A. P. Motailo, Cand. Sc. (Eng.)

BASES OF BIPYRAMID

Research is devoted to the study of functions interpolation, applying the method of finite elements in the area of quadrilateral b bipyramid. Main task of the study is to obtain general formula of finite-element basis of bipyramid and analysis of approximation properties of basis functions on this polyhedron.

In the given study bipyramid is considered as a finite element, obtained from the regular octahedron as a result of linear deformations of its semi-axes. Depending on the number of directions of deformation bipyramid may have one/two/three moving nodes. Such method of bipyramid formation is the result of adaptation of gratings of tetrahedrally-octahedral structure to the boundaries of areas, which have complex geometric form.

The paper contains general formulas for the bases of bipyramid with six and seven interpolation nodes, which are parametric functions of three variables. Values of parameters correspond to the elongation compression factors of the semi-axes of bipyramid and weight coefficient for six-node basis functions.

For the functions of bipyramid bases with seven and six interpolation nodes geometric properties of functions which are associated with the vertices of a polyhedron, are studied. The given functions are presented by the surfaces of zero level. Dependence of the type of surface on the value of parameters of bases functions of bipyramid is determined.

Approximation properties of seven node base of bipyramid functions are investigated in the paper. Quality criterion of approximation minimal trace of the stiffness matrix on a polyhedron is chosen. Critical values of elongation /compression parameters of bipyramid semi-axes which deliver minimum functionality to the trace of the stiffness matrix of bipyramid with seven nodes of interpolation are found. Additionally interval assessments for the parameters of seven node base function of bipyramid are obtained, they determine the boundaries of finite element deviation in the form of bipyramid from regular octahedron.

Perspective of further studies is mathematical substantiation of bipyramids application in the method of finite elements, determination of quality indices of bipyramid, used in the systems of finite-element analysis for asymmetric 3D-elements, obtaining of cubature formulas.

Key words: *finite element, regular octahedron, bipyramid, interpolation nodes, basis, basis functions, condensation method, stiffness matrix.*

Introduction

Method of finite elements (MFE) is widely used for the solution of engineering problems of mechanics, elasticity theory, field theory, etc. Modern programming complexes, realizing MFE enable to select finite elements (FE), set boundary conditions, change the order of solution accuracy, control computational time. In some cases it is impossible to obtain approximate solution, which would satisfy the set accuracy or computation time exceeds the admissible limits. Constant growth of the complexity of mathematic models of real states and processes needs the development of more efficient algorithms of numerical analysis and increase of the memory volume of computers.

One of the ways of optimization of finite-element calculation is correction of the discrete model, sometimes by selecting FE, which are not presented in the software libraries of MFE. In particular, in 3D-space gratings of tetrahedrally-octahedral structure are used, they are more efficient for the solution of field theory problems by time complexity as compared with tetrahedral gratings, it is proved by the publications [1, 2].

It is obvious, that the condition of discretization of 3D area of complex geometrical form by the grating, which contains only regular polyhedrons can not be performed. Some nodes have to be taken outside the boundary of the area, then octahedra are transformed into bipyramids. Thus, problem of studying the conditions of MFE functions approximation in the area of bipyramid remains relevant.

Analysis of the previous publications

In the study [3] quadrilateral bipyramid which is irregular polyhedron and can be obtained from octahedron (regular polyhedron) by means of linear deformation of one of its semi-axes. Node of bipyramid, located at random distance from the point of intersection of its diagonals, in this study is called mobile. Authors [3] constructed the systems of basis functions of quadrangular bipyramid with one moving node. Application of different approaches to the construction of the basis of bipyramid with seven and six interpolation nodes shows the advantages of geometric and method of the internal condensation, which, as compared with algebraic methods, have less computational complexity. Geometrical properties of basis function, which corresponds to the center of polyhedron is studied but for the functions, associated with the vertices of bipyramid, geometrical interpretation and its analysis are not available. In the given study approximation qualities of six-nodal model of bipyramid are investigated, the best basis, for which trace of the stiffness matrix is minimal, is determined. Theoretical conclusions, regarding the efficient use of bipyramids with six node of interpolation in MFE is verified by the problem of thermal conductivity for the bar. Similar research for the function of seven nodal model of the bipyramid in the given study is not available.

In studies [4, 5] bases of quadrangular bipyramid with two and three moving nodes for the polyhedron model with seven and six interpolation nodes is constructed. Obtained functions are polynomials of the second order, which depend on elongation/compression parameters of the semi-axes of the bipyramid. Basis functions of six node model of bipyramid contain additional parameter in the form of weight coefficient. In the research [4] the study of geometric properties of finite-element functions of bipyramid with two moving nodes is limited by central function of seven-node model of polyhedron. In the work [5] geometrical properties of bipyramid basis functions with three moving nodes are not investigated. The problem of approximation quality of functions of finite- element basis is considered in [4, 5] only for six-node model of bipyramid. Characteristic features of approximation on FE in the form of bipyramid with seven interpolation nodes are not considered.

