І. А. Polishchuk

SETTING OF PID-REGULATOR BASED ON DIRECT SYTHESIS METHOD FOR THE SECOND ORDER OBJECTS PLUS TIME DELAY

For the construction of modern systems of automation in greater part of cases PІ-, PID-regulators are used, this is stipulated by their simplicity, lack of static regulation error and availability of the programming units in the automation devices being used. Setting of such regulators is a relevant problem as the available methods do not meet the requirements of the needed quality or are rather complicated for practical application.

The given paper contains the method for PID-regulator parameters setting, based on the direct-synthesis method for the second-order objects with delay on the channel task-output. The method is based on the reduction of the transfer function of the closed-loop system on the task-output channel to the simplified form. This occurs by means of Pade approximation of the first order for the delay time element and establishing *time integration coefficient and time of PID-regulator differentiation, connected with the coefficients of the control object model. The paper presents mathematical analysis of the simplified transfer function of the closed-loop system. Ranges of the general gain coefficient of PID-regulator at which the transient process of the closed-loop system is presented by the oscillating, conservative elements or by the aperiodic element of the second order and its stability is provided, are determined. Taking into account the substantiated requirements, concerning the quality of the transient process in the task-output channel the expression for the calculation of the general gain coefficient of PID-regulator is analytically obtained.*

The study of the suggested method and comparison with the methods of Zigler-Nickols, CHP and IMC for tuning PID-regulator of the objects of the second order with delay has been performed. According to the results of the study it is established that the suggested method improves quality indices of the transient *process namely: reduces the dynamic error and regulation time and can be applied for the objects of control with different response time. Results of the study show the great potential of the suggested method for rapid tuning of PID-regulator parameters, using only parameters of the object model for their calculation.*

Key words: PID-regulator, transient process, object of control, transfer function.

Introduction

Operation of any automation system depends on the reliable functioning of the equipment and usage of the reliable efficient control algorithms. Reliability of the operation of the equipment mainly depends on its quality. That is why, in the process of the systems of automation design special attention is paid to the selection of the equipment and characteristics of its reliability. In its turn, control algorithms determine the efficiency of the system operation, its ability to provide the designed indices. One of the basic functions, performed by the automation system, is the support of the mode parameters at the set values by means of the regulators. There exists a great number of various regulators but PI- and PID-regulators became extremely popular. This is stipulated by their simplicity and possibility to provide the support of the set parameter without static error. Besides, modern programmable logic controllers, as the basic type of devices for the construction of the automation systems have built-in programmable units, which realize these types of regulators. The only problem, arising in the process of PI- and PID-regulators operation is the setting of their parameters.

Analysis of the recent research and publications

Scientific Works of VNTU, 2023, \mathbb{N}^2 2 1 The majority of the methods of PI-, PID-regulators parameters setting are based on their calculation by means of the coefficients of the model of the controlled object, which is approximated by the preset function. The example of such methods of Ziegler-Nichols [1] or CHR (Chien-Hrones-Reswick) method [2]. In real conditions the obtaining of the accurate model of the controlled object is a complicated tasks, that is why, setting of the regulators as a rule, does not

meet needed quality indices of the transient processes. Besides, greater part of the methods use the simplified models of the objects, this also worsens the setting quality. Practical experience shows, that due to these reasons the engineers in the process of testing and commissioning often use manual setting of the regulators parameters, instead of using the methods, which do not bring the desired result.

To overcome the above-mentioned problems there exist methods, using more accurate models of the object of control and provide the robustness of the control system. In [3, 4] it is recommended to use as the example of the object model the aperiodic element of the second order with delay and application of IMC (Internal Model Control) approach for obtaining the settings of PID-regulator. Another example is the setting of the parameters of PID-regulator by means of the direct synthesis of the desired transfer function of the closed loop system [5]. The drawback of these methods is the complexity of setting, as it is necessary to determine optimal value of the filter parameter, which is the additional parameter. The accuracy of filter parameter determination also depends on the model of the object and influences the final result.

