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ANALYTICAL RESEARCH OF RESIDUAL RESOURCE MODEL DURING MATERIAL DIAGNOSTICS

There has been analysed and analytically researched the model of a residual resource during two-stage deformation. There had been determined the mathematically admissible changes of parameters α_{12}, *n*, I_{12} *models which are included in its criterion correlation and with which it acquires only valid values.*

Keywords*: tensor model; accumulation of damages; two-stage deformation; function of damaged; a residual resource; analytical research; non-stationary deformation*

Statement of a problem and the analysis of latest researches

A two-stage deformation is the important class of deformation for two principal reasons. First, this class is the simplest representative of unstationary deformation characterized by numerous brightly expressed effects in dependences of ultimate strain. Second, in a number of cases the mode of deformation in billets during the processing under pressure may be considered, with certain approximation, exactly as a two-stage deformation. In such a case the analysis of billet serviceableness to accept the given technological operation is simplified.

The purpose of the given work is complete analytical research of model of a residual resource which comes up from the tensor-nonlinear theory with power function of damaged during the two-stage deformation.

Consequently, the research task consists in determination the mathematically admissible limits of parameters' change α_{12}, I_{12}, *n* which are included in criterion correlation(1).

The two-stage deformation was examined in many works, in particular in [1], but to this time the results of complete research of criterion correlation proceeding from a tensor-nonlinear model with the power function of damaged are not described in literature.

As for the two-stage deformation from a tensor-nonlinear model [1] we get the criterial correlation:

$$\psi_{*2} = \left[\psi_1^n \cdot \left(\alpha_{12}^n - I_{12}\right) + \sqrt{\psi_1^{2 \cdot n} \cdot \left(I_{12}^2 - 1\right) + 1}\right]^{1/n} - \psi_1 \cdot \alpha_{12}, \tag{1}$$

where $\psi_{*2} = \dfrac{\varepsilon_*^{(2)}}{\varepsilon_{*2}}$ − a residual resource of ultimate strain during the second stage of deformation; $\psi_1 = \dfrac{\varepsilon_u^{(1)}}{\varepsilon_{*1}}$ − the used resource of ultimate strain at the first stage of deformation; $\alpha_{12} = \dfrac{\varepsilon_{*1}}{\varepsilon_{*2}}$ − parameter which characterizes the sequence of deformation conditions; $I_{12} = k_{12} \cdot a^{(1)} \cdot a^{(2)} + I_1 \cdot a^{(1)} \cdot b^{(2)} + I_2 \cdot a^{(2)} \cdot b^{(1)} + \left(I_3 - \dfrac{1}{3}\right) \cdot b^{(1)} \cdot b^{(2)}$, $k_{12} = \beta_{ij}^{(1)} \cdot \beta_{ij}^{(2)}$ is a cosine of angle of deformation trajectory fracture of; $a^{(i)}, b^{(i)}$ − value of parameters a and b on unpaired and pair stages of deformation; I_1, I_2, I_3 − invariants of product the tensors, and $I_1 = \beta_{ij}^{(1)} \cdot \beta_{jk}^{(2)} \cdot \beta_{ki}^{(2)}$, $I_2 = \beta_{ij}^{(1)} \cdot \beta_{jk}^{(1)} \cdot \beta_{ki}^{(2)}$, $I_3 = \beta_{ij}^{(1)} \cdot \beta_{jk}^{(1)} \cdot \beta_{kl}^{(2)} \cdot \beta_{li}^{(2)}$; $\varepsilon_*^{(2)}$ − value of residual deformation to destruction on the second stage of deformation; $\varepsilon_u^{(1)}$ − value of the accumulated deformation at the first stage of deformation; $\varepsilon_{*1} = \varepsilon_{*c}\left(\eta^{(1)}, D^{(1)}\right)$, $\varepsilon_{*2} = \varepsilon_{*c}\left(\eta^{(2)}, D^{(2)}\right)$ − ultimate strain to destruction during the stationary deformation (the diagram of plasticity); n − parameter which characterizes properties of a material and a mode of loading.

Papers [2, 3, 4] analyze different invariants to be used as arguments of a surface of ultimate strain.

