UDC 621.325.5

O. Semenova, Cand. Sc. (Eng.); O. Wojciechowska

STRUCTURAL SYNTHESIS OF PULSE-FREQUENCY FUZZY-LOGIC ELEMENTS

In this article a method of fuzzy-logic elements structural synthesis has been proposed. The elements are made of basis circuits, which perform operations of addition, subtraction, partition, and division by two. Block diagrams of elements have been synthesized. The elements perform such operations as implication, contradiction, and equivalence. Information in the developed elements is presented as pulse-frequency signals.

Keywords: *fuzzy-logic, synthesis, element.*

Using in telecommunication systems fuzz-logic devices allows performing a precise control in changing conditions with low expenses. A part of the devices is elements performing fuzzy-logic functions.

In the case, when information signals of the system are pulse-frequency ones, using of special converters leads to complication of circuits, increasing of sizes, and decreasing of accuracy. That is why in such cases in order to perform fuzzy-logic operations it is necessary to use the pulse-frequency elements.

The aim of the work is fuzzy-logic pulse elements design efficiency improvement.

In order to improve the fuzzy-logic pulse elements design efficiency it is necessary to develop a method of fuzzy-logic elements structural synthesis, that will allow design of basis fuzzy-logic function elements: complement, minimum, maximum; and of auxiliary fuzzy-logic function elements: contradiction, tautology, prohibition, implication, Peirce's arrow, Sheffer's dash, exclusive OR, equivalence.

In order to reach the aim one should solve such **tasks**:

– to define a generalized block diagram of the fuzzy-logic elements;

– to develop an algorithm of the method of fuzzy-logic elements synthesis.

Basis physical circuits types. In the work [1] the method of pulse-frequency logic elements synthesis has been proposed. According to it, each element corresponds to a set of basic physical circuits and bounds between them. An operational description is composed on the base of logic functions, after it structural diagrams of elements is built.

In this case the method of fuzzy-logic elements structural synthesis is proposed, in it the basic elements are physical circuits, performing operation of addition (A-element), subtraction (S-element), partition (P-element) and division by two (D-element).

Combining in series and in parallel different basic physical elements we will get diagrams to perform fuzzy-logic functions. We will describe such function as sequence of corresponding operations. We use such a structure of description for it:

$$\underbrace{z\overset{1}{\uparrow}x_1\overset{2}{\uparrow}x_2\overset{3}{\uparrow}\ldots x_m\overset{m+1}{\uparrow}}_{\text{input field}}:\underbrace{\qquad\qquad}_{\text{operator field}}:\underbrace{\overset{n}{\downarrow}y}_{\text{output field}}$$

So, because in the operational description all the signals and basic physical elements are included, as well as sequence of elements conjunctions, a structural diagram of an element is built on the base of the description.

The algorithm of the method of fuzzy-logic elements structural synthesis. On the base of developed block diagrams of complement, minimum, maximum elements we construct a generalized block diagram of a fuzzy-logic element (fig.1). This diagram consists of two complement operations, two minimum operations and one maximum operation.

The generalized operational description of the generalized block diagram of the fuzzy-logic element has the following form:

$$z\overset{1}{\uparrow}x_1\overset{2}{\uparrow}x_2\overset{3}{\uparrow}:\left(\overset{1}{\downarrow}P\overset{4}{\uparrow}\overset{5}{\uparrow}\overset{2}{\downarrow}P\overset{6}{\uparrow}\overset{7}{\uparrow}\overset{8}{\uparrow}\overset{3}{\downarrow}P\overset{9}{\uparrow}\overset{10}{\uparrow}\overset{11}{\uparrow}\right)\left(\overset{4}{\downarrow}\overset{6}{\downarrow}SP\overset{12}{\uparrow}\overset{13}{\uparrow}\overset{5}{\downarrow}\overset{9}{\downarrow}SP\overset{14}{\uparrow}\overset{15}{\uparrow}\right)$$