Alternative solution, concerning the setting of PI-, PID-regulators may be the application of fuzzy logic [6], but in this case accurate input data, based on the experience and large data sample are needed. That is why, practical application of such methods is problematic.

Having performed the analysis of the available setting methods of PI-, PID-regulators, the conclusion can be drawn that there exists the need in developing new methods, which would provide the comparatively simple and rapid way of obtaining these settings.

Objective and tasks of the paper

Objective of the paper is the development of the method for setting the parameters of PIDregulator by means of the direct synthesis for the object of the second order with delay.

Main part

One of the approaches to tuning PID-regulator parameters is the application of the direct synthesis method, which enables to reduce the transfer function of the closed-loop system to the desired form. In this case the obtained transfer function depends on one or several tuning parameters, which are to be found for obtaining the needed result. In the given study the model of the second-order delayed object, transfer function of which has the following form, is considered:

$$
W_o(s) = \frac{k_o e^{-\tau s}}{(T_1 s + 1)(T_2 s + 1)},
$$
\n(1)

where k_o – is the transfer coefficient; T_1 and T_2 – are time constants; τ – is transport delay.

Transfer function of PID-regulator has the following form:

$$
W_p(s) = k_p \left(1 + \frac{1}{T_i s} + T_{\partial} s\right),\tag{2}
$$

where k_p – is gain factor; T_i – is time of integration; T_o – is time of differentiation.

Structural diagram of the closed-loop system (Fig. 1), where $y -$ is the output value, $y^* -$ is the task.

Fig. 1. Structural diagram of the closed-loop system on the channel task-output

Transfer function of the closed-loop system on the link task-output:

$$
W_{3c}(s) = \frac{W_o(s) \cdot W_p(s)}{1 + W_o(s) \cdot W_p(s)}.
$$
\n(3)

Having substituted (1) and (2) into (3) and performed transformations with fractions, we obtain:
\n
$$
W_{3c}(s) = \frac{\frac{k_o e^{-rs}}{(T_1 s + 1)(T_2 s + 1)} \cdot k_p (1 + \frac{1}{T_i s} + T_o s)}{1 + \frac{k_o e^{-rs}}{(T_1 s + 1)(T_2 s + 1)} \cdot k_p (1 + \frac{1}{T_i s} + T_o s)}
$$
\n
$$
= \frac{k_o k_p e^{-rs} (T_o T_i s^2 + T_i s + 1)}{T_i s (T_1 T_2 s^2 + (T_1 + T_2) s + 1) + k_o k_p e^{-rs} (T_o T_i s^2 + T_i s + 1)}.
$$
\n(4)

If for time integration and differentiation parameters the following values are set:

$$
T_i = T_1 + T_2,\tag{5}
$$

$$
T_o = \frac{T_1 \cdot T_2}{T_1 + T_2} \tag{6}
$$

and having substituted them into (4), then the simplification of this expression to the following form is obtained:

$$
W_{3c}(s) = \frac{k_o k_p e^{-\tau s}}{(T_1 + T_2)s + k_o k_p e^{-\tau s}}.
$$
\n(7)

As it is seen from (7), the obtained transfer function of the closed-loop system on the link task-output depends only on one parameter of the regulator k_p , but at the same time this parameter is common for PID-regulator and influences its proportional, integral and differential components. Such an approach provides the possibility of the simplified analytical searching for only one tuning parameter k_p independently on the range of parameter values of the object model, this is an important advantage. For the solution of the problem of the presence in the denominator the transport delayed element, Pade approximation of the first order is used:

$$
e^{-\tau s} \approx \frac{1 - \frac{\tau}{2} s}{1 + \frac{\tau}{2} s}.\tag{8}
$$

Then, having substituted in (7) the expression (8) in the denominator and having performed transformation with fractions, we obtain:
 $k_o k_p \left(1 + \frac{\tau}{2} s\right) e^{-\tau s}$ $\left(1 + \frac{\tau}{2} s\right) e^{-\tau s}$ (9)

