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VECTOR DIAGRAM AND MATHEMATICAL MODELS OF WIND WHEEL WITH THE VERTICAL ROTATION AXLE

There had been constructed vector diagrams of both, the airstream speed which activate the blades of the wind wheel with the vertical rotation axle, as well as the forces, created by these streams. There had been suggested the mathematical models, which connects these speeds and forces during the rotation of the wind wheel.

Keywords: *wind electric station, wind motor with vertical rotation axis, wind speed, blades turn angle of attack aerodynamic force.*

Problem Statement

Paper [1] shows that the wind wheels with the vertical rotation axle may have the own very essential sphere of application in win farm construction. However today neither the classical aerodynamics [2], nor the publications, known to the authors, answer the questions of how to build the simple and efficient mathematical models which allow to analyze the forces which appear during the rotation of the wind wheel with the vertical rotation axle, which, in turn, does not allow to stabilize or optimize the operation of the wind farm with the wind motor of such type.

In the given paper the authors present the own answer to the above question, based on general principles of electrodynamics.

Task solution

As is known [2], angle of attack α_α of the wing is determined as the angle between the plane of the wing and the speed vector of wind flow, attacking that wing. That is quite clear that the angle cannot exceed 90^o, as being so, the wing looses the carrying capacity. The analogical situation may be observed when considering the blade of the wind wheel with the horizontal rotation axle, as is shown on fig.1, taken from [3].

On this figure: ω – angular velocity of wind wheel around the axle, V_B – the speed of the wind flow, R – radius of wing wheel, ωR – peripheral velocity of the blade, $w_в$ – speed pf the resulting wind flow, springing up the blade, $\varphi_л$ – degree of blade rotation relatively to plane of rotation of wind wheel, α_α – blade angle of attack by the resulting wind flow, $\boldsymbol{F_a}$ – power of aerodynamic pressure of the resulting wind flow on the blade, F_n – aerodynamic carrying capacity, which influences the blade, F_Σ – resulting aerodynamic power, which influences the blade, $F_л$ – blade wind ram power, F_m – tractive force, which creates the rotational moment.