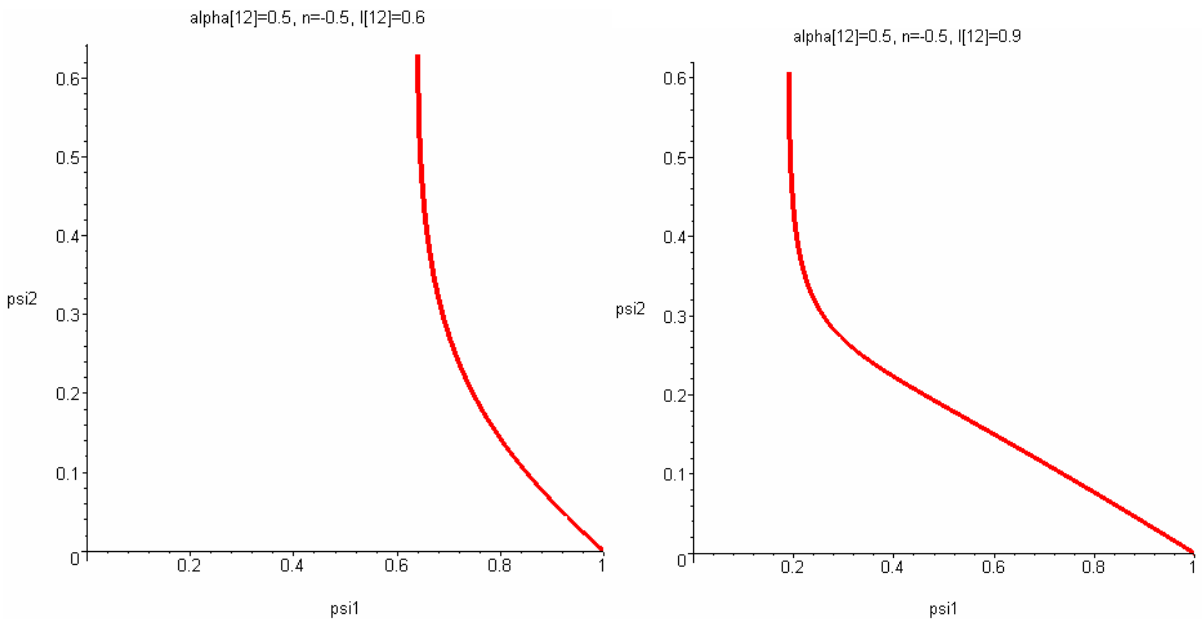
The basic part

Let's determine mathematically admissible limits of change of parameters α_{12}, I_{12}, n, which are part criterion correlation (1).

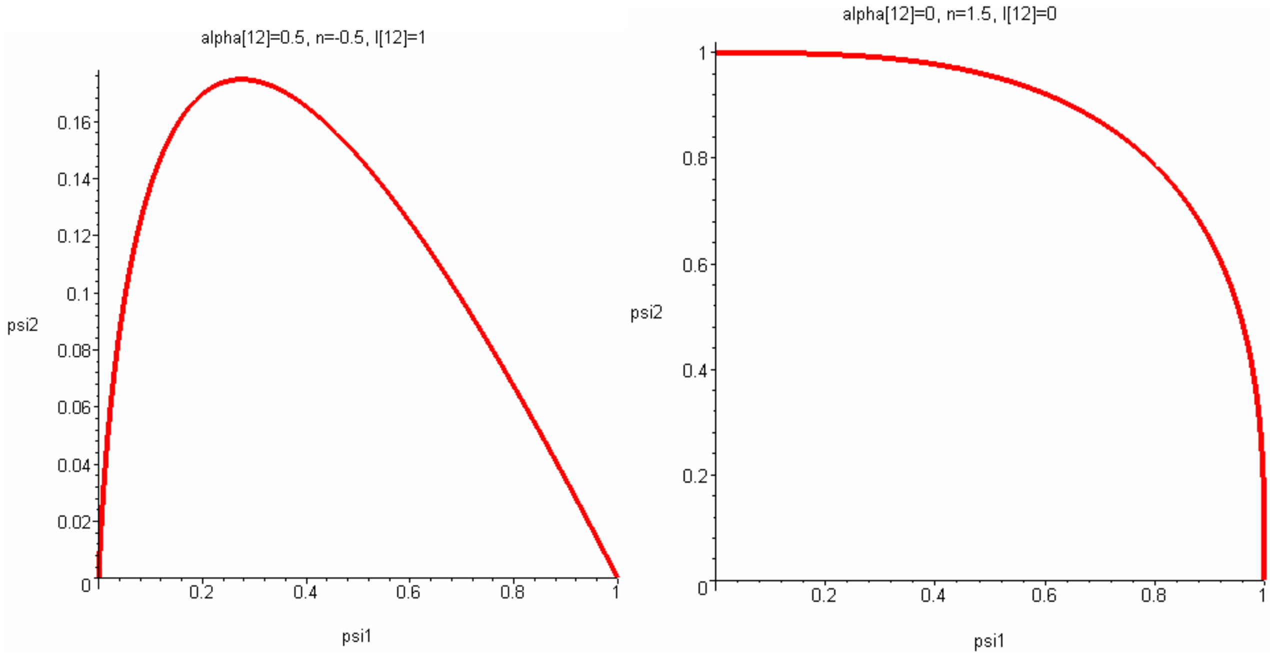
Following the determination $\psi_1 \in [0;1]$, during the a two-stage deformation: $\psi_1 \in [0;1)$. Taking into account that the function is certain in the sphere of real numbers, if a denominator does not change into a zero and the basis of the power function is more than a zero, we draw a conclusion, that for criteria correlation (1), the parameter of n does not equal a zero, and also $\alpha_{12} > 0$. Consequently, limitations which are imposed on a variable ψ_1 , parameters of n and α_{12} shall be expressed by the set of inequalities:

$$\begin{cases} n \neq 0 \\ 0 \leq \psi_1 \leq 1\,. \\ \alpha_{12} \geq 0 \end{cases} \qquad (2)$$