$$\left(\overset{10}{\downarrow}\overset{12}{\downarrow}A\overset{16}{\uparrow}\overset{11}{\downarrow}\overset{13}{\downarrow}S\overset{17}{\uparrow}\overset{7}{\downarrow}\overset{14}{\downarrow}A\overset{18}{\uparrow}\overset{8}{\downarrow}\overset{15}{\downarrow}S\overset{19}{\uparrow}\right)\left(\overset{16}{\downarrow}\overset{17}{\downarrow}SDP\overset{20}{\uparrow}\overset{21}{\uparrow}\overset{18}{\downarrow}\overset{19}{\downarrow}SDP\overset{22}{\uparrow}\overset{23}{\uparrow}\right)$$

$$\left(\overset{20}{\downarrow}\overset{21}{\downarrow}A\overset{24}{\uparrow}\overset{21}{\downarrow}\overset{23}{\downarrow}S\overset{25}{\uparrow}\right)\left(\overset{24}{\downarrow}\overset{25}{\downarrow}AD\overset{26}{\uparrow}\right):y\overset{26}{\downarrow}.$$

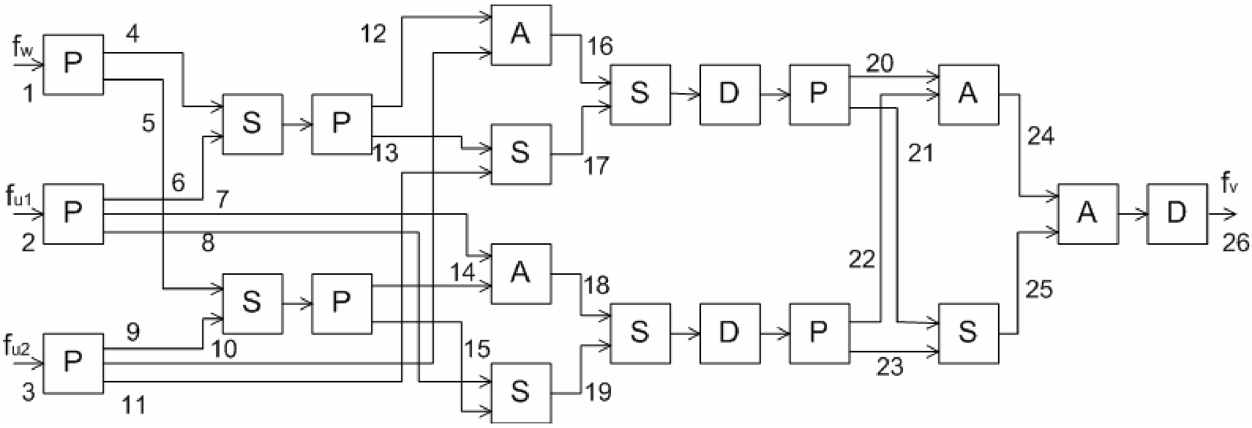

Fig. 1. The generalized block diagram of the fuzzy-logic element

In order to pass from the element's formula to its operational description, it is necessary to fill in an operational Table 1.

Table 1

The operational table

	1. Absent	2. First	3. Second
1. Input			
2. Complement			
3. Minimum			
4. Maximum			

The operational Table is analyzed by the following data:

If a cell 1.3. (the first row, the third column) is empty, then in the first parenthesis of the operational description of the element the third operator P will be absent.

If a cell 2.2. is empty, then in the first parenthesis the first operator P will be absent, and in the second parenthesis the first operators S and P will be absent.

If a cell 2.3. is empty, then in the first parenthesis the first operator P will be absent, and in the second parenthesis the second operators S and P will be absent.

If a cell 3.1. or 4.1. is filled and a cell 2.2. is empty, then in the first parenthesis after the second operator P there will be two arrows.

If a cell 3.1. or 4.1. is filled, and a cell 2.2. is filled, then in the first parenthesis the second operator P will be absent.

If a cell 3.1. or 4.1. is filled, and a cell 2.3. is empty, then in the first parenthesis after the third operator P there will be two arrows.

If a cell 3.1. or 4.1. is filled, and a cell 2.3. is filled, then in the first parenthesis the third operator P will be absent.

