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SEARCH OF THE MINIMUM OF THE FUNCTIONS WHICH HAVE GAPS OF PARTIAL DERIVATIVES

Possible solutions of search of minimum of functions which have partial derivative gap were analyzed. First order method of optimization for solution of the given problem was developed and proved.

Keywords: *optimization methods, function extremum , minimum of function, gradient, steepest descent, Armijo rule, conjugate gradients, gap of derivative*

Actuality of the problem

The modern methods of optimization are able to solve large amount of problems which can be reduced to the search of extremum of certain objective function. Among basic tasks it is possible to distinguish the following: direct search of a minimum (maximum) of certain objective function of the criterion, solution of the systems of linear equations with the large number of unknown values, solution of approximation problems.

While solving of more complex mathematical problems there is a necessity to increase optimization methods application on the different kinds of objective functions searching of extremum of which is impossible to perform using the existing methods. The essence of numeral methods of optimization is in gradual approaching to extremum of objective function with every following step (iteration) starting from the certain initial approximation (or interval of search).

There exists a lot of different methods of optimization and their modifications. All these methods can be divided into three groups:

methods of zero order (dichotomy, gold section method, Fibonacci method, and others);

methods of the first order (gradient method, steepest descent, conjugate gradients);

methods of the second order (Newton method).

According with the order of method the values of objective function, first partial derivatives (gradient of function) and second partial derivatives (Hessian functions) are used. Modifications of these methods allow to decrease the amount of iterations (or to reduce them to the finite amount for certain types of objective functions) for finding of objective function extremum.

The aim of this research is extension of optimization methods application for searching of a minimum of functions which have gaps of partial derivatives.

The task of research is to develop a first order optimization method which will allow optimization of the functions which have gaps of partial derivatives which pass through the point of minimum.

Let us try to make an attempt to apply the known methods for the given problem solution . The methods of a zero order are not used in practice for solving of multidimensional tasks because of little coefficient of convergence in comparison with the methods of higher orders.

Among the methods of the first order we should pay attention the steepest descent method. This method consists in the sequence of calculation of antigradient of function at a certain step and linear search in this direction. Repetition of iteration is performed until achievement of certain preset accuracy [1, 2]. If a function has a curvilinear (or straight) break line of first partial derivative which passes through a local minimum, this method is reduced to the first crossing with the break line.

As the result of research it was revealed that it is possible to increase the efficiency of this method using fuzzy linear descent - Armijo rule [3]. Armijo rule and principle of its operation are presented in Figure 1. It can be explained that this rule allows quickly but not exactly to calculate minimum at the stage of line search. It provides a possibility to define more exactly the gradient before achieving break line . But this approach is also not effective enough for functions with large number of arguments: it often has the same drawbacks as the approach using exact line search .

Also this method is rather slow and needs a great number of iterations especially for solution of multidimensional problems.