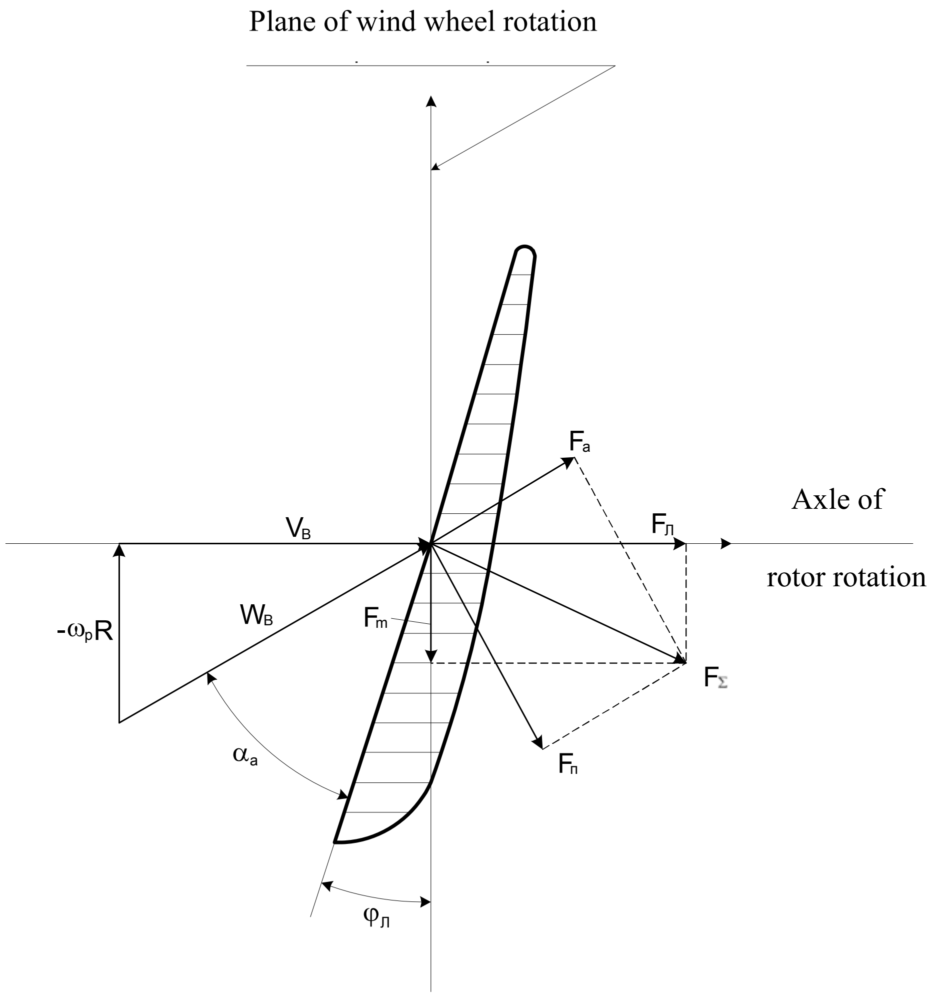
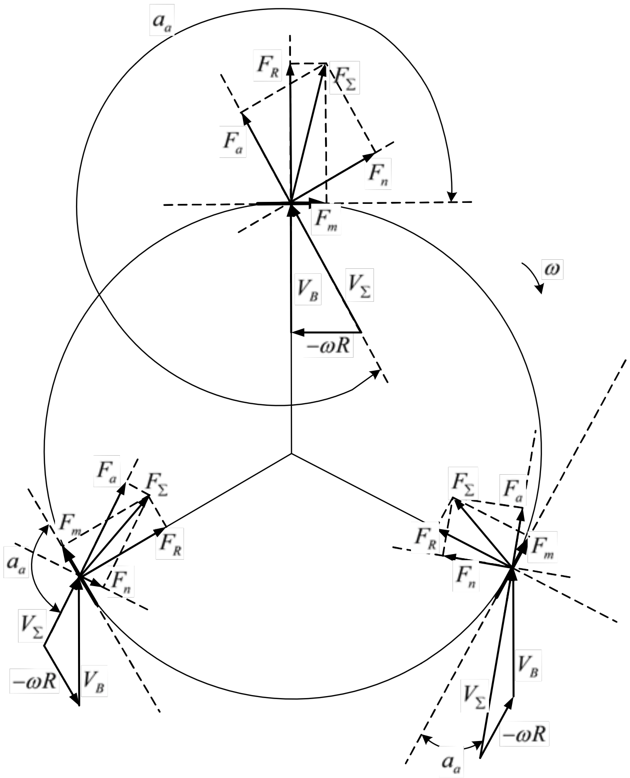


Fig 1. Vector diagram of powers and speeds for cross-section of the wind wheel blade with the horizontal rotation axle [3]

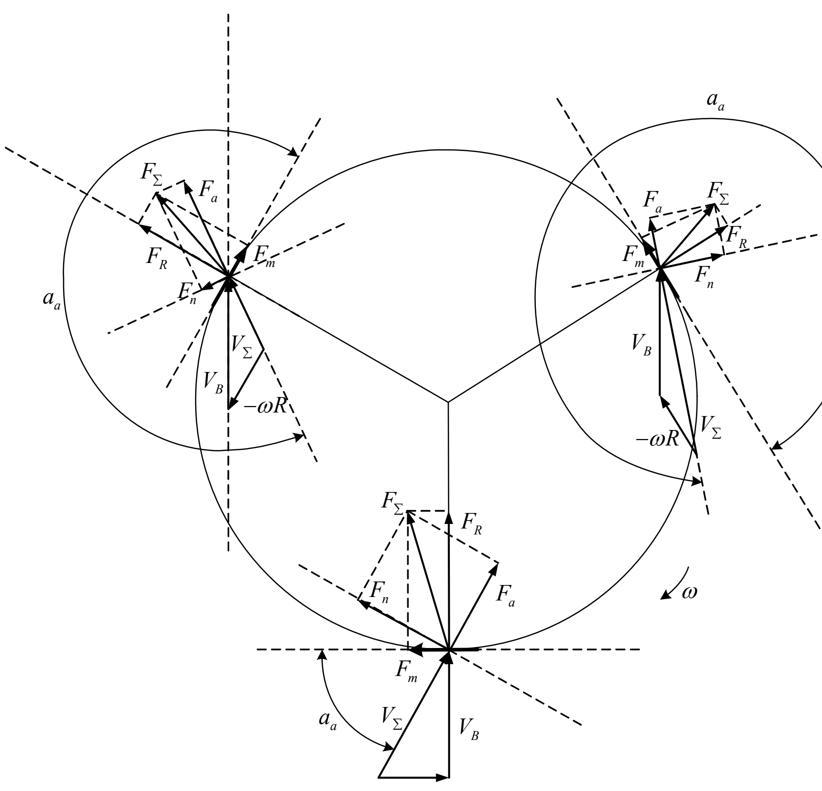
Fig.1 shows that even when the wind wheel stops and becomes $\omega=0$, the angle of attack will remain less 90^o, and with the increasing of rotation angular velocity of the wind wheel it will decrease, but will never be less then the angle of degree of rotation of the blade, which is bigger then zero.