During the numerical research of correlations (1) with parameters, which satisfy inequalities (2), there had been found the areas, when a function $\psi_{*2} = \psi_{*2}(\psi_1)$ in the sphere of real numbers is indefinite. So when the values of parameters $\alpha_{12} = 0{,}5$; n = -0,5; $I_{12} = 0{,}6$ a function is certain on an interval $\psi_1 \in [0{,}64;1)$. During the increase of value of parameter I_{12} to 0,9; the range of definition of function is increases and is determined by an interval $\psi_1 \in [0{,}19;1)$ (see fig. 1).



a) b)



c) d)

Fig. 1. Curve of dependences between resources of plasticity at two-stage deformations, when α_{12} = 0,5; n= -0,5; a I_{12} :
a) I_{12} = 0,6; b) I_{12} = 0,9; c) I_{12} = 1; d) I_{12} = 0

Proceeding from the obtained results of numerical research of criterion correlation (1), it is possible to draw conclusion, that the above limitations of parameters (2) are insufficient. To find the additional restrictions of parameters of mathematical model it is necessary to research it analytically.

First of all it is necessary to research the expression:

$$\sqrt{\psi_1^{2 \cdot n} \cdot \left(I_{12}^2 - 1\right) + 1}\,. \tag{3}$$

By determining the square root, the radicand should be not less than zero so that the value of expression would get the valid values. That is:

$$\psi_1^{2 \cdot n} \cdot \left(I_{12}^2 - 1\right) + 1 \geq 0\,. \tag{4}$$

Whence:

$$I_{12}^2 \geq 1 - \frac{1}{\psi_1^{2 \cdot n}}\,. \tag{5}$$

Let's consider a case, when n > 0. Then, when $0 \leq \psi_1 \leq 1$ the right part of an inequality (5) is within $-\infty < 1 - \frac{1}{\psi_1^{2 \cdot n}} \leq 0$, that is the non positive number. I_{12}^2 - is the number positive, hence, the inequality (5) when n > 0 is holds for $\forall I_{12} \in \left(-\infty;\infty\right)$. Therefore when n > 0 no additional restrictions on the parameters are imposed.

Let's consider an inequality (5) when n < 0. Then changing ψ_1 from 0 up to 1 the expression $1 - \frac{1}{\psi_1^{2 \cdot n}}$ monotonously descends from 1 up to 0. That is, the execution of an inequality (5) requires $\left|I_{12}\right| \geq 1$. If $\left|I_{12}\right| < 1$, always for $\psi_1 \in [0;1)$ on the curve of the function $\psi_{*2} = \psi_{*2}(\psi_1)$ will be always observed the area, with which the function is indefinite. According to an inequality (5) the range of definition of function $\psi_{*2} = \psi_{*2}(\psi_1)$ is determined by an inequality:

$$\sqrt[2n]{\frac{1}{1-I_{12}^{2}}} \le \psi_1 \le 1\,. \tag{6}$$

The area determined by an inequality (6) completely coincides with the values ψ_1 on fig. 1, for which ψ_{*2}, that is calculated by the criterion correlation (1), is within the area of real numbers.

Now let us analytically research the expression of model, exponentiated 1/n. This expression must be less than zero. That is:

$$\psi_1^{n}\cdot\left(\alpha_{12}^{n}-I_{12}\right)+\sqrt{\psi_1^{2\cdot n}\cdot\left(I_{12}^{2}-1\right)+1}\ge 0\,. \tag{7}$$

From expression (7) we get the irrational inequality:

$$\sqrt{\psi_1^{2\cdot n}\cdot\left(I_{12}^{2}-1\right)+1}\ge \psi_1^{n}\cdot\left(I_{12}-\alpha_{12}^{n}\right). \tag{8}$$