Objective of this research is generalization of the information concerning the bases of quadrangular bipyramid, acquisition of information regarding geometric and approximation properties of bipyramid basis and determination of the conditions of bipyramids usage with one/two/three moving nodes in MFE algorithm.

For achieving the set goal the following problems are to be solved:

– obtain general formulas for the functions of seven and six-node bases of the bipyramid, their usage enables to reduce time complexity of MFE algorithm;

– study geometrical properties of the functions of finite-element bases of bipyramid, associated with the vertices of polyhedron and method of visualization of the surfaces of the basis functions level, determine their types, depending on the number of bipyramid interpolation nodes and values of the available parameters;

– study approximation properties of the functions of seven-node bases of the bipyramid, considering minimal trace of rigidity matrix on this polyhedron to be approximation quality criterion and check interval assessments for the elongation/compression parameters of bipyramid semi-axes, satisfying the conditions of using asymmetric 3D elements in MFE.

Bases of bipyramid with seven interpolation nodes

Study [5] considers quadrangular bipyramid, interpolation nodes of which are located in the vertices of the polyhedron and intersection point of the diagonals (Fig. 1a). It is assumed that points K_3, K_4, K_6 are located at the distance a from K_0, and sections K_0K_1, K_0K_2, K_0K_5 may have random length:

$$K_0K_1 = r \cdot a = t, K_0K_2 = p \cdot a = b, K_0K_5 = q \cdot a = c, \tag{1}$$

where $r, p, q > 0$ and $r, p, q \in R$.

Condition (1) means that nodes 1, 2, 5 of bipyramid (Fig. 1b), which correspond to vertices K_1,

K_2, K_5, are moving, i.e., they may be moved along semi-axes of the bipyramid.

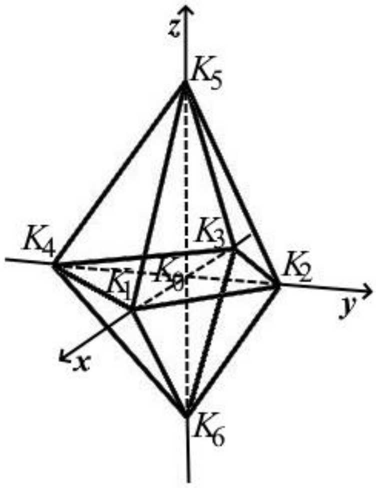


Fig. 1a. Bipyramid as a finite element

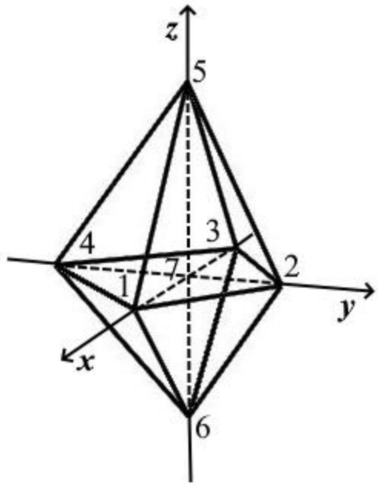


Fig. 1b. Model of bipyramid with seven interpolation nodes

Functions of finite-element basis of bipyramid with three moving nodes [5], associated with the vertices of bipyramid are expressed by the formula:

$$NS_1=\frac{1}{t(a+t)}x(x+a);\ NS_3=\frac{1}{a(a+t)}x(x-t);$$
$$NS_2=\frac{1}{b(a+b)}y(y+a);\ NS_4=\frac{1}{a(a+b)}y(y-b); \tag{2}$$
$$NS_5=\frac{1}{c(a+c)}z(z+a);\ NS_6=\frac{1}{a(a+c)}z(z-c).$$

Function, corresponding to the central node can be found from the equality, which is one of the conditions of the completeness of finite-element basis [6]:

$$NS_0=1-\frac{1}{pqra^2}\Big(pq\big(x^2+a(1-r)x\big)+rq\big(y^2+a(1-p)y\big)+rp\big(z^2+a(1-q)z\big)\Big). \tag{3}$$

Equalities (2), (3) are basis functions of bipyramid with two moving nodes [4], when r=1, and functions of bipyramid with one moving node [3], when r=1, p=1.

It should be noted that the basis functions of bipyramid, determined by the formulas (2), (3), are quadratic functions of three variables x, y, z with the parameters a, r, p, q. Functions NS_i (i=0..6) belong to the class C^2-smooth functions irrespective of the admissible parameters value, that provides performing of the necessary conditions of MFE coincidence [6] in the area of bipyramid.

