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COMPLEX SYSTEMS FUNCTIONING QUALITY ANALYSIS BY CRITERIA MODELS

The paper presents the method, based upon the combination of the theory of Markov processes and principles of the criteria simulation, which allows to choose the best variant of functioning the underreserach system as for one criteria without the determination of the absolute (numerical) value of the output effect. This method allows also to consider the reliability of the system during the decision making process as for its optimization.

Keywords: *functioning quality, Markov process, criteria model.*

Introduction

Development of the modern science and technique as well as the modern economic conditions initiate the new tasks in the sphere of controlling over the electric energy systems (EES). The increasing abilities of the calculating and microprocessing equipment allow to automate the optimal controlling over regimes [1,2], the objective of which is to improve the reliability of electric power supply and reduction of its losses during its producing, transportation and distribution.

Nowadays the controlling systems within the on-line scale are widely used in the sphere of electric energy [3]. The system of technological process controlling in EES is known as the automated system of the dispatcher control (ASDC). It is responsible for the monitoring and controlling over the electric energy objects of generating, transporting and distributive companies.

ASDC belongs to the complex systems, that is, it can continue operating with some elements disabled, but with the reduced efficiency, that is, to be in several operating modes. This peculiarity of the automated controlling system requires the determination of functioning quality as the factor for evaluating its operating efficiency, which directly influences the level of the reached optimal regime of the object controlling (electric energy system) [4]. Functioning quality is the level of system adaptability to the performing of its several functions. The quantity factor, which shows the level of the systems' usefulness for the consumer, is called the indicator or the functioning quality criteria. It usually considers the systems reliability as one of the factor, which influences the result of the task to be solved [4,5].

With this in view, the development of the mathematical methods of ASDC researches with the distributed architecture is an important task. The objective of the paper is to develop the mathematical model of the systems' functioning quality of the optimal controlling over the state of the dynamic systems as EES, to improve their operation.

Mathematical model of quality of the systems' functioning

The system, which is under consideration, can acquire different states during its operation. These states are operating, but differ by quality of performing of their functions. The changing of the state is stipulated by the change in the reliability level of the systems' elements. In such a case, the optimal criteria when comparing the systems' functioning variants is its maximum remaining in the state, when its parameters are within the allowed values. The change of the systems' state may be demonstrated with the help of graphs, the example of which is shown on fig.1. The graph (fig.1) helps build the system of differential equations by Kholmogorov [6]. Assuming the non-consideration of the dynamics of the transitional processes between the separate states ($\frac{dp_i}{dt} = 0$), the system of the differential equations will look like:

$$\left.\begin{array}{l}\sum_{i=1}^{m}\nu_{ji}p_i = 0,\ j=\overline{2,n} \\ \sum_{i=1}^{m}p_i = 1,\end{array}\right\}, \tag{1}$$

where pi – is vector of probability of the states of the under research system; ν_{ij} – elements of the matrix ν, which is the matrix of transition intensively from one state into the other; m – the number of possible states of the under research system; n – number of the directions which are out of the operating state 1 (see fig.1).

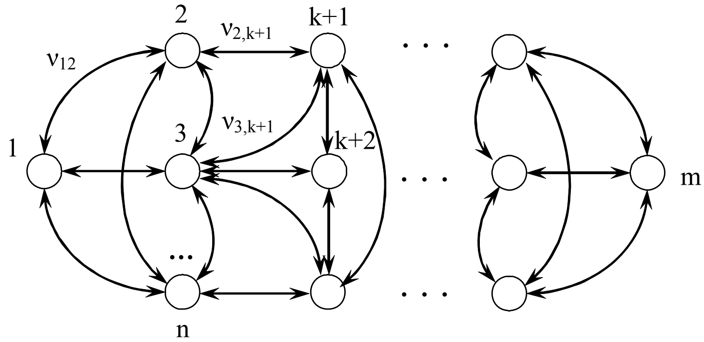

Fig. 1. Graph of systems' state transition

To determine the probabilities of the operating conditions and to evaluate the functioning quality of the under research system it is necessary to solve the algebraic equation system (1), which is written as:

$$\mathbf{v}\cdot\mathbf{p}=\mathbf{b}. \tag{2}$$

In the criteria programming the equation system of orthogonal and standardization may be written as [7]

$$\boldsymbol{\alpha}\cdot\boldsymbol{\pi}=\mathbf{b}, \tag{3}$$

where α – the matrix of indexes; $\boldsymbol{\pi}$ – vector of similarity criteria.