Then, having substituted in (7) the expression (8) in the denominator and having performed
transformation with fractions, we obtain:

$$
W_{3c}(s) = \frac{k_o k_p \left(1 + \frac{\tau}{2}s\right) e^{-\tau s}}{(T_1 + T_2)s \left(1 + \frac{\tau}{2}s\right) + k_o k_p \left(1 - \frac{\tau}{2}s\right)} = \frac{\left(1 + \frac{\tau}{2}s\right) e^{-\tau s}}{\frac{(T_1 + T_2)\tau}{2k_o k_p} s^2 + \left(\frac{T_1 + T_2}{k_o k_p} - \frac{\tau}{2}\right)s + 1}
$$
(9)

or

$$
W_{3c}(s) = \left(W_1(s) + \frac{\tau}{2} s W_1(s)\right) e^{-\tau s},
$$
\n(10)

where

Scientific Works of VNTU, 2023, $\mathcal{N} \subseteq 2$ 3

$$
W_1(s) = \frac{1}{\frac{(T_1 + T_2)\tau}{2k_o k_p} s^2 + \left(\frac{T_1 + T_2}{k_o k_p} - \frac{\tau}{2}\right) s + 1},
$$
\n(11)

As it is seen from the expression (11), transfer function $W_1(s)$ is the inertial element of the second order with the transfer coefficient $k = 1$, and depending on the coefficients in the denominator can be presented by the oscillating, conservative or aperiodic element of the second order. It follows from the expression (10) that the transfer function of the closed-loop system is dependent on $W_1(s)$, which has the form of the typical element and its properties are convenient for further studies, that is necessary for the method of the direct synthesis.

As the obtained transfer function of the closed-loop system depends only on one tuning parameter k_p , as it was mentioned above, then it is necessary to find optimal value of this parameter to provide the desired transient process. For this reason it is necessary to set the quality criterion. For different systems different quality criteria can be chosen but, usually, minimal or zero dynamic error is tried to get with possibly less regulation time. At such quality criteria regulated parameter will change smoothly, without oscillations, that is very important for the processes with accurate support of mode parameter Smooth regulation also results in less wear of the executive mechanism and regulating organ.

For the analysis of the above-mentioned quality criteria the transition from the transfer function

(b) to time transient response of the closed-loop system is performed, i. e., the dependence of the

put value on the tim (10) to time transient response of the closed-loop system is performed, i. e., the dependence of the output value on the time at the stepwise excitation. The image of the output value is :
 $Y(s) = W(s) \cdot X(s) = \left(W_1(s) + \frac{\tau}{2} s W$ output value on the time at the stepwise excitation. The image of the output value is :

to time transient response of the closed-loop system is performed, i. e., the dependence of the
t value on the time at the stepwise excitation. The image of the output value is :

$$
Y(s) = W(s) \cdot X(s) = \left(W_1(s) + \frac{\tau}{2} s W_1(s)\right) e^{-\tau s} \cdot X(s) = W_1(s) X(s) e^{-\tau s} + \frac{\tau}{2} s W_1(s) X(s) e^{-\tau s}, \quad (12)
$$

where *X*(*s*) – is the image of the input value. Having performed inverse Laplace transform and
having applied theorems of linearity, delay and differentiation at zero initial conditions, time
transient response of the c having applied theorems of linearity , delay and differentiation at zero initial conditions, time transient response of the closed-loop system is obtained:
 $y_{3c}(t-\tau) = L^{-1}[Y(s)] = L^{-1}\left[W_1(s)X(s)e^{-\tau s} + \frac{\tau}{2} sW_1(s)X(s)e^{-\tau s}\right] = y_$) – is the image of the input value. Having performed inverse Laplace transform and
lied theorems of linearity, delay and differentiation at zero initial conditions, time
ponse of the closed-loop system is obtained:
 $-\tau$)