If cells 3.2. and 4.1. are filled, then in the third parenthesis the second operators A and S will be absent; in the fourth parenthesis the second operators S, D, P will be absent; also all the operators from the fifth and the sixth parenthesis will be absent.

If cells 4.2. and 3.1. are filled, then in the third parenthesis the second operators A and S will be absent; in the fourth parenthesis the first operator S is substituted on an operator A, the second operators S, D, P will be absent; also all the operators from the fifth and the sixth parenthesis will be absent.

If cells 4.2., 4.3. and 2.2. are filled, then in the fourth parenthesis operators S are substituted on operators A; in the sixth parenthesis an operator A is substituted by an operator S.

If cells 3.2., 3.3. and 4.2. are filled, then the operational description does not change.

The algorithm of the method of structural synthesis is the following:

1. The operational Table is filled.
2. The operational Table is analyzed.
3. On the base of the generalized operational description and according to the performed analysis of the operational Table the operational description is composed.
4. On the base of the operational description a block diagram of the element is composed.

Synthesis of the contradiction element block diagram. Fuzzy-logic operation of contradiction is performed according to the formula [2, 3]:

$$\min(x, 1-x).$$

1. The operational Table is filled:

	1. Absent	2. First	3. Second
1. Input		+	
2. Complement		+	
3. Minimum		+	
4. Maximum	+		

2. The operational Table is analyzed:

The cell 1.3. is empty, so in the first parenthesis of the operational description of the contradiction element the third operator P is absent.

The cell 2.3. is empty, so in the first parenthesis the first operator P is absent, and in the second parenthesis the second operators S and P are absent.

The cells 3.2. and 4.1. are filled, so in the third parenthesis the second operators A and S are absent; in the fourth parenthesis the second operators S, D, P are absent; also all the operators from the fifth and the sixth parenthesis are absent.

3. On the base of the performed analysis the contradiction element operational description is composed:

$$z\overset{1}{\uparrow}x\overset{2}{\uparrow}\left(\overset{2}{\downarrow}P\overset{3}{\uparrow}\overset{4}{\uparrow}\overset{5}{\uparrow}\right)\overset{1}{\downarrow}\overset{3}{\downarrow}AP\overset{6}{\uparrow}\overset{7}{\uparrow}\left(\overset{4}{\downarrow}\overset{6}{\downarrow}A\overset{8}{\uparrow}\overset{5}{\downarrow}\overset{7}{\downarrow}S\overset{9}{\uparrow}\right)\overset{8}{\downarrow}\overset{9}{\downarrow}SD\overset{10}{\uparrow}:y\overset{10}{\downarrow}.$$

4. On the base of the operational description the block diagram of the contradiction element is composed and presented in Fig. 2.

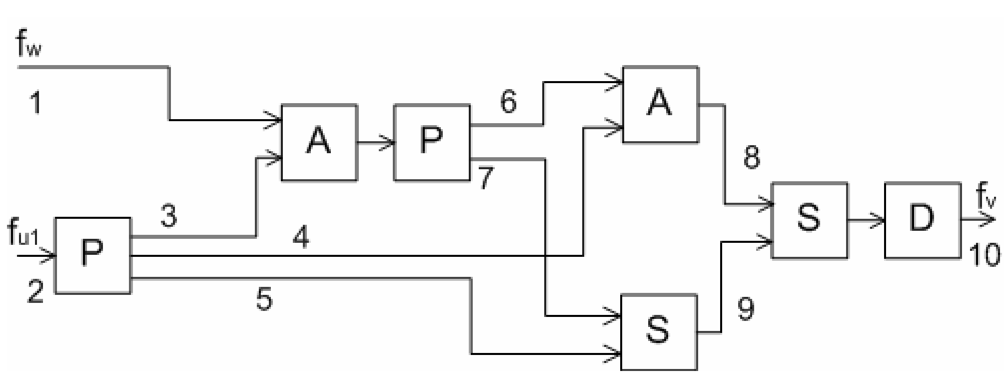

Fig. 2. The contradiction element block diagram

Synthesis of the implication element block diagram. Fuzzy-logic operation of implication is performed according to the formula [2, 3]:

$$\max(1 - x_1, x_2)\ .$$

1. The operational Table is filled:

	1. Absent	2. First	3. Second
1. Input		+	
2. Complement		+	
3. Minimum	+		
4. Maximum		+	

2. The operational Table is analyzed:

The cell 2.3. is empty, so in the first parenthesis the first operator P is absent, and in the second parenthesis the second operators S and P are absent.