Having analyzed the equation systems (2) and (3), it may by noted that the matrix of factors $\mathbf{v}$ in the equation system (2) is analogical to the matrix of regularity α of the system of the equations (3), which is applied to the similarity theory [7,8,9], and the vector **p**, the components of which are the weighing factors of the state of the under research process, answer the vector of similarity criteria $\boldsymbol{\pi}$, the elements of which are measureless correlations of the systems parameters even in cases when they are determined by the method of integral analogues, are ponderable factors of target functions components (standardized to the one) [7]. Consequently, it is possible to draw a parallel between the system of equation (2) and (3).

To prove the analogy (one of the similarity types) between the system of equations of orthogonally and the system of equations by Kholmogorov, we use the theorems of theory and similarity. To do this, we build the multinomial from the matrixes α and $\mathbf{v}$.

If we use the interpolation multinomial [10], the matrix $\boldsymbol{\alpha}$ of the orthogonally system equation (2) of the criterial programming and the matrix of transition $\mathbf{v}$ of the equation system (4) may be brought to the matrix multinomial. Let us use for this purpose the exponential function $f(z)=e^{zt}$. If the minimal multinomial (in this case this is the characteristical multinomial $\Delta(z)$) consists of linear multipliers $(z-z_k)$, than it is sufficient to determine the function $f(z)$ in the characteristic points $z_1, z_2, \ldots, z_m$. The system of equations for the factors of interpolation multinomial looks like:

$$f(z_k)=a_0+a_1z_k+\ldots+a_{m-1}z_k^{m-1} \tag{4}$$

or in the matrix form

$$\begin{bmatrix} f(z_1) \\ f(z_2) \\ \cdots \\ f(z_m) \end{bmatrix} = \begin{bmatrix} 1\ z_1\ z_1^2\ \ldots\ z_1^{m-1} \\ 1\ z_2\ z_2^2\ \ldots\ z_2^{m-1} \\ \ldots\ldots\ \ldots\ldots\ \ldots \\ 1\ z_m\ z_m^2\ \ldots\ z_m^{m-1} \end{bmatrix} \cdot \begin{bmatrix} a_0 \\ a_1 \\ \cdots \\ a_{m-1} \end{bmatrix}$$

Having solved this system as for $a_0, a_1, \ldots, a_{m-1}$, we get

$$f(A) = \sum_{i=0}^{m-1} a_i A^i \ .$$

Consequently, the matrix $\boldsymbol{\alpha}$ will look like the multinomial of the following type:

$$f(\boldsymbol{\alpha}) = \sum_{i=0}^{m-1} a_i \boldsymbol{\alpha}^i \ .(5)$$

And the matrix $\mathbf{v}$:

$$f(\mathbf{v}) = \sum_{i=0}^{m-1} a_i \mathbf{v}^i \ . \tag{6}$$

Having made such a transformation we may use all the peculiarities of the scalar multinomial, including the results of the similarity theory.

It is known [11] that for the determination of the similarity between the original and the model instead of the conditions:

$$\pi_i = \frac{a_i \prod_{j=1}^{n} u_j^{\alpha_{ji}}}{f} = \mathrm{idem}, \tag{7}$$

the equivalent expressions may be used

$$\mu_i = \frac{\mu_{a_i} \prod_{j=1}^{n} \mu_{u_j}^{\alpha_{ji}}}{\mu_f}, \tag{8}$$

where π_i − similarity criteria, determined by the integral analogues method; μ_i − indicators of the similarity, determined by the scales of the corresponding factors and models parameters.

Using these conditions allow to prove the similarity of matrix multinomial and matrixes, corresponding to them.

For the matrix multinomials (5) and (6), the condition (8) may be written:

$$\frac{\mu_{a_1}}{\mu_f} = 1; \ \frac{\mu_{a_2} \mu_{\alpha/\nu}}{\mu_f} = 1; \ \frac{\mu_{a_3} \mu_{\alpha/\nu}}{\mu_f} = 1 \text{ etc.,}$$

where $\mu_{a_i} = \dfrac{a_{i\alpha}}{a_{i\nu}}$; $\mu_{\alpha/\nu} = \boldsymbol{\alpha} \cdot \mathbf{v}^{-1}$; $\mu_f = \dfrac{e^{|\boldsymbol{\alpha}|t}}{e^{|\mathbf{v}|t}}$.