having applied theorems of linearity, delay and differentiation at zero initial conditions, time
transient response of the closed-loop system is obtained:

$$
y_{sc}(t-\tau) = L^{-1}[Y(s)] = L^{-1}\left[W_1(s)X(s)e^{-\tau s} + \frac{\tau}{2}sW_1(s)X(s)e^{-\tau s}\right] = y_1(t-\tau) + \frac{\tau}{2} \int_{0}^{\pi} y_1(t-\tau), \quad (13)
$$

where $y_1(t-\tau)$ – is time transient response of the element $W_1(s)$ at the stepwise excitation of the input value, displaced at time τ , $y_1(t-\tau)$ – of its derivative.

It follows from the expression (13) that for the analysis of the time transient response of the closed-loop system it is necessary to find time transient response $y_1(t-\tau)$. As $W_1(s)$ depending on the coefficients, can be represented by different types of the inertia elements, then first of all, it is necessary to determine the type of the element $W_1(s)$ by means of the characteristic equation from the expression (11):

$$
\frac{(T_1 + T_2)\tau}{2k_o k_p} s^2 + \left(\frac{T_1 + T_2}{k_o k_p} - \frac{\tau}{2}\right) s + 1 = 0.
$$
\n(14)

As the characteristic equation is quadratic, to find its roots the discriminant is calculated, it depends on one tuning parameter *k р* :

$$
D(k_p) = \left(\frac{T_1 + T_2}{k_o k_p} - \frac{\tau}{2}\right)^2 - 4 \cdot \frac{(T_1 + T_2)\tau}{2k_o k_p} = \left(\frac{T_1 + T_2}{k_o k_p}\right)^2 - \frac{3(T_1 + T_2)\tau}{k_o k_p} + \frac{\tau^2}{4}.
$$
 (15)

Scientific Works of VNTU, 2023, $\mathcal{N} \subseteq 2$ 4

Then the roots of the characteristic equation (14) are:
\n
$$
s_{1,2} = \frac{-\left(\frac{T_1 + T_2}{k_o k_p} - \frac{\tau}{2}\right) \pm \sqrt{D(k_p)}}{2 \cdot \frac{(T_1 + T_2)\tau}{2k_o k_p}} = \frac{k_o k_p}{2(T_1 + T_2)} - \frac{1}{\tau} \pm \frac{k_o k_p \sqrt{D(k_p)}}{(T_1 + T_2)\tau}.
$$
\n(16)

If $D(k_p) < 0$, then $W_1(s)$ is the oscillatory or conservative element, otherwise $W_1(s)$ is aperiodic element of the second order. Thus, for the determination of the type of the inertia link it is necessary to perform the analysis of $D(k_p)$. From the expression (15) the equation is composed to find the roots:

$$
\left(\frac{T_1 + T_2}{k_o k_p}\right)^2 - \frac{3(T_1 + T_2)\tau}{k_o k_p} + \frac{\tau^2}{4} = 0.
$$
\n(17)

Having multiplied left and right parts of the equation (17) by k_p^2 k_p^2 , the quadratic equation is obtained:

$$
\frac{\tau^2}{4}k_p^2 - \frac{3(T_1 + T_2)\tau}{k_o}k_p + \left(\frac{T_1 + T_2}{k_o}\right)^2 = 0.
$$
\n(18)

Then the discriminant for the equation (18):
\n
$$
D = \left(-\frac{3(T_1 + T_2)\tau}{k_o}\right)^2 - 4 \cdot \frac{\tau^2}{4} \cdot \left(\frac{T_1 + T_2}{k_o}\right)^2 = 8\left(\frac{(T_1 + T_2)\tau}{k_o}\right)^2.
$$
\n(19)

Roots of the equation (18):