Basis functions of pyramid, constructed in the works [3 – 5] allow for geometric interpretation. Surfaces of function NS_i (i=1..6) level are pairs of parallel planes, passing across all the nodes of bipyramid, except homonymous node i. Surface of the level of the given functions at fixed values of parameters r, p, q and a=1 in the local coordinates system, the beginning of which is in the point K_0 is shown in Fig. 2.

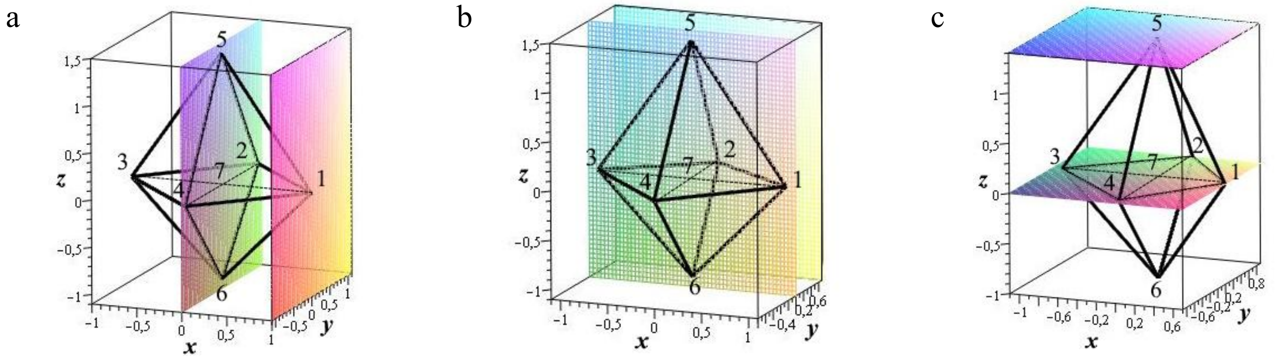


Fig. 2. Surfaces of the level of the function $NS_i(x,y,z,a,r,p,q)=0$: a) $i=3$; $r=p=1$; $q=1{,}4$; b) $i=4$; $r=1$; $p=0{,}7$; $q=1{,}2$; c) $i=6$; $r=0{,}9$; $p=0{,}7$; $q=1{,}4$

Functions, corresponding to the centre of bipyramid K_0, are also quadratic from three variables x, y, z with the parameters a, r, p, q. Surfaces of the level are ellipsoids with shifted (along axis Oz/y in the plane Oyz/y space $Oxyz$ centers, geometrical location of which depends on the number of moving nodes of bipyramid (one/two/three). The surfaces of the level of NS_0 function of bipyramid with one, two and three moving nodes at fixed values of elongation/compression parameters of polyhedron semi-axes for $a=1$ are shown in Fig. 3. All the constructions are executed in the local coordinates system with the start in the point K_0.

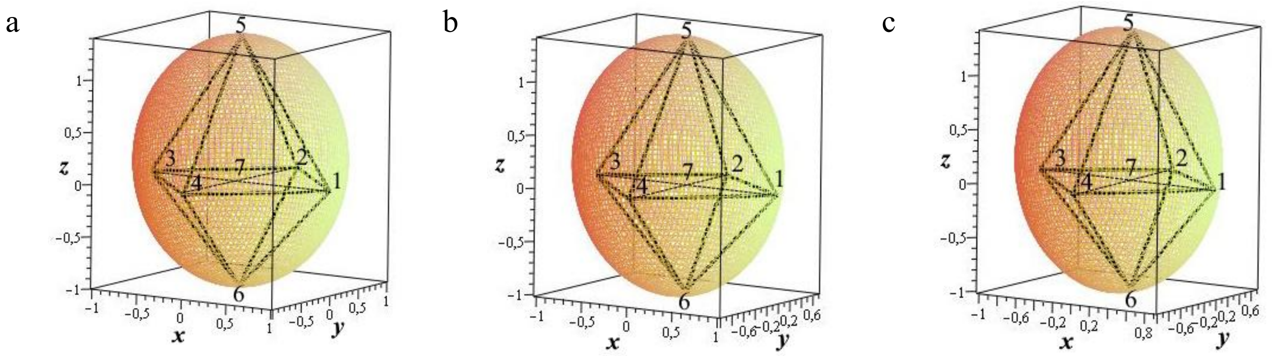


Fig. 3. Surfaces of the function level $NS_0(x,y,z,a,r,p,q)=0$: a) $r=p=1$; $q=1{,}4$; b) $r=1$; $p=0{,}7$; $q=1{,}4$; c) $r=0{,}9$; $p=0{,}7$; $q=1{,}4$

Analyzing geometrical representation of the functions of finite-element basis of the bipyramid with seven interpolation nodes it should be noted that the type of the surfaces of second order, which correspond to node i do not depend on elongation/compression parameters of semi-axes of the pyramid. Thus, geometrical properties of functions NS_i ($i=0..6$) are determined by the order of polynomials (2), (3) and correspondence to the node i of the bipyramid.