Irrational inequality (8) shall be factorized on the aggregate of sets of inequalities, which look like:

$$\left[\begin{array}{ll}\begin{cases}\psi_1^{2\cdot n}\left(I_{12}-1\right)+1\ge 0\\ I_{12}-\alpha_{12}^{n}\ge 0\\ I_{12}\ge\dfrac{\psi_1^{2n}+\psi_1^{2n}\cdot\alpha_{12}^{2n}-1}{2\cdot\psi_1^{2n}\cdot\alpha_{12}^{n}}\end{cases} ; & (9)\\ \begin{cases}\psi_1^{2\cdot n}\left(I_{12}-1\right)+1\ge 0\\ I_{12}-\alpha_{12}^{n}<0\end{cases} ; & (10)\end{array}\right. \tag{11}$$

If the system of inequalities (9) would not include the third inequality, the set of systems of inequalities (11) would have the solution, coinciding with the decision of the first inequality of systems (9) and (10). It is explained by the fact that the second inequity of the system (9) and the second inequality of system (10) supplement each other, that is in the aggregate they create restrictions on neither parameter. Let us determine whether the third inequality of system (9) imposes the new restrictions on parameters α_{12}, I_{12}, *n* if two previous inequalities are held.

Proceeding from the above researches, the If there was not the third inequality in the set of inequalities (9), the aggregate of sets of inequalities (11) would have a decision which coincides with the decision of the first inequality (9) and (10) systems. It is explained, that the second inequality of the system (9) and second inequality of the system (10) complement each other, that at aggregate does not create limits on not a single parameter. Let's define, whether imposes the third inequality of system (9) new restrictions on parameters α_{12}, I_{12}, *n*, if two previous inequalities are carried out.

Going out from the researches conducted higher, the first inequality of set (9) will be executed at n > 0 or at n <0 and $\left|I_{12}\right|\ge 1$.

At first will explore the set of inequalities:

$$\begin{cases} I_{12}\ge\alpha_{12}^{n}\\ I_{12}\ge\dfrac{\psi_1^{2n}+\psi_1^{2n}\cdot\alpha_{12}^{n}-1}{2\cdot\psi_1^{2n}\cdot\alpha_{12}^{n}}, \end{cases} \tag{12}$$

at $n>0$.

Let's designate the right part of the second inequality of system (12):

$$f(\psi_1) = \frac{\psi_1^{2n} + \psi_1^{2n} \cdot \alpha_{12}^n - 1}{2 \cdot \psi_1^{2n} \cdot \alpha_{12}^n}. \tag{13}$$

Will explore as will move itself $f(\psi_1)$ at change ψ_1 from 0 to 1. Will define or there are extremism's on this interval of this function:

$$\left(\frac{\psi_1^{2n} + \psi_1^{2n} \cdot \alpha_{12}^n - 1}{2 \cdot \psi_1^{2n} \cdot \alpha_{12}^n} \right)' = \frac{n}{\alpha_{12}^n \cdot \psi_1^{2 \cdot n+1}} > 0. \tag{14}$$

Hence, are not present the extremism's at $\psi_1 \in [0;1)$, function is monotonously growing, therefore the maximal value reaches at $\psi_1 = 1$:

$$f_{\max} = f(1) = \frac{\alpha_{12}^n}{2}.$$

Considering, that $\alpha_{12} > 0$:

$$\alpha_{12}^n > \frac{\alpha_{12}^n}{2},$$

and it means that after the condition of implementation of the first inequality of the system (12), the second inequality is automatically executed, that at $n > 0$ the second inequality does not impose additional limits on none of parameters.

Will explore the set of inequalities (12) at $n < 0$ and $|I_{12}| \geq 1$. In view of (14) a function $f(\psi_1)$ is droningly descending, and, consequently, achieves the maximal value at $\psi_1 = 0$:

$$f_{\max} = \lim_{\psi_1 \to 0} \frac{\psi_1^{2n} + \psi_1^{2n} \cdot \alpha_{12}^n - 1}{2 \cdot \psi_1^{2n} \cdot \alpha_{12}^n} = \frac{\alpha_{12}^n}{2}.$$

In view of the previous researches we do a conclusion, that at $n < 0$; $|I_{12}| \geq 1$ any additional restrictions of parameters the criterion of correlation (1), the second inequality of set (12) does not create.

The analysis of results which are received from inequality (4) and aggregate of sets of inequalities (11) allows to make generalization, that mathematically legitimate values of parameters of criterion correlation (1) determined an aggregate:

$$\left[\begin{array}{l} \begin{cases} \alpha_{12} > 0; \\ n > 0; \\ -\infty < I_{12} < +\infty; \end{cases} \\ \begin{cases} \alpha_{12} > 0; \\ n < 0; \\ |I_{12}| > 1. \end{cases} \end{array} \right. \tag{15}$$

The corresponding generalization completely is coordinated with results of numerical research of the proper criterion the correlation (see fig. 1).

Conclusion

Laws in change of a residual resource and reception of restriction (15) on changing of parameters of criterion correlation (1) will allow to simplify diagnostics of a material at a two-stage deformation.

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