(18):
\n
$$
k_{p_{1,2}} = \frac{\frac{3(T_1 + T_2)\tau}{k_o} \pm \frac{2\sqrt{2}(T_1 + T_2)\tau}{k_o}}{2 \cdot \frac{\tau^2}{4}} = \frac{(6 \pm 4\sqrt{2})(T_1 + T_2)}{k_o \tau}.
$$
\n(20)

To provide the strength of the closed- loop system it is necessary that the real part of the roots of the characteristic equation (14) were negative or equal zero. Then having used the expression (16), we have the following condition:

n:
\n
$$
\frac{k_{o}k_{p}}{2(T_{1}+T_{2})} - \frac{1}{\tau} \le 0 \to k_{p} \le \frac{2(T_{1}+T_{2})}{k_{o}\tau}.
$$
\n(21)

Fig. 2 shows the dependence graph of the equation (18) discriminant on k_p .

Fig. 2. Dependence graph of the equation (18) discriminant on k_p

Having analyzed Fig. 2, taking into account the condition (21), the conclusion can be made that $W_1(s)$ is an aperiodic element of the second order if $k_p \in \left(0, \frac{(6-4\sqrt{2})(T_1+T_2)}{k\sigma}\right)$ *o* $k_p \in \left(0;\frac{(6-4\sqrt{2})(T_1+T_2)}{T_1+T_2}\right)$ k_{o}^{\dagger} ϵ (0; $\frac{(6-4\sqrt{2})(T_1+T_2)}{k_o\tau}$), oscillating element if $k_p \in \left(\frac{(6-4\sqrt{2})(T_1+T_2)}{k_o \tau}; \frac{2(T_1+T_2)}{k_o \tau}\right)$ $k_p \in \left(\frac{(6-4\sqrt{2})(T_1+T_2)}{T_1+T_2}; \frac{2(T_1+T_2)}{T_1+T_2}\right)$ $\left(\frac{2}{k_o \tau} \cdot \frac{2(T_1 + T_2)}{k_o \tau}\right)$ $\left(\frac{(6-4\sqrt{2})(T_1+T_2)}{(T_1+T_2)}\right)$ and $\in \left(\frac{(6-4\sqrt{2})(T_1+T_2)}{k \tau}; \frac{2(T_1+T_2)}{k \tau}\right)$, and $\left(\frac{(6-4\sqrt{2})(T_1+T_2)}{k_o\tau}; \frac{2(T_1+T_2)}{k_o\tau}\right)$, and if $k_p = \frac{2(T_1+T_2)}{k_o\tau}$ $k_p = \frac{2(T_1 + T_2)}{T_1}$ $k_o \tau$ $=\frac{2(T_1+T_2)}{T_1}$ – is conservative element.

Having obtained the ranges of the tuning parameter k_p , at which $W_1(s)$ is represented by different elements, it is necessary to determine the dynamic error for the output value of closed-loop system in each case.

o

As it is known from theory [7], time transient characteristic of the conservative element is continuous. In this case the dynamic error of the output value $\Delta y_{3c} > 0$, this does not meet the requirements of the quality criteria.

In its turn, oscillating element represents damped oscillating process at a single stepwise disturbance from $y(t) \rightarrow k$ if $t \rightarrow \infty$, where k – is the coefficient of the oscillating element transfer [7]. Dynamic error Δy for such element is measured in the point t_1 (Fig. 3), in such event $\Delta y > 0$. As in the point t_1 the derivative from the output value of the oscillating element $\overline{y}(t_1) = 0$, then it follows from the expression (13) that $y_{3c}(t_1) = y_1(t_1)$. As it was mentioned above, transfer
coefficient $k = 1$, that is why, the condition $y_1(t_1) > k \rightarrow y_1(t_1) > 1 \rightarrow y_{3c}(t_1) > 1$ is met and coefficient $k=1$, that is why, the condition $y_1(t_1) > k \rightarrow y_1(t_1) > l \rightarrow y_{2c}(t_1) > l$ is met and there exists such moment of time, when the value of the output quantity of the closed-loop system

is greater than the set value $y_{3c}(\infty)$. Thus, the dynamic error of the output quantity $\Delta y_{3c} > 0$, if $W₁(s)$ is oscillating element, that does not correspond to quality criteria, described above.