Quite opposite is observed on the vector diagram of speeds and powers , built for the wind wheel with the vertical rotation axle and three blades, placed with the angle of $2\pi/3$, horizontal section of which is shown on fig. 2 for the two positions of wind wheel, shifted as for each other by 180^o.

On this figure only two values are marked by the symbols, different from those, used on fig. 1: first, power $F_л$, which, in case of horizontal rotation axle of wind wheel, tries to bent the pole, to the top of which this axle is connected, and in case of vertical rotation axle of the wind wheel, if tries to bend the axle itself, marked by the symbol F_R, to underline its directioning along the radius of the wind wheel; and second, the speed $w_в$ of the resulting wind flow, springing up the blade is marked by more adequate symbol V_Σ. All the other symbols and their definitions on fig. 2 coincide with the analogical symbols and their definitions, shown on fig. 1.



a)



б)

Fig. 2 Vector diagram of the wind flow speeds, affecting the blades of the wind wheel with the vertical rotation axle, and powers, created by these flows when φл = 0.

The diagram, shown on fig . 2 .allows to see, that the angle of attack α_α of the resulting wind flow on the blade of the wind wheel with the vertical rotation axle, gets the additional growth by 360°. And this enables us in time intervals, within which the speed vector of the wind does not change its direction, to apply the analogy with three – phase AC generator; for construction of mathematical model for all the forces, applied to wind wheel with three blades shifted by the angle $2\pi/3$ relatively each other, for phase electromotive forces e_A, e_B, e_C for which the known model [4] is valid.

$$\begin{aligned} e_A &= E_m \sin \omega t, \\ e_B &= E_m \sin\left(\omega t - 2\pi/3\right), \\ e_C &= E_m \sin\left(\omega t - 4\pi/3\right). \end{aligned} \tag{1}$$

According to theoretical principles of electrotechnology, the circular rotation of rotor coils of three phases in the directed magnetostaticfield, created by the flux of poles, causes the electromotive force in each of these coils, vector of which is behind or leads the vectors of electromotive force of other two phases by angel of $2\pi/3$, and changes by the law of sine, as is shown in the expression (1), where E_m – amplitude value of phase electromotive force, and ω – angular frequency of electromagnetic field, the value of which with the one pair of poles in the generator coincides with the value of angular velocity of rotor rotation.

