longest way without repetitions). In socion such case will be realized too, when separate types of 2AIA perform communication with most of other types of 2AIA. Socion is just such an object which must be created to produce all the aggregate of possible modes (means, algorithms, methods) for realization of management in random socio-economic system.

In other words, socion is the object which *is identical tol* maximally possible coalition in the conditions of symmetric information. Thus, the following theorem is valid.

Theorem 8. «Information capacity» of the given aggregate of people is assigned by topological features of fundamental group, constructed on the set of those words, which can be created on the basis of those types of 2AIA, which correspond to people from this aggregate, and it can not be greater than information capacity of the socion.

Remark. These results can be obtained by «geometrical» method, when corresponding operators are presented as graphs.

It is interesting to note, that, as it follows from Theorem 7, optimum for functioning of the socion tree-like graph can be described as the object which has 1 «*input »-it* is an asymmetric operator, 1 «output»-it is an asymmetric operator and 5 non-directed edges – symmetric operators.

It is obvious, that in general case, the random type of 2AIA can function within in thesocion only in case, when the graph of 7±2 derivatives –edges is located on it . This statement is, probably, the first mathematical proof for known from psychology and management fact, that communication

between people is possible only between 7±2 communicators [9]. Also it can serve as the variant of proof of Ingve hypothesis [10].

Application and approbation

The method of application of the results obtained is described in [4, 11 – 14].

Joint administrative activity *of pair* of people is described most completely, that is, separate operators e_i from $\{e_i\}$ *are described* [11 – 13]. Theoretical forecast are obtained for the level of efficiency of joint activity of politicians L.Kuchma and D.Tabachnik [11, 12], M. Shaimiev andV. Putin [13], and co-operations between a number of other politicians [14], due to what it became to describe a number of specific effects while their joint administrative activity.

Words are also developed for realization of effective joint activity of people which consist of *a few* operators. For example, such effective chain for a *purposeful* management a concrete man was successfully approved [11, 12]. For already some years at one of private firms Vinnitca for organization of management of a concrete man – the director of firm uses the word e_{13}•e_4, which was offered as optimum one proceeding from the analysis of concrete composition of 2AIA types for the employees of the firm. The necessity of it was caused by the fact , that the types *of customer* (man which sets management) and director of firm are bound by the asymmetric relation e_6, that is, *the customer can not* pass the information to the director (information can be passed only from the director to our customer). For a management the third man was used. Note, , that the director of the firm is even unaware, that for many years he plays the role of the «target» for management (that is, management is performed so «naturally» and comfortably for him, that the director takes his decisions for his own). We will note that moral and ethical aspects of this problem were specially stipulated by us with the *customer*.

Conclusions

1. Mathematical apparatus for modeling of joint economic behaviour of people has been elaborated.

2. Classification of relations between the types of economic behaviour of people has been constructed.

3. The use of the formalism developed in the article for description of method joint administrative decisions making by concrete people is published in [11 – 14]. Theoretical results are confirmed.

REFERENCES

1. Mas-Collel A., Whinston M.D., Green J.R. Microeconomic Theory. – Oxford: Oxford University Press, 1995. – 977 p.
2. Нуреев Р.М. Общественный выбор: теория и практика. – М.: Изд-во ГУ ВШЭ, 2005. – 532 с.
3. Мулен Э. Кооперативное принятие решений: Аксиомы и модели. – М.: Мир, 1991. – 464 с.
4. Шиян А.А. Економічна кібернетика: вступ до моделювання соціальних і економічних систем. – Львів: «Магнолія 2006», 2007. – 228 с.
5. Шиян А.А. Информационное пространство и классификация стратегий управленческой деятельности в теории игр и принятия решений // Інформаційні технології та комп'ютерна інженерія. – 2007. – № 3 (10). – С. 131 – 139.
6. Ван дер Варден Б.Л. Алгебра. – М.: Наука,1976. – 648 с.
7. Маслов В.П. Операторные методы. – М.: Наука,1973. – 544 с.
8. Рохлин В.А., Фукс Д.Б. Начальный курс топологии. Геометрические главы. – М.: Наука, 1977. – 488 с.
9. Miller G.A. The Magic Number Seven Plus or Minus Two: Some Limits on Our Capacity for Processing Information // Psychology Review, 1956. – March. – P. 81 – 97.
10. Словарь по кибернетике / под ред. В.С. Михалевича. – Киев: Гл. ред. УСЭ им. М.П. Бажана, 1989. – С. 751.
11. Шиян А.А. О роли коммуникантов в обеспечении психологического комфорта: от стресса к суициду // Прикладная психология (Москва). – 2000. – № 4. – С. 67 – 79.
12. Курносов Ю.В., Конотопов П.Ю. Аналитика: методология, технология и организация информационно-аналитической работы. – М.: РУСАКИ, 2004. – 512 с.
13. Мингазов Р., Киямов И. Президентский характер: личность М. Ш. Шаймиева в ракурсе современных социальных технологий // Татарстан. – 2003. – № 1. – С. 4 – 9.
14. Шиян А.А. Соціально-психологічні портрети політиків: О.О. Мороз, Н.М. Вітренко та В.П. Горбулін // Нова Політика. – 1998. – № 4. – С. 24 – 28.

Shiyan A. A. – associate professor of the Department of the Design of medical-biological equipment, LMaximus@yandex.ru,http://soctech.narod.ru..

Vinnytcia National Technical University