The cell 3.1. is filled, and the cell 2.2. is filled, so in the first parenthesis the second operator P is absent.

The cell 3.1. is filled, and the cell 2.3. is empty, so in the first parenthesis after the third operator P are two arrows.

The cells 4.2. and 3.1. are filled, so in the third parenthesis the second operators A and S are absent; in the fourth parenthesis the first operator S is substituted by an operator A, the second operators S, D, P are absent; also all the operators from the fifth and the sixth parenthesis are absent.

3. On the base of the performed analysis the implication element operational description is composed:

$$z\overset{1}{\uparrow} x_1 \overset{2}{\uparrow} x_2 \overset{3}{\uparrow} : \overset{3}{\downarrow} P \overset{4}{\uparrow}\overset{5}{\uparrow}\overset{1}{\downarrow}\overset{2}{\downarrow} SP \overset{6}{\uparrow}\overset{7}{\uparrow}\left(\overset{4}{\downarrow}\overset{6}{\downarrow} A \overset{8}{\uparrow}\overset{5}{\downarrow}\overset{7}{\downarrow} S \overset{9}{\uparrow}\right)\overset{8}{\downarrow}\overset{9}{\downarrow} AD \overset{10}{\uparrow} : y \overset{10}{\downarrow}\ .$$

4. On the base of the operational description the block diagram of the implication element is composed and presented in Fig. 3.

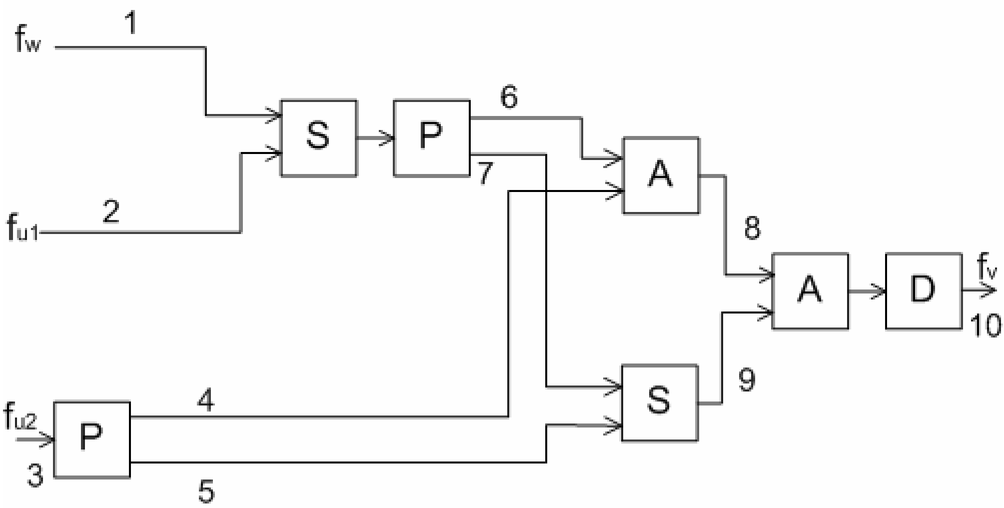

Fig. 3. The implication element block diagram

Synthesis of the equivalence element block diagram. Fuzzy-logic operation of equivalence is performed according to the formula [2, 3]:

$$\min[\max(1 - x_1, x_2), \max(x_1, 1 - x_2)]\ .$$

1. The operational Table is filled.

	1. Absent	2. First	3. Second
1. Input		+	+
2. Complement		+	+
3. Minimum		+	
4. Maximum		+	+

2. The operational Table is analyzed:

The cells 4.2., 4.3. and 2.2. are filled, so in the fourth parenthesis operators S are substituted by operators A; in the sixth parenthesis an operator A is substituted by an operator S.