The case when the wind affects all the three blades of the wind wheel, blowing from one direction, is just the analogue to one pair of poles of electric generator, and the influence of the vector of peripheral velocity of the blade ωR on the vector of total velocity V_Σ is the analogue of the influence of generator rotor reaction. Therefore we have perfect analogy, and so for the force F_Σ, which is created by the aerodynamic pressure of the total air follow, we can write the mathematical model in the king of (1), that is:

$$\begin{aligned} F_{\Sigma 1} &= F_{\max} \sin\left(\omega t + \alpha_{an}\right), \\ F_{\Sigma 2} &= F_{\max} \sin\left(\omega t + \alpha_{an} - 2\pi/3\right), \\ F_{\Sigma 3} &= F_{\max} \sin\left(\omega t + \alpha_{an} - 4\pi/3\right) \end{aligned} \tag{2}$$

where α_{an} – value of the angle of attack of the first blade at the moment of time reckoning; $F_{\Sigma 1}, F_{\Sigma 2}, F_{\Sigma 3}$ – total sum of the aerodynamic pressure accordingly on the first, second and the third blade, and F_{max} – its amplitude value, which may be found, for instance, from the first equation of the system (2) under condition that

$$\omega t + \alpha_{an} = \pi/2 \tag{3}$$

The vector diagram on fig. 2 clearly shows that for each blade in the speed system of coordinates -

$$F_\Sigma^2 = F_a^2 + F_n^2 \tag{4}$$

and in connected system of coordinates -

$$F_\Sigma^2 = F_R^2 + F_m^2 \tag{5}$$

Force F_m, which creates the total moment, and force F_R, which tries to bend the axle of the wind wheel and to destroy the journal bearing, are the most important of all the forces for designers and operatives. Relying upon the theory, presented in work [2] and on figure 2, it is possible to state that the value of the force F_R on the blade 1 may be found from the following correlation:

$$F_{R1} = S_л (\Pi p_R V_\Sigma) \rho (\Pi p_R V_\Sigma) k_F^R = k_F^R S_л \rho V_в^2 \tag{6}$$

where $S_л$ – sectional area of the blade (m^2), $\Pi p V_\Sigma$ – projection of speed vector of the resulting wind flow on the axle, which is the continuation of the wind wheel radius (m/s), ρ – specific density of air (kg/m^3), $V_в$ – wind speed (m/s), k_F^R – coefficient

k_F^R – coefficient, less then unit, which characterizes the difference of the «corridor» of the air stream movement, directed to the blade, from the pipe with the rectangular cross-section, equal to the cross-section of this blade.

And on the basis of the mentioned above assumptions concerning force F_Σ, for force F_{R1} it is possible to write that

$$F_{R1} = F^R_{\max} \sin(\omega t + \varphi_n) \tag{7}$$

where φ_n – the initial angle of turn of vector force F_{R1} in the moment of time $t=0$, coordinated with the value of the initial angle of attack α_{an}, and F^R_{ma} – amplitude value of this force, which may be determined from the equation (7) under condition that

$$\omega t + \varphi_n = \pi / 2 \tag{8}$$

By analogy with the equation (2) for the system of forces F_{R1}, F_{R2}, F_{R3} it is possible to write that

$$\begin{aligned} F_{R1} &= F^R_{\max} \sin(\omega t + \varphi_n), \\ F_{R2} &= F^R_{\max} \sin(\omega t + \varphi_n - 2\pi/3), \\ F_{R3} &= F^R_{\max} \sin(\omega t + \varphi_n - 4\pi/3) \end{aligned} \tag{9}$$

Relying upon the theory, presented in work [2], and on fig.2, it is possible to state that the value of the force F_a on the blade 1 may be found from the following correlation

$$F_{a1} = S_л \cos\angle(F_m, F_n) V_\Sigma \rho V_\Sigma k^a_F = k^a_F S_л \rho \frac{V_в^2}{\cos\angle(F_m, F_n)}, \tag{10}$$

where $- k_F^a$ coefficient, less then unit, which characterizes the difference of the «corridor» of the air stream movement, directed to the blade, from the pipe with the rectangular cross-section, equal to the cross-section of this blade, $a <(F_m, F_n)$ – angle between the corresponding axis of speed and connected system of coordinates.