Fig. 3. Transient characteristic of the oscillatory element

In case of the aperiodic element of the second order time transient characteristic depends on the roots of the characteristic equation (14). If the roots are equal, then time transient characteristic is represented as [8]:

$$
y(t) = k \cdot \Delta U \cdot (1 - (1 + \frac{t}{\tau}) \cdot e^{-\frac{t}{\tau}}) = k \cdot \Delta U \cdot (1 + e^{at} (at - 1)),
$$
\n(22)

where $k - i$ s the transfer coefficient, $\Delta U - i$ s the amplitude of the input stepwise disturbance, $a = -\frac{1}{\tau}$ – is the root of the characteristic equation. Having substituted the root from (16) and transfer coefficient $k = 1$ in the expression (22) at a single input disturbance, taking into account time τ , displacement, we obtain:

$$
y_1(t-\tau) = 1 + e^{s_1(t-\tau)} (s_1(t-\tau)-1).
$$
 (23)

Having performed differentiation of the expression (23), we obtain:
\n
$$
\int_{y_1}^{\square} (t-\tau) = s_1 e^{s_1(t-\tau)} (s_1(t-\tau)-1) + s_1 e^{s_1(t-\tau)} = s_1^2(t-\tau) e^{s_1(t-\tau)}.
$$
\n(24)

Having substituted the expressions (23) and (24) into (13), we obtain:
\n
$$
y_{3c}(t-\tau) = 1 + e^{s_1(t-\tau)} \left(s_1(t-\tau) - 1 \right) + \frac{\tau}{2} s_1^2(t-\tau) e^{s_1(t-\tau)}.
$$
\n(25)

For finding the maximum of the output value, it is necessary to find maxima and minima of the function in the interval $[\tau; \infty)$ and the value in the points of maxima and minima. For this reason the derivative is to be found:
 $y_{3c}(t-\tau) = s_1 e^{s_1(t-\tau)} (s_1(t-\tau) - 1) + s_1 e^{s_1(t-\tau)} + \frac{\tau}{2} s_1^2 e^{s_1(t-\tau)} + \frac{\tau}{2} s_1^3(t$ reason the derivative is to be found: ne interval $[\tau; \infty)$ and the value in the points of maxima and minima. For this
ive is to be found:
 $(-\tau) = s_1 e^{s_1(t-\tau)} (s_1(t-\tau) - 1) + s_1 e^{s_1(t-\tau)} + \frac{\tau}{2} s_1^2 e^{s_1(t-\tau)} + \frac{\tau}{2} s_1^3(t-\tau) e^{s_1(t-\tau)} =$

$$
\begin{aligned}\n\text{derivative is to be found:} \\
\int_{y_{3c}}^{\square} (t-\tau) &= s_1 e^{s_1(t-\tau)} (s_1(t-\tau)-1) + s_1 e^{s_1(t-\tau)} + \frac{\tau}{2} s_1^2 e^{s_1(t-\tau)} + \frac{\tau}{2} s_1^3(t-\tau) e^{s_1(t-\tau)} \\
&= s_1^2 e^{s_1(t-\tau)} \cdot \left((1+\frac{\tau}{2} s_1)(t-\tau) + \frac{\tau}{2} \right).\n\end{aligned} \tag{26}
$$

Scientific Works of VNTU, 2023, № 2 7 7 As in case of equal roots of the characteristic equation (14) we have $D(k_p) = 0$, then, taking into account (21) we will have only one root from the expression (20), that corresponds to the condition for the aperiodic element of the second order:

$$
k_p = \frac{(6 - 4\sqrt{2})(T_1 + T_2)}{k_o \tau}.
$$
\n(27)