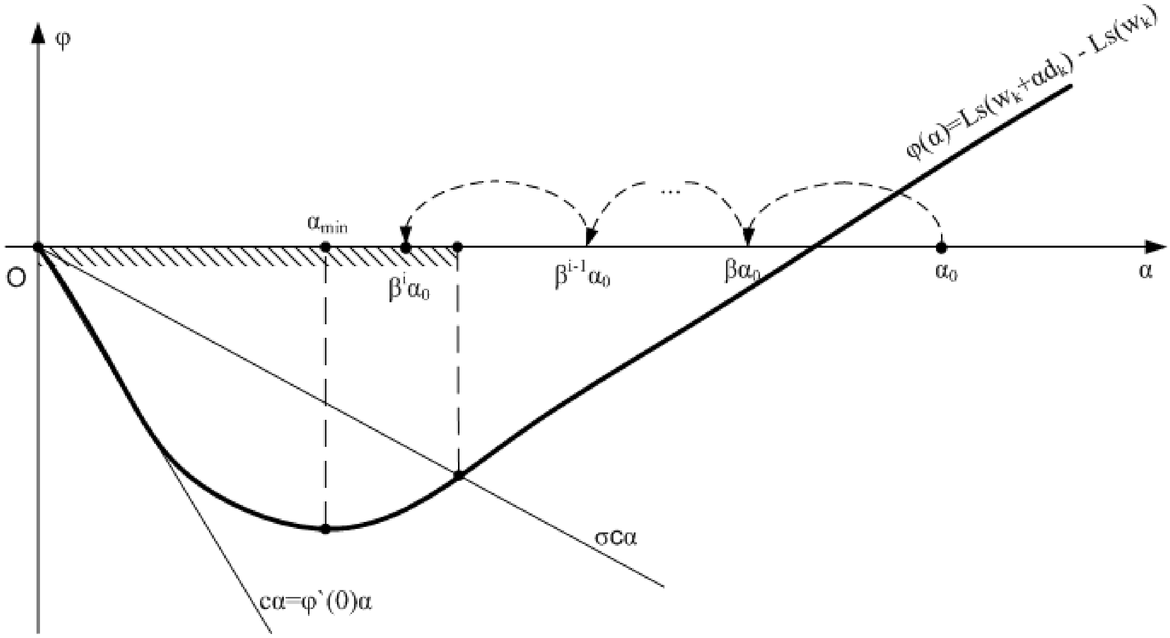

Fig. 1 Armijo rule

Another method of the first order which found wide application is conjugate gradients method. One of its advantages is that it has small and finite iterations count for searching of quadratic functions minimum with positive-definite matrix of linear transformation. It has two modifications according to the algorithms of conjugate direction calculation: Fletcher-Reeves method and Polak-Ribière method [1, 4]. Using this method to solve the problem with gap of first partial derivative it turned out that the algorithm begins to diverge after achievement of break line because of incorrect computation of conjugate direction on it.

Among the methods of the second order the Newton method has additional limitations regarding the type of function and requires the calculation of Hessian matrix that is not always possible and reasonable from the point of algorithm speed [1].

Mathematical model

Proceeding from analysis and research of existing methods, their application area does not cover functions which have gaps of first partial derivative.

The research of existing methods showed such basic problems as divergence and stop of algorithms take place on the border of partial derivative break. Hence if a point of minimum is on this border then after its reaching further motion is to be performed along the border. Direction of such motion can be set as interpolation of several points which are located on the border. The calculation of these points can be done by the steepest descent method.

In terms of the developed method it is suggested to choose three initial approximations and in further iterations to narrow the obtained triangle in the direction of a function minimum. The choice of points is performed near the minimum of function so that the gradient lines of any two points were superimposed.

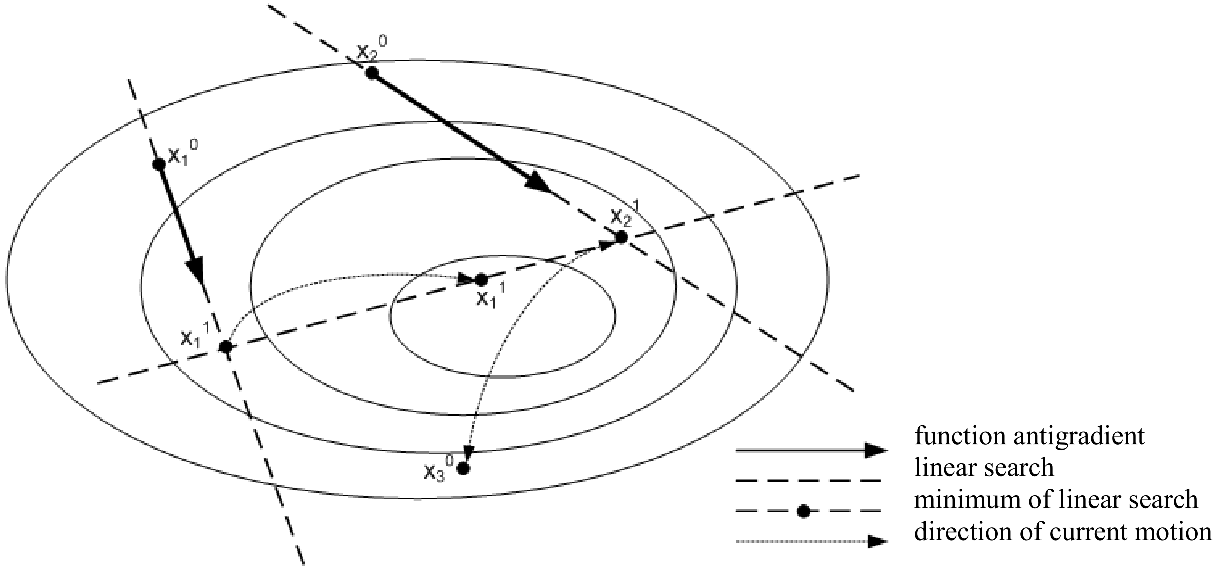

Fig. 2 First step of the method

After the selection of three approximations two points - x_1 and x_2 must be selected among them and linear search must be performed in direction of function antigradients in these points:

$$d_i^k = -\nabla f(x_i^k), \tag{1}$$

where f – function minimum of which is to be found; d_i^k – linear search direction; x_i^k – κ approximation of i starting point.

Linear search can be performed using Newton method for the function of one variable (2).

$$\alpha_i^k = \arg\min_{\alpha}(f(x_i^k + \alpha \cdot d_i^k)). \tag{2}$$

The Newton method for one dimensional problem is based on approximation of function by parabola in the point of approach and moving the argument in point of parabola minimum. Approximation is performed by decomposition of function in Taylor's series the coefficients of which are first and second derivatives of function in the initial point. Function derivatives in the chosen direction are calculated numerically, using difference formulas. Because the Newton method is used for finding of a function minimum with continuous partial derivatives it is need to set the proper terms for its application while solving the task [1]. Such a condition is that the difference step of method must not exceed the method accuracy. This condition provides by the correct difference calculation of first and second derivatives in points located near the break line of function derivative.