3. On the base of the performed analysis the equivalence element operational description is composed:

$$z\overset{1}{\uparrow}x_1\overset{2}{\uparrow}x_2\overset{3}{\uparrow}:\left(\overset{1}{\downarrow}P\overset{4}{\uparrow}\overset{5}{\uparrow}\overset{2}{\downarrow}P\overset{6}{\uparrow}\overset{7}{\uparrow}\overset{8}{\uparrow}\overset{3}{\downarrow}P\overset{9}{\uparrow}\overset{10}{\uparrow}\overset{11}{\uparrow}\right)\left(\overset{4}{\downarrow}\overset{6}{\downarrow}SP\overset{12}{\uparrow}\overset{13}{\uparrow}\overset{5}{\downarrow}\overset{9}{\downarrow}SP\overset{14}{\uparrow}\overset{15}{\uparrow}\right)$$

$$\left(\overset{10}{\downarrow}\overset{12}{\downarrow}A\overset{16}{\uparrow}\overset{11}{\downarrow}\overset{13}{\downarrow}S\overset{17}{\uparrow}\overset{7}{\downarrow}\overset{14}{\downarrow}A\overset{18}{\uparrow}\overset{8}{\downarrow}\overset{15}{\downarrow}S\overset{19}{\uparrow}\right)\left(\overset{16}{\downarrow}\overset{17}{\downarrow}ADP\overset{20}{\uparrow}\overset{21}{\uparrow}\overset{18}{\downarrow}\overset{19}{\downarrow}ADP\overset{22}{\uparrow}\overset{23}{\uparrow}\right)$$

$$\left(\overset{20}{\downarrow}\overset{21}{\downarrow}A\overset{24}{\uparrow}\overset{21}{\downarrow}\overset{23}{\downarrow}S\overset{25}{\uparrow}\right)\left(\overset{24}{\downarrow}\overset{25}{\downarrow}SD\overset{26}{\uparrow}\right):y\overset{26}{\downarrow}.$$

4. On the base of the operational description the block diagram of the equivalence element is composed and depictured in Fig. 4.

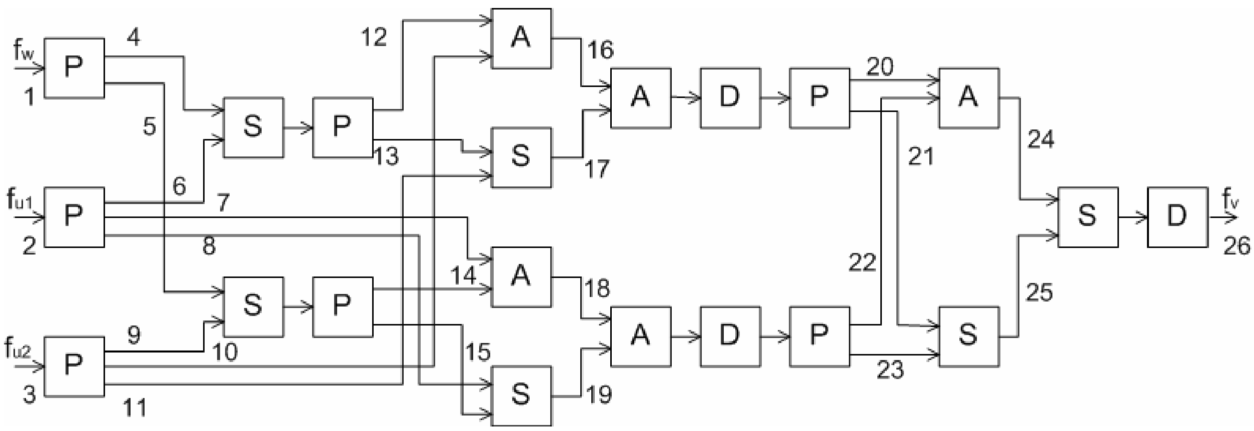

Fig. 4. The equivalence element block diagram

The method of structural synthesis efficiency estimation. Table 2 contains all fuzzy logic operations, some of them are implemented in an inference block of a fuzzy-controller. Let us indicate the operations implemented by elements with different information presentation.