Basis of the bipyramid with six interpolation nodes

In the study [5] functions of six-node basis of quadrangular bipyramid with three moving nodes are obtained. At the same time, the numbering order of nodes, associated with the vertices of bipyramid remains (Fig. 1b). Constructed basis functions are expressed by the formulas:

$$\begin{gathered} NC_1 = NS_1 + \alpha_1 \cdot NS_0,\ NC_3 = NS_3 + \alpha_2 \cdot NS_0, \\ NC_2 = NS_2 + \beta_1 \cdot NS_0,\ NC_4 = NS_2 + \beta_2 \cdot NS_0, \\ NC_5 = NS_5 + \gamma_1 \cdot NS_0,\ NC_6 = NS_6 + \gamma_2 \cdot NS_0, \end{gathered} \tag{4}$$

where $\alpha_i, \beta_i, \gamma_i (i=1,2)$ – are weight coefficients, which satisfy the condition $\alpha_1 + \alpha_2 + \beta_1 + \beta_2 + \gamma_1 + \gamma_2 = 1$.

Number of weight coefficients in the formulas (4) can be reduced to one, taking into consideration their linear dependence and satisfying the conditions of completeness [6] of finite-element basis:

$$\sum_{i=1}^{6} x_i NC_i = x, \sum_{i=1}^{6} y_i NC_i = y, \sum_{i=1}^{6} z_i NC_i = z, \sum_{i=1}^{6} NC_i = 1,$$

where (x_i, y_i, z_i) – are the coordinates of the bipyramid vertices in the local coordinates system (Fig. 1a).

Actually, weight coefficients can be presented by the functions α_1 and elongation/compression parameters r, p, q of the bipyramid:

$$\alpha_2 = r\alpha_1; \beta_1 = \frac{1-\alpha_1(r+1)}{p+1}; \beta_2 = p\beta_1; \gamma_1 = \frac{1-\alpha_1(r+1)}{q+1}; \gamma_2 = q\gamma_1. \quad (5)$$

Then, if $0 \le \alpha_1 \le 1$, limitations for other weight coefficients have the form:

$$0 \le \alpha_2 \le r; 0 \le \beta_1 \le \frac{1}{p+1}; 0 \le \beta_2 \le \frac{p}{p+1}; 0 \le \gamma_1 \le \frac{1}{q+1}; 0 \le \gamma_2 \le \frac{q}{q+1}.$$

Thus, basis of quadrangular bipyramid with three moving nodes, located in points K_1, K_2, K_5, is determined by the formulas (4), (5). Concurrently, basis functions of bipyramid with two moving nodes K_2, K_5 [4] can be obtained from the equalities (4), (5) if r=1 and $\alpha_1 = \alpha_2$. Basis functions of the bipyramid with one moving node K_5 correspond to the values of r=p=1 parameters and weight coefficients $\alpha_1 = \alpha_2 = \beta_1 = \beta_2$.

Obtained functions of finite-element basis of bipyramid with six interpolation nodes, which satisfy formulas (4), (5), are quadratic functions of three variables x, y, z with the parameters a, r, p, q, α_1. Functions NC_i (i=1..6) belong to the class C^2-smooth in the area of bipyramid functions irrespective of the parameters a, r, p, q and weight coefficient α_1, this guarantees the fulfillment of the conditions of one of MFE convergence criteria [6] in the area of this polyhedron.

Basis functions of quadrangular bipyramid with six interpolation nodes, constructed in the studies [3 – 5] admit geometric interpretation. Fig. 4 shows level surfaces for basis function NC_1 if r=0,9, p=0,7, q=1,4, a=1 in the local coordinates system.