Having substituted the expression (27) into (16), we obtain the root of the characteristic equation (14):

$$
s_1 = \frac{k_o}{2(T_1 + T_2)} \cdot \frac{(6 - 4\sqrt{2})(T_1 + T_2)}{k_o \tau} - \frac{1}{\tau} = \frac{2 - 2\sqrt{2}}{\tau}.
$$
 (28)

For finding the points of maxima and minima of the output value we make equivalent its derivative to zero, then from (26) and (28) we will have:
\n
$$
\int_{y_{3c}}^{\pi} (t-\tau) = 0 \rightarrow s_1^2 e^{s_1(t-\tau)} \left(\left(1 + \frac{\tau}{2} s_1 \right) (t-\tau) + \frac{\tau}{2} \right) = 0 \rightarrow
$$
\n
$$
e^{s_1(t-\tau)} = 0 \rightarrow e^{-\frac{2-\sqrt{2}}{\tau}} (t-\tau) = 0 \rightarrow t \rightarrow \infty,
$$
\n
$$
\left(1 + \frac{\tau}{2} s_1 \right) (t-\tau) + \frac{\tau}{2} = 0 \rightarrow \left(1 + \frac{\tau}{2} \frac{2-\sqrt{2}}{\tau} \right) (t-\tau) + \frac{\tau}{2} = 0 \rightarrow t = \left(1 - \frac{1}{4-\sqrt{2}} \right) \tau.
$$
\n(29)

As the second point of the extremum $t = \left(1 - \frac{1}{\cdots}\right)$ $4 - \sqrt{2}$ $t = \left(1 - \frac{1}{4 - \sqrt{2}}\right) \tau$ does not enter the interval $[\tau; \infty)$, it may

not be taken into account, thus we have only one extremum of the output value if $t \rightarrow \infty$, that corresponds to the set value of the output quantity $y_{3c}(\infty)$. That is why, in case, when $W_1(s)$ is the aperiodic element with equal roots of the characteristic equation, the dynamic error of the output value $\Delta y_{3c} = 0$, that corresponds to the set quality criteria.

From the analogous arguments, in case when $W_1(s)$ is the aperiodic element with different roots of the characteristic equation (14), the dynamic error of the output value is also absent, but the time of regulation will be greater than at equal roots [8]. Thus, optimal settings of PID-regulator, which correspond to the set quality criteria are calculated from the expressions (5), (6) and (27).

For the analysis of the obtained results the study for the object is performed [9]:

$$
W_o(s) = \frac{e^{-2s}}{0.7s^2 + 1.7s + 1}.
$$
\n(30)

Calculated values of PID-regulator settings for the suggested method of the direct synthesis (MDS PID), Ziegler-Nichols methods, CHR and IMC method are presented in the Table 1. For IMC method, the parameters of PID-regulator tuning are obtained from the research [4].

Table 1

Parameters for PID-regulator tuning for the object (30)

Fig. 4 shows the transient processes for the object (30) in the channel task-output.

Fig. 4. Transient process for the object (30) on the channel task-output

As it is seen from Fig. 4, the suggested method enables to obtain the least regulation time and unlike another methods the dynamic error is missing.

For the additional study of the suggested method, another object with the transfer function was chosen [10]:

$$
W_o(s) = \frac{2e^{-s}}{(10s+1)(5s+1)}.\tag{31}
$$

Tuning parameters of PID-regulator are given in Table 2. For IMC method the tuning parameters are taken from the study [10].

Table 2

Tuning parameters of PID-regulator for the object (31)

Fig. 5 presents the transient processes for the object (31) on the channel task-output.

Fig. 5. Transient process for the object (31) on the channel task-output

If Fig. 5 is analyzed, the conclusion could be made that the suggested method provides the least dynamic error and the regulation time is analog to IMC method. Availability of the dynamic error for the suggested method can be explained by the application of Pade approximation for the delay link.