The theory of matrixes contains the section of matrix transformations [10]. According to it, the equivalent transformation may be considered as the transition to the new coordinate bases for the vector x and y, that is $x' = Q^{-1}x$ and $y' = Py$. That is, the transformation $\widetilde{A} = PAQ$ answers the independent transformation of coordinates, determined by the matrixes Q^{-1} and P (nonspecial squared matrixes).

If the vectors x and y are transferred to the one coordinate bases, we may write $P = Q^{-1}$. That is, we switch over to similarity transformation $\widetilde{A} = Q^{-1}AQ$. The important peculiarity of similarity transformation is that the matrix determinant is invariant as for this transformation:

$$\det \widetilde{A} = \det A .$$

Consequently, such a transformation does not change the own matrix values, which allows to write:

$$\det[zE - \widetilde{A}] = \det[zE - A] .$$

The result of solution of the equation system (4) for the matrixes $\widetilde{A}$ and A will be the same.

The role of the transformational matrix Q belongs to the modal matrix H [10], that is, $\widetilde{A} = H^{-1}AH$. It may be determined as the aggregate of columns $\mathrm{h}^{(i)}$ which are the solution to the homogeneous equations:

$$(z_i E - A)h^{(i)} = 0 \quad i = \overline{1,n} , \tag{9}$$

where n – rank of the A matrix.

Building of the matrix α and ν allows to find the matrix H, which would meet the system of homogenous equations (9). Consequently, $\mu_{a_i} = \dfrac{a_{i\alpha}}{a_{i\nu}} = 1$; $\mu_{\alpha/\nu} = \alpha \cdot \nu^{-1} = 1$; $\mu_f = \dfrac{e^{\left|\alpha^t\right|}}{e^{\left|\nu^t\right|}} = 1$, And the conditions (8) come true which prove the matrix similarity of the orthogonal criteria programming and transition of the Kholmogorov equation system.

Similarity of the simulation of the Markov processes and criteria simulation allows to apply the principles of criteria programming to the equation system (2).

System of the equations (3) in the criteria programming answers the direct task [7]

$$\min \left\{ y(x) = \sum_{i=1}^{m} a_i \prod_{j=1}^{n} x_j^{\alpha_{ji}} \right\}, \tag{10}$$

where *y(x)* – some generalized technique and economic factor, which characterizes the process under research, x_j – system applied parameters, the values of which are being optimized; a_i, α_{ji} – stable factors values of which are determined by the peculiarities of the system; m – number of members of the target function; n – number of variables.

Following the analogue, the target function of the criteria program for the equation system (2) will be written

$$\min \left\{ f(x) = \sum_{i=1}^{m} c_i \prod_{j=1}^{n} x_j^{\nu_{ji}} \right\}, \tag{11}$$

where *f(x)* – the function of failures, which reflects the influence of the elements of the system on the ability to perform the task set; c_i – constant factors (in the tasks of the considered type c_i=1); x_j – independent parameters, which characterize the state of the system.

Thus, there had been obtained the dependence (11) instead of the equation system, which reflect the functioning of the underresearch system.

Further using the analogies to the direct task of PCP allows to write the double task [7]. For the equation (11) the double task will look like:

$$d(P) = \prod_{i=1}^{m} \left(\frac{c_i}{P_i} \right)^{P_i}, \tag{12}$$

where P_i – similarity criteria, which is the probability of the systems remaining in the state i.

Consequently, due to the used similarity, instead of the equation system (1), there had been obtained the two functional dependences (11) and (12), which allow to evaluate the peculiarities of the system, like failure and functioning quality.