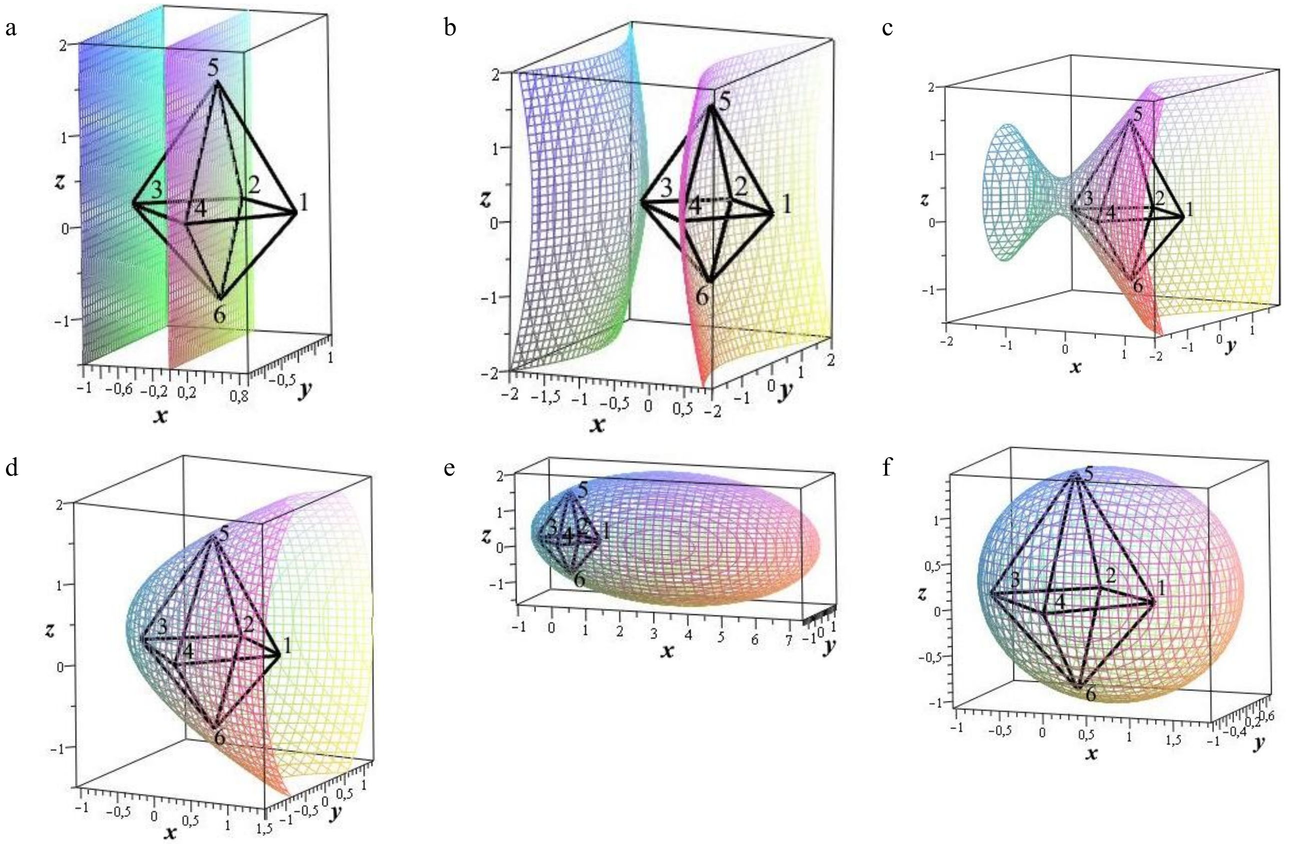


Fig. 4. Surfaces of the function $NC_1(x,y,z,a,r,p,q,\alpha_1)=0$ level: a) $\alpha_1=0$, pair of the parallel planes; b) $\alpha_1=0{,}1$, double-empty hyperboloid; c) $\alpha_1=0{,}3$, single-empty hyperboloid; d) $\alpha_1=0{,}5$, elliptical paraboloid; e) $\alpha_1=0{,}6$, ellipsoid; f) $\alpha_1=1$ ellipsoid

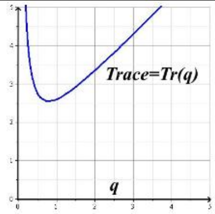
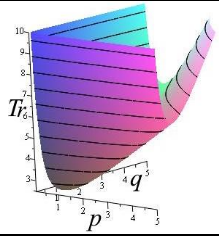
Analyzing geometrical properties of finite-element basis bipyramid with six interpolation nodes function, it should be noted that depending on the value of weight coefficient α_1 level surface, associated with node i, are different types of surfaces of the second order.

Bases of the bipyramid with minimal trace of the rigidity matrix

In the research [3 – 5] approximation properties of bipyramid bases with six interpolation nodes are investigated. It is assumed that the best one is the basis with minimum trace of the rigidity on CE [7]. According to this characteristics critical values of elongation/compression parameters of bipyramid semi-axes, which provide minimum function of the trace of the rigidity matrix $Trace=Tr(r,p,q)$ (Table 1) are found. All calculations are performed for $a=1$.