Table 2. Fuzzy logic operations

Operation	Amplitude presentation of information	Pulse-frequency presentation of information
Complement	+	+
Minimum	+	+
Maximum	+	+
Contradiction	+	+
Tautology	+	+
Prohibition	+	+
Implication	+	+
Peirce's arrow,	+	+
Sheffer's dash	+	+
Exclusive OR	+	+
Equivalence	+	+
AND		+
OR		+
PROD		+
Total amount pf operations to be implemented by the elements	11	14

As one can see from Table 2, pulse-frequency fuzzy-logic elements can implement 14 functions of 14 possible ones. Amplitude fuzzy-elements can implement only 11 functions.

Thus, the pulse-frequency fuzzy-logic elements and devices can implement more fuzzy logic functions than amplitude fuzzy-logic elements and devices can.

The proposed method of fuzzy-logic elements structural synthesis allows synthesizing 11 fuzzy elements out of 14 possible ones.

Table 3 contains all fuzzy logic and binary logic operations, that can be implemented by the elements developed using the method of structural synthesis.

Table 3

The synthesized elements and the operations implemented by them

Logic element	Fuzzy logic	Binary logic
Complement (inversion)	$1-\mu$	$\overline{x}$
Minimum (conjunction)	$\min[\mu_1, \mu_2]$	$x_1 \cdot x_2$
Maximum (disjunction)	$\max[\mu_1, \mu_2]$	$x_1 + x_2$
Contradiction	$\min(x,\ 1-x)$	$\overline{x}$
Tautology	$\max(x,\ 1-x)$	x
Prohibition	$\min(x_1, 1-x_2)$	$x_1 \cdot \overline{x}_2$
Implication	$\max(1-x_1, x_2)$	$\overline{x}_1 + x_2$
Peirce's arrow,	$\min(1-x_1, 1-x_2)$	$\overline{x}_1 \cdot \overline{x}_2$
Sheffer's dash	$\max(1-x_1, 1-x_2)$	$\overline{x}_1 + \overline{x}_2$
Exclusive OR	$\max[\min(1-x_1, x_2), \min(x_1, 1-x_2)]$	$\overline{x}_1 \cdot x_2 + x_1 \cdot \overline{x}_2$
Equivalence	$\min[\max(1-x_1, x_2), \max(x_1, 1-x_2)]$	$(\overline{x}_1 + x_2) \cdot (x_1 + \overline{x}_2)$

Thus, the main advantage of the proposed method of structural synthesis is that elements, developed by this method can implement not only fuzzy-logic functions, but also corresponding binary logic functions.

Summary

1. The method of fuzzy-logic elements structural synthesis has been developed. Its point is that for each fuzzy-logic function the operational Table is to be filled and analyzed; on the base of this analysis and the generalized operational description the operational description for each element is developed; the block diagram is synthesized by the operational description.

2. The block diagrams of pulse-frequency elements have been synthesized. The presented elements perform operations of contradiction, implication and equivalence.

REFERENCES

1. Кичак В.М. Синтез частотно-імпульсних елементів цифрової техніки. Монографія. – Вінниця: УНІВЕРСУМ – Вінниця, 2005. – 266с.

2. Бардаченко В.Ф., Кичак В.М. Основи теорії таймерних обчислювально-вимірювальних пристроїв. – Вінниця: ВДТУ. – 2003. – 106с.

3. Бардаченко В.Ф., Колесницький О.К., Василецький С.А. Таймерні нейронні елементи та структури. Монографія. – Вінниця: УНІВЕРСУМ-Вінниця, 2004. – 126с.

Semenova Olena – assistant professor of the Department for Telecommunication Systems and Television;

Wojciechowska Olga – engineer of the Department for Telecommunication Systems and Television.
Vinnytsya National Technical University