By analogy with the equation (2) for the system of forces F_{a1}, F_{a2}, F_{a3} it is possible to write that

$$\begin{aligned} F_{a1} &= F^a_{\max} \sin(\omega t + \psi_n), \\ F_{a2} &= F^a_{\max} \sin(\omega t + \psi_n - 2\pi/3), \\ F_{a3} &= F^a_{\max} \sin(\omega t + \psi_n - 4\pi/3), \end{aligned} \tag{11}$$

where ψ_n – the initial angle of turn of vector force F_{a1} in the moment of time $t=0$, coordinated with the value of the initial angle of attack α_{an}, a F^a_{max} – amplitude value of this force, which may be determined from the first equation of the system (11) under condition t

$$\omega t + \psi_n = \pi / 2 \tag{12}$$

We have created the method for finding the forces F_a, F_R, but, as it already had been mentioned above, we have to know the forces F_m, F_R. Let us now switch over to creation of method for finding the tractive force F_m, which creates rotation moment of the wind wheel.

The vector diagram shown on fig. 2, demonstrates, that the following system of two equations is true for the blade 1:

$$\begin{aligned} F_{m1} &= F_{n1} \cos\angle(F_m, F_n) - F_{a1} \sin\angle(F_m, F_n), \\ F_{R1} &= F_{n1} \sin\angle(F_m, F_n) + F_{a1} \cos\angle(F_m, F_n) \end{aligned} \tag{13}$$

with two unknown F_{m1}, F_{n1}, solving which, we easily find them.

As the vector of force F_{m1} is behind the vector of force F_{R1} by the angle of $\pi/2$ (see fig.2), we may write that:

$$\begin{aligned} F_{m1} &= F^m_{\max}\sin(\omega t+\varphi_n-\pi/2), \\ F_{m2} &= F^m_{\max}\sin(\omega t+\varphi_n-\pi/2-2\pi/3), \\ F_{m3} &= F^m_{\max}\sin(\omega t+\varphi_n-\pi/2-4\pi/3) \end{aligned} \tag{14}$$

Considering the system (14), we see, that in order to find F^m_{max} we need to use the condition:

$$\omega t+\varphi_n-\pi/2=\pi/2 \tag{15}$$

There remains one question to be solved: "How the system of equations (14) allows to determine the active value of the tractive force F_{m1}, which creates the rotation moment of the wind wheel in the whole?" For this, we go back to the theoretical principles of electrotechnology [4], which prove that the active value of electromotive force in the circuit of AC of the sinusoidal form is by $\sqrt{2}$ times less then the amplitude one.

By analogy for our task we may write that the active value $F^{д}_m$ of the tractive force may be found from the equation

$$F^{д}_m=\frac{1}{\sqrt{2}}F^m_{\max} \tag{16}$$

And the active value $M^{д}_{об}$ of the rotational moment – from the equation

$$M^{д}_{об}=F^{д}_m R \tag{17}$$

In which the radius of the wind wheel R is an arm, to which the force $F^{д}_m$ is applied.

It should be noted that the availability of three blades in the wind wheel does not increase the effective value of tractive force, but only ensures for rotation of the point it is attached to and the sinusoidal character of alteration of instantaneous value on each of the blades.

Conclusions

There had been built the vector diagrams of the air streams speeds, which affect the blades of the wind wheel with the vertical rotation axle, and forces, created by these streams, when the angle of blade rotation relative to the tangent in the point of blade coupling to the rim of the wind wheel equals zero.

There had been suggested the new approach to the building of the mathematical model of forces which appear on the blades of the wind wheel with the vertical rotation axle when affected by the air streams, which is based on the analogy of three-phase electric system of alternating current of sinusoidal character.

There had been developed the method for identification of the suggested mathematical models for forces which appear on the blades of the wind wheel with the vertical rotation axle.

REFERENCES

1. Мокін Б.І., Мокін О.Б., Жуков О.А. До питання вибору вітрових двигунів і електричних генераторів вітрових електричних станцій // Вісник Вінницького політехнічного інституту. – 2007. – №6. – С. 52 – 62.

2. Мхитарян А.М. Аэродинамика.– М.: Машиностроение, 1970. – 428 с.

3. Васько В.П. Керування нестаціонарними режимами роботи вітроустановок промислових вітроелектричних станцій: Автореф. дис... к-та техн. наук / Інститут електродинаміки НАН України. – Київ, 2003. – 17 с.

4. Бессонов Л.А. Теоретические основы электротехники. – М.: Высшая школа, 1978. – 528 с.

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