Table 1

Results of the investigation on minimum function of the trace of the rigidity matrix of the bipyramid with six interpolation nodes

Number of moving nodes of the bipyramid	Critical values of basis functions parameters, which satisfy the formulas (4), (5)			Minimum function of the rigidity matrix trace	Geometric presentation of the function of the rigidity matrix trace
	r	p	q		
one	1	1	0.7584	2.4776	
two	1	0.744	0.744	2.475	
three	0.64917	0.70588	0.70593	2.358	Surface of the level *Trace*=2.358 is point (r,p,q)=(0.64917; 0.70588; 0.70593)

For further analysis of the approximation quality on CE in the form of bipyramid it is necessary to find the values of r, p, q parameters, at which the trace of rigidity matrix is minimal.

By the formula:

$$Trace_7 = \sum_{i=0}^{6} \iiint_V \left(\left(\frac{\partial NS_i}{\partial x} \right)^2 + \left(\frac{\partial NS_i}{\partial y} \right)^2 + \left(\frac{\partial NS_i}{\partial z} \right)^2 \right) dv,$$

where V – is bipyramid, NS_i – are basis functions of bipyramid (1 – 3), – function $Trace_7=Tr_7(r,p,q)$ can be presented by the polynomial of one of parameters r, p, q. Then the development of the function of the trace of the rigidity matrix in powers of q has the form:

$$Trace_7 = \frac{a}{r^2 p^2 q^2 (1+r)(1+p)(1+q)} \sum_{i=0}^{4} A_i(r,p) q^i, \tag{6}$$

where

$$\begin{aligned}
A_0 &= 4(r^2p^4 + r^2p^2 + r^4p^4 + r^4p^2) + 8(r^2p^3 + r^4p^3 + r^3p^2 + r^3p^4) + 16r^3p^3, \\
A_1 &= 5(r^2p^4 + r^2p^2 + r^4p^4 + r^4p^2) + 10(r^2p^3 + r^4p^3 + r^3p^2 + r^3p^4) + 20r^3p^3, \\
A_2 &= 12(r^2p^4 + r^2p^2 + r^4p^4 + r^4p^2) + 21(r^2p^3 + r^4p^3 + r^3p^2 + r^3p^4) + 5(rp^4 + rp^2 + r^4p + r^2p) + \\
&\quad + 4(r^2 + p^2 + p^4 + r^4) + 10(r^3p + rp^3) + 36r^3p^3, \\
A_3 &= 36(r^3p^4 + r^3p^2 + r^4p^3 + r^4p^2) + 21(r^2p^4 + r^2p^2 + r^4p^4 + r^4p^2) + 10(rp^4 + rp^2 + r^4p + r^2p) + \\
&\quad + 8(r^2 + p^2 + p^4 + r^4) + 20(r^3p + rp^3) + 16(r^3 + p^3) + 60r^3p^3, \\
A_4 &= A_2 - 10(r^3p + rp^3) + 8(r^3 + p^3).
\end{aligned} \tag{7}$$

For exploring the function $Trace_7=Tr_7(r,p,q)$ for extremum it is necessary to find critical points of this function, satisfying the conditions:

$$\begin{cases} \partial Tr_7 / \partial r = 0, \\ \partial Tr_7 / \partial p = 0, \\ \partial Tr_7 / \partial q = 0. \end{cases} \qquad (8)$$

Unique solution of the equation system (8) is point $(r_0,p_0,q_0)\approx(0.78996;\ 0.78996;\ 0.78996)$, its coordinates are found by the numerical method with the accuracy of up to 10^{-4} for a=1 (Fig. 5).