The additional study of the suggested method usage for the objects, characterized by average and high response time was carried out:

$$
W_o(s) = \frac{0.4e^{-20s}}{800s^2 + 60s + 1}.
$$
\n(32)

$$
W_o(s) = \frac{0.15e^{-200s}}{1430000s^2 + 2400s + 1}.
$$
\n(33)

Transient processes for the objects (32) and (33) are presented in Fig. 6 and Fig. 7 correspondingly.

Fig. 6. Transient process for the object (32) on the channel task-output

Fig. 7. Transient process for the object (33) on the channel task-output

Conclusions

The suggested method enables by means of the formulas (5), (6) and (27) to provide rapid tuning of the parameters of PID-regulator with minimal dynamic error and regulation time on the channel task-output. The obtained results prove the possibility of the application of the method for the objects of the second order with the delay with different response time. It should be noted that the suggested method provides the tuning of PID-regulators only in the channel task-output, for the channel excitation-output other methods should be applied.

REFERENCES

1. Ziegler J. G. Optimum Settings for Automatic Controllers / J. G. Ziegler, N. B. Nichols // Translation ofthe ASME. – 1942. – Vol. 64. – P. 759 – 768.

2. Chien K. L. On The Automatic Control of Generalized Passive Systems / K. L. Chien, J. A. Hrones, J. B. Reswick // Translations of the ASME. – 1952. – Vol. 74, № 2. – P. 175 – 183.

3. PID Tuning Method Based on IMC for Inverse-Response Second-Order Plus Dead Time Processes / D. Castellanos-Cárdenas, F. Castrillón, R. E. Vásquez [et al.] // Processes. – September 2020. – Vol. 8, № 9. – P. 1183.

4. Arya P. P. A Modified IMC Design for Second Order Plus Time Delayed Processes / P. P. Arya // IFAC-PapersOnLine. – 2022. – Vol. 55, № 1. – P. 843 – 847.

5. [Chen](https://pubs.acs.org/action/doSearch?field1=Contrib&text1=Dan++Chen) D. PI/PID Controller Design Based on Direct Synthesis and Disturbance Rejection / D. [Chen,](https://pubs.acs.org/action/doSearch?field1=Contrib&text1=Dan++Chen) D. E. Seborg // Industrial & Engineering Chemistry Research. – August 2002. – Vol. 41, № 19. – P. 4807 – 4822.

6. Garasimiv Гарасимів V. М. Algorithm of PID-regulator coefficients tuning, using fuzzy logic methods / V. М. Garasimiv // Methods and means of quality control. – 2020. – № 2 (45). – P. 102 – 108. (Ukr).

7. Popovych M. G. Theory of automatic control : manual $[2^{nd}$ edition] / M. G. Popovych, O. V. Kovalchuk. – Kyiv : Lybid, 2007. – 656 p. (Ukr).

8. Bequette B. W. Second-Order Behavior / [B. W. Bequette](https://www.informit.com/authors/bio/2a16c941-cf6b-4375-82f4-55e230f4c798) // Process Control : Modeling, Design and Simulation. – Prentice-Hall Professional, 2003. – P. 138 – 147.

9. Lee J. Simple Analytic PIDController Tuning Rules Revisited / J. Lee, W. Cho, T. F. Edgar // Industrial & Engineering Chemistry Research. – April 2014. – Vol. 53, № 13. – P. 5038 – 5047.

10. Shamsuzzoha M. IMC Filter Design for PID Controller Tuning of Time Delayed Processes / M. Shamsuzzoha, M. Lee // PID Controller Design Approaches – Theory, Tuning and Application to Frontier Areas. – InTech, 2012. – P. $253 - 286$.

Editorial office received the paper 15.05.2023. The paper was reviewed 26.05.2023.

Polishchuk Igor – Senior Lecture with the Department of automation of energy processes of Education-Scientific Institute of Nuclear and Thermal Power Engineering.

National Technical University of Ukraine «Igor Sikorskyi Kyiv Polytechnical Institute».