New approximation is obtained using the formula (3).

$$x_i^{k+1} = x_i^k + \alpha_i^k \cdot d_i^k. \tag{3}$$

After finding of the following approaches for the chosen points linear search is performed in the direction of d^k, along the line ($x_1^k\ x_2^k$) with the initial approach of x^κ which corresponds smaller of two values of the function:

$$
\begin{aligned}
d^k &= x_2^k - x_1^k, \\
x^k &= \arg\min_x (f(x_2^k), f(x_1^k)).
\end{aligned}
\tag{4}
$$

The minimum obtained as a result of linear search replaces approach of x_1^k or x_2^k depending on the value of function and another approach exchanges with the remained - x_3. The first iteration process is presented in Figure 2. The developed method can also be applied for solution of problems dealing with continuous multidimensional functions minimum search . It requires small amount of iterations and has the same finite convergence for quadratic functions as conjugate gradients method. The general algorithm of the method is presented in Figure 3.

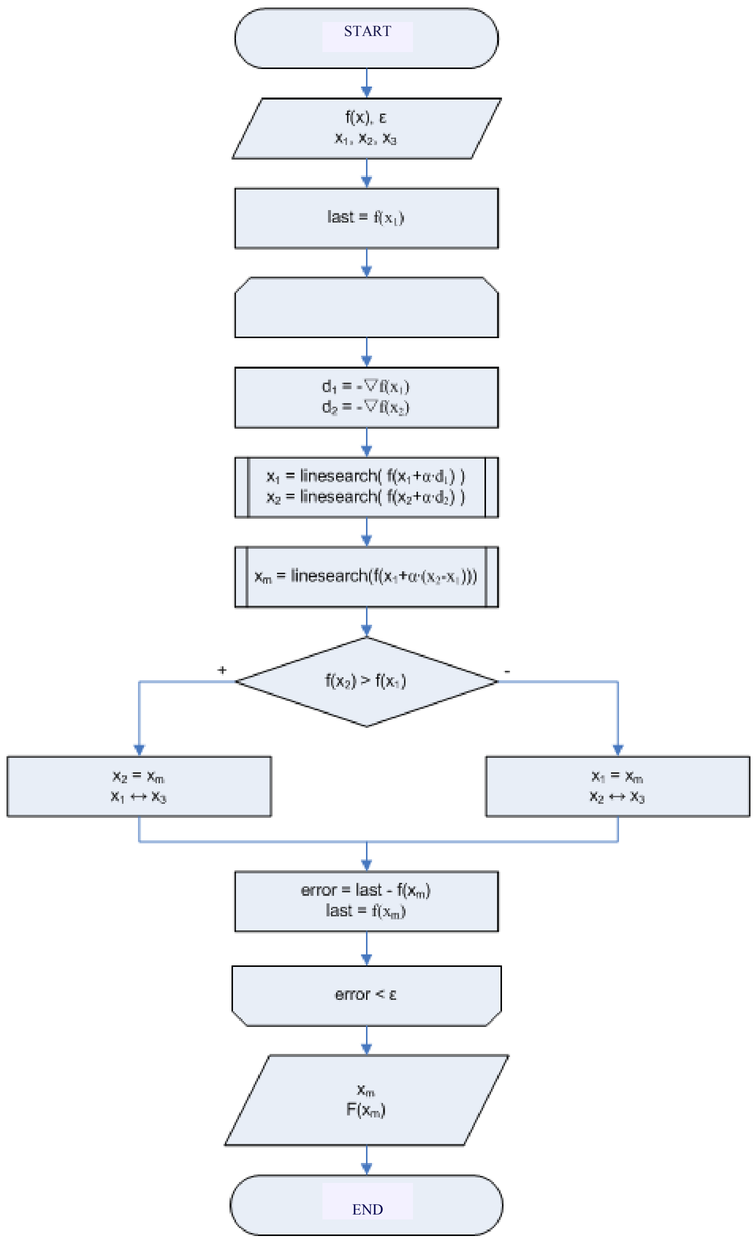

Fig. 3 The algorithm of the method

Conclusions

As a result of research optimization method of the first order which allows to find a minimum of functions which have gaps of first partial derivative was developed. Also on the basis of the method the algorithm and corresponding software was elaborated as a set of MATLAB package functions. The developed algorithm is general and can be applied for the search of minimum functions with continuous partial derivatives. This method can be applied in the problems of multidimensional interpolation and approximation the solution of which is built on the least square criterion.

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