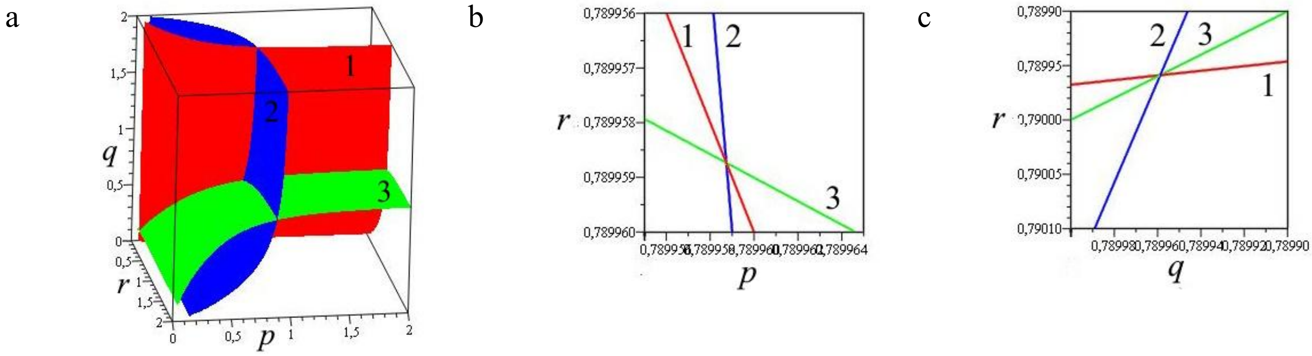


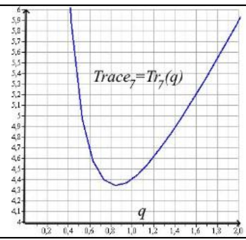
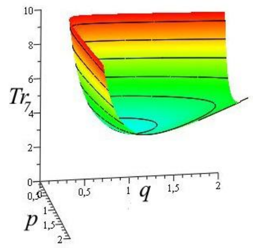
Fig. 5. Graphic presentation of system solution (8): a) in the space *Orpq*; b) in projection on *Orp*; c) in projection on *Oqr*, where 1 – is the surface $\partial Tr_7/\partial r=0$; 2 – is the surface $\partial Tr_7/\partial p=0$; 3 – is the surface $\partial Tr_7/\partial q=0$

All the major minors of partial derivatives of the second order of the function $Trace_7=Tr_7(r,p,q)$ are positive in the found point. Thus, according to Sylvester criterion [8] (r_0,p_0,q_0) is the point of minimum function of the trace of the bipyramid rigidity matrix. Corresponding minimum of the function of the trace of the rigidity matrix with the accuracy up to 10^{-4} equals 4.15007.

The obtained formulas (6), (7) enable to carry out similar study for the extremum of the function of the trace of the rigidity matrix of the bipyramid, which has two and one moving nodes. Corresponding functions $Trace_7=Tr_7(r,p,q)$ are considered separately at fixed values of elongation/compression parameters of semi-axes of bipyramid. Table 2 contains basic results of verification of the conditions of minimum functions $Tr_7(p,q)$ and $Tr_7(q)$ existence. All computations are performed with the accuracy of up to 10^{-4}.

Table 2

Results of research for minimization function of the trace of the rigidity matrix of the bipyramid with seven interpolation nodes

Number of moving nodes of the bipyramid	Critical values of basis functions parameters, which satisfy the formulas (4), (5)			Minimum function of the trace of the rigidity matrix (6), (7)	Geometric presentation of the function of the trace of the rigidity matrix
	r	*p*	*q*		
one	1	1	0.84990	4.33918	
two	1	0.82447	0.82447	4.2592	

Analyzing the results of the study for the extremum of the function of the trace of the rigidity matrix of bipyramid with six and seven interpolation nodes, it should be noted that regardless of the number of moving nodes of the bipyramid there exist admissible values of r, p, q, parameters at which values of the rigidity matrix trace are minimal. Hence, according to this characteristic, the best among seven-node are bases of the bipyramid, set by the formulas (2), (3) with the parameter values in Table 2 and in Fig. 5. Accordingly, the best among six-node are bases of the bipyramid, set by the formulas (4), (5) with the parameter values in Table 1.

Conditions of bipyramid application in the method of finite elements

Study of approximative features of the bipyramid is directed on the determination of the conditions of this polyhedron usage at algorithmization of MFE. As the bipyramid (Fig. 1a) is not a regular polyhedron, there exists the problem of determining the boundaries of its usage as the cell of finite-element grating. In the studies [3 – 5] on the base of the analysis of bipyramid volume deviation from the volume of octahedron the interval assessments for the elongation/compression parameters of semi-axes of bipyramid r, p, q are obtained. Table 3 contains the corresponding limitations for the bipyramid with one, two and three moving nodes.

Table 3

Interval assessments for the bipyramid parameters

Number of moving nodes of the bipyramid	Values of elongation/compression parameters of the bipyramid			Permissible deviation limits of the bipyramid volume from the volume of the octahedron	
	r	p	q	by 10%	by 25%
one	1	1	>0	$0.66 \le q \le 0.86$	$0.51 \le q \le 1.0$
two	1	>0	>0	$0.8 \le (p+1)q \le 1.2$	$0.5 \le (p+1)q \le 1.5$
three	>0	>0	>0	$3.6 \le rpq+r(p+1)+p(q+1)+q(r+1) \le 4.4$	$3 \le rpq+r(p+1)+p(q+1)+q(r+1) \le 5$

Analyzing corresponding assessments for the bipyramid parameters it should be noted that critical values of r, p, q (Table 1, 2, Fig. 5) meet the requirements of bipyramid volume deviation from the volume of regular polyhedron (octahedron) by 10 %, this provides high accuracy of the obtained MFE solution. But the data in Table 3 require clarification by calculating the quality indices for 3D elements, used in ANSYS [9, 10] and other systems of finite-element analysis [11].

Conclusions

1. In the research finite-element bases of the bipyramid are presented by two different systems of functions, which correspond to FE in the form of the bipyramid with seven and six interpolation nodes. Basis functions of each of two systems are quadratic, this provides C^2-smoothness of approximate functions in the area of the bipyramid. Functions of bipyramid basis contains undetermined parameters, enabling to generalize the expressions for basis functions with one/two/three moving nodes on this polygon.