Criteria simulation of the qualities of the system's functioning

Function (11) is easy to use if transferred into the criteria type by devision by basis. The value of the function (11) is taken as the basis for the system on the moment of its putting into operation. Criteria failure function looks like:

$$f(x_*) = \sum_{i=1}^{m} P_i \prod_{j=1}^{n} x_{*j}^{\nu_{ji}} . \tag{13}$$

Fig. 2 shows the function in graphic. Similarity criteria P_i is determined from the equation system (1). Finding the relative value of influencing factors x_{*j} requires to determine the opposite matrix $\tilde{\nu}$, to the transponented matrix ν^t [8], the last column in which consists of -1. Then, the correlation between the variable values x_j (after the last system testing) and x_{0j} (after the testing before putting into operation) are to be determined.

$$x_{*j} = \frac{\prod_{i=1}^{m} P_i^{\tilde{\nu}_{ji}}}{\prod_{i=1}^{m} P_{0_i}^{\tilde{\nu}_{0ji}}} .$$

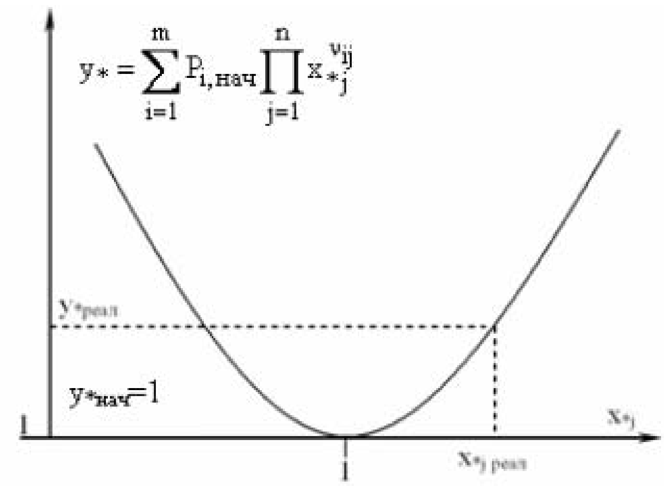

Fig. 2. Criteria failure function

During the operation, the reliability characteristics of the system's elements change, which, in turn, lead to the change of the system's state as a whole. That is, the change of the possibility of the system's remaining in this or that state takes place. The verifying of the possibilities is executed by the Bayes theorem:

$$P_{i,реал} = P_{i,нач} \frac{p(\bar{x}_1,...,\bar{x}_n / s_i)}{\sum_{j=1}^{m} P_{j,поч} p(\bar{x}_1,...,\bar{x}_n / s_j)},$$

where $P_{i,реал}$ - apposteor possibilities; $P_{i,нач}$ – possibility of apriority, which are determined from the equation system (1); $p(\bar{x}_1,...,\bar{x}_n / s_i)$ – density of the possibilities of the state determination s_i for the determined indications (parameters) of the system $\bar{x}$, if the indicators are continuous, and the distribution of possibilities, if the indicators are discrete.

Function of failures may be used both for the evaluation of the real state of the system and for specifying the insensitivity of the zone, considering the level of reliability, during the realization of the controlling law, for instance in the complex systems like the electric energy one.

To evaluate the quality of the system's functioning and to develop the strategy as for the restoration of the system's elements, we use function (12).

For the convenience in using functions of quality functioning, it should look as follows:

$$d_*(P_*) = \prod_{i=1}^{m} \frac{P_i^{P_i}}{P_{0i}^{P_{0i}}} .$$

Fig. 3 shows the function in graphic.

Considering the task of works planning on the quality improvement of the system's functioning stipulates for the consideration of the yeargraph of the qualities functions (see fig. 4). The changes of reliability and reparability are set along the axes. These parameters are conditional. Consequently, the margin of the yeargraph is divided into five sections as for the reliability and reparability. These sections are also conditional.

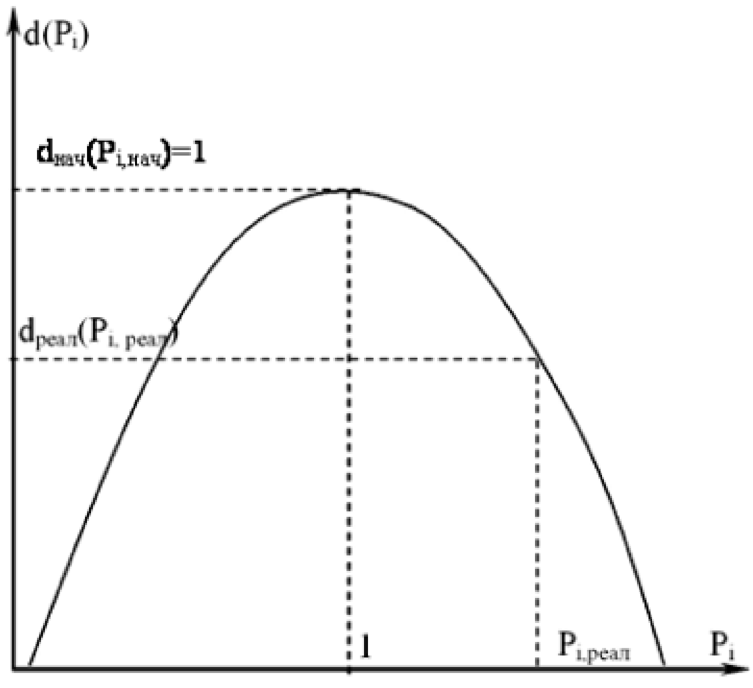

Fig. 3. Function of quality functioning

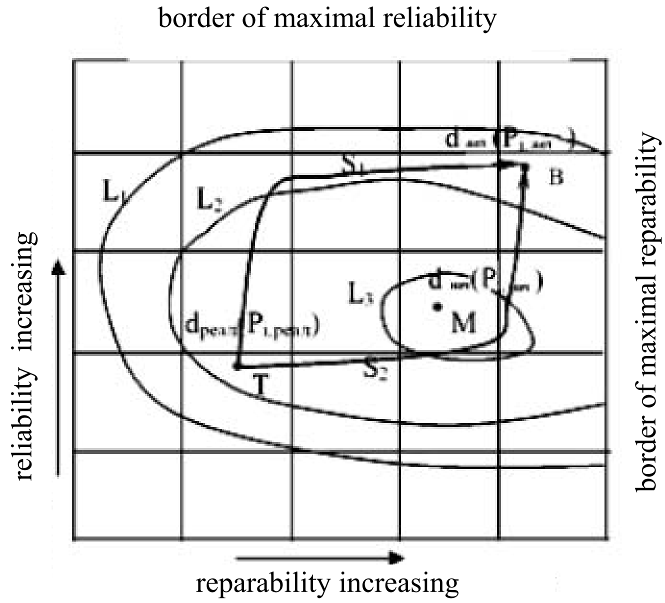

Fig. 4. Distribution of the requirements to the elements' reliability

On fig.4 the point *T* – the achieved level of the system's reliability; *M* – point in which the function of the quality functioning reaches its maximum value; *B* – point, which answers the desired level of the system's reliability; curves L_1, L_2, L_3 – lines of the yeargraphs of the function $d(P_i)$, that is the geometrical place of the points, in which the function preserves its value. Each line of the yeargraph L_i answers the definite level of the quality functioning.

Each plan of the activities, directed on the changing of the indicators of the reliability of the elements of the system with the aim of improving the quality of its functioning, is the trajectory S_i. For instance, on fig. 4 the trajectory S_1 stipulates, first of all, for the realization of activities, directed on the reliability improvement, and later- the reparability. On the other hand, the trajectory S_2, first of all, improves the characteristics of reparability. If the function of recourses is known, that is the function of costs of the specific recourses for the achievement of the desired quality level of functioning, it then allows to determine the costs for the trajectory execution.

Conclusions

Using such an approach allows to solve the following tasks:

Determine the sphere of values of the reliability and reparability of the elements, which ensure the set level of the quality of the system's functioning (spheres are limited by the lines of the yeargraphs).

Determine the minimum requirements to the reparability and reliability of the elements, which ensure the set level of the quality of the functioning system.

Evaluate the degree of influence of the elements' reliability factors on the quality of the system's functioning.

Determine the strategy of the improvement of the quality of the system's functioning to the set level under the conditions of natural aging of the elements and limited recourses.

Evaluate the necessity and determine the sequence of activities on elements' upgrading.

Evaluate the necessity and to determine the content of the activities for the reconstruction of the system as for the criteria of the system's functioning.

Evaluate the necessity and determine the sequence of the activities for the continuation of the residual recourse of the elements.

The fact that the result obtained is close to the initial state is a substantial one while solving the similar tasks. It allows to reduce the number of the calculations for the comparing the states.

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