2. Geometrical features of basis functions of the bipyramid with seven and six interpolation nodes are studied. The differences in representation by the surfaces the level of functions, associated with vertices of the bipyramid are revealed. Surfaces of the functions level of seven-node basis of the bipyramid are degenerated surfaces of the second order in the form of the pair of parallel planes. Surfaces of functions level of six-node basis of bipyramid are surfaces of the second order of different types, which depend on the value of the weight coefficient as the parameter of functions of finite-element basis.

3. Approximation properties of the system of basis functions of bipyramid with seven and six interpolation nodes are investigated. Among the functions of finite-element basis of bipyramid the best are found, critical values of elongation/compression parameters of semi-axes of bipyramid, correspond to these functions, that deliver minimum functions of the rigidity matrix trace. The

obtained values of parameters satisfy the interval estimates, which determine the limits of the bipyramid volume deviation from the volume of octahedron by 10 %, this enables to predict high accuracy of the solutions if bipyramids are used in MFE.

4. Prospects of further studies are:

– study on convergence of MFE on FE in the form of bipyramid in the assemble with tetrahedrons and octahedrons for mathematical substantiation of bipyramid application in the grates of tetrahedral-octahedral structure;

– determination of the quality indices of the bipyramid, which use the known systems of finite-element analysis for the elements in 3D in order to specify the interval estimates for the elongation/compression parameters of the bipyramid semi-axes, that provide the approximation quality on the bipyramid;

– construction of cubic formulas on the bipyramid, which has two, three moving nodes, this enables to use the bipyramid in finite-elements computations.

REFERENCES

1. Greiner G. Hierarchical Tetrahedral-Octahedral Subdivision for Volume Visualization / G. Greiner, R. Grosso // The Visual Computer. – 2000. – № 16 (6). – P. 357 – 369. DOI: https://doi.org/10.1007/PL00007214.

2. de Bruijn H. Numerical Method for 3D Ideal Flow [Electronic resource] / Access mode : http://hdebruijn.soo.dto.tudelft.nl/jaar2010/octaeder.pdf.

3. Motailo A. P. Construction of the basis of bipyramid / A. P. Motailo, A. N. Khomchenko, G. J. Tuluchenko // Radioelectronics, informatics, mahagement. – (2016). – № 4 (39). – P. 29 – 36. DOI: https://doi.org/10.15588/1607-3274-2016-4-4. (Ukr).

4. Motailo A. P. Construction of the basis of bipyramid with two movable nodes / A. P. Motailo // Systems and technologies. – 2023. – V. 65, №1. – P. 7 – 12. DOI: https://doi.org/10.32782/2521-6643-2023.1-65.1. (Ukr).

5. Motailo A. P. Basis of bipyramid with three movable nodes / A. P. Motailo // Towards a Holistic Understanding: Interdisciplinary Approaches to Tackle Global Challenges and Promotion of Innovative Solutions: Materials of Ist International Scientific Practical Internet Conference (c. Dnipro, March 14 − 15 2024). Dnipro, 2024. – P. 158 – 160. (Ukr).

6. Zienciewicz O. C. Introductory Lectures on the Finite Elements Method / O. C. Zienciewicz. – Springer, 2014. – 99 p. DOI: https://doi.org/10.1007/978-3-7091-2973-9.

7. Sekulović M. Metod konačnih elemenata [The Finite Element Method] / M. Sekulović. – Beograd : Gradevinska knjiga, 1984. – 649 p.

8. Gilbert George T. Positive definite matrices and Sylvester's criterion / George T. Gilbert // The American Mathematical Monthly. – 1991. – № 98 (1). – P. 44 – 46. DOI: https://doi.org/10.2307/2324036.

9. How to calculate Element Quality Criteria in Ansys Meshing. Skewness [Electronic resource]. – Access mode : https://www.youtube.com/watch?v=WgOMSYGMZfc.

10. Lecture7: Mesh Quality & Advanced Topics – FEA Tips [Electronic resource]. – Access mode: https://featips.com/wp-content/uploads/2021/05/Mesh-Intro_16.0_L07_Mesh_Quality_and_Advanced_Topics.pdf.

11. How Element Quality is Calculated – Altair [Electronic resource]. Access mode: https://2021.help.altair.com/2021/hwdesktop/hm/topics/pre_processing/meshing/element_quality_how_its_calculated_c.htm.

Editorial office received the paper 28.05.2024.
The paper was reviewed 10.06.2024.

Motailo Anzhelika – Cand. Sc. (Eng.), Associate Professor, Associate Professor with the Department of Natural-Science Training, e-mail: akillehzna@gmail.com.

Kherson State Maritime Academy.