

**O. V. Bereziuk, Cand. Sc. (Eng.), Assistant Professor**

## **ANALYTICAL STUDY OF THE IMPROVED MATHEMATICAL MODEL OF THE VIBRATION DRIVE FOR SOLID HOUSEHOLD WASTE ADDITIONAL COMPACTION IN THE DUST CART**

*On the base of the analysis, carried out, of the numerical studies of the complete. improved mathematical model of the vibrational compaction of the solid waste in the dust cart , using the pressure pulse generator of the relay differential action and taking into account the density and relative humidity of the dehydrated and pre- compacted solid waste the simplified improved mathematical model was suggested . In the process of comparison of the results, obtained using the complete and simplified improved mathematical models of the vibrational additional compaction of the solid household waste in the dust cart , in particular, the degree of their compaction, the error was 4.35%, this value is acceptable for the realization of the previous design calculations of the basic parameters of the drive. For the analytical study of the simplified mathematical model the working cycle of the improved vibrational drive for the additional compaction of the solid household waste in the dust cart is presented by six basic phases, each of them is described by the corresponding system of the differential equations with the initial and boundary conditions. For the analytical study of the simplified, improved mathematical model of the vibrational drive for the additional compaction of the solid household waste in the dust cart such methods are used: solution of the system of ordinary linear differential equations, applying the Laplace transformation, linearization of the non-linearities, decomposition of the expression into the common fractions, solution of the incomplete cubic equation, applying Cardano formulas, computer simulation. Analytical study of the simplified mathematical model of the vibrational drive for the additional compaction of the solid household waste in the dust cart enabled to obtain the analytical interdependences of its basic parameters and dependences of the frequency and amplitude on the basic parameters of the given drive with the error not exceeding 5.34%, they can be used for the realization of the preliminary design calculations of its parameters as one of the components for the solution of the problem, dealing with the development of the scientific-engineering fundamentals of the design of the efficient tools of the machines for the collection and primary processing of the solid household waste.*

**Key word:** *dust cart, compaction, solid household waste, mathematical model.*

### **Introduction**

Every year, approximately 53 million tons of the solid household waste (SHW) are generated in Ukraine, threatening the health of the population and environment [1]. According to the preliminary calculation, more than 45 thousand tons of fuel per year are needed for the transportation by the dust cart to the sites of treatment at minimal distance of 30 km, this distance corresponds to the size of the sanitary protection zone. Wear of the dust carts pool of the municipal enterprises of Ukraine is approximately 70 % [2].

### **Problem set up**

According to the Decree of the Cabinet of Ministers of Ukraine № 265 [3] one of the priority directions of the waste management in Ukraine is the application of modern efficient dust carts, that is why the analytical study of the improved mathematical model of the vibration drive for the solid waste compaction in the dust cart for the obtaining of the interrelations of its basic parameters is important scientific-engineering problem as one of the components for the solution of the problem, aimed at creation of scientific-engineering fundamentals for the design of the efficient tools of the machines for collection and primary treatment of the solid municipal waste.

### **Analysis of the recent research and publications**

The paper [4], contains the analytical study of the simplified mathematical model of the overturning

drive of the waste container in the dust cart. Research [5] contains the analytical study of the mathematical model of the group hydraulic drive with serial connection of the hydraulic motors of the attachable sweeping equipment. In the paper [6] mathematical model of the vibration drive for SHW compaction is studied analytically, the given model does not take into account the relative humidity of the waste and characteristic features of the pressure pulse generator of the relay differential action, protected by the patents of Ukraine, the latest patent is 92720 U [7]. In [8], the mathematical model of the valve-pulsator operation for the hydraulic drives of the vibration mining machines is studied analytically. In the research [9] the improved mathematical model of the vibration drive for the compaction of SHW in the dust cart is suggested, its numerical study is carried out.

### Aim and tasks of the research

**The aim of this research** is the analytical study of the improved mathematical model of the vibration drive for SHW compaction in the dust cart for the obtaining the interdependences of its basic parameters and dependences of the frequency and the amplitude on the basic parameters of the given drive.

### Methods and materials

For the analytical study of the simplified improved mathematical model of the vibration drive for SHW compaction such methods are used: solution of the system of linear differential equations by means of Laplace transform, linearization of nonlinearities, decomposition of the expression into simpler fractions, solution on the incomplete cubic equation by Cardano formula, computer modeling.

### Results of the analytical study

Analysis of the studies, carried out, of the complete improved mathematical model of the vibration compaction of SHW [9] showed that  $p_1 \approx p_2 \approx p_{12}$ , and the impact of the pressure in the drain pipelines, viscous friction forces, permanent component of dry friction forces and the weight of pressure pulse generator of the relay differential action gate on the operation of the vibrational drive is of negligible importance. That is why, the simplified improved model of the vibrational drive for SHW compaction, using pressure pulse generator of relay differential action (PPGRDA) has the form:

$$\begin{cases} Q_H = \dot{x}S_{II1} + \sigma p_{12} + KW_{12}\dot{p}_{12} + \dot{y}\pi[d_3^2 - \mathbf{1}(h_{hoc} - y)d_1^2]/4 + \mathbf{1}(y - h_n)\mu\pi d_3(y - h_n)\sqrt{2p_{12}/\rho_{pp}} + \\ + \mathbf{1}(y)\mu\pi d_4^2\sqrt{2p_{12}/\rho_{pp}}/4 + \mathbf{1}(y - h_e)\mu\pi d_6^2\sqrt{2p_{12}/\rho_{pp}}/4; \end{cases} \quad (1)$$

$$\begin{cases} p_{12}S_{II1} = m_p\ddot{x} + (c_1e^{6,094Q_H t/(x_{max}S_{II1})} + c_0)S_{III1}; \end{cases} \quad (2)$$

$$\begin{cases} p_{12}\pi\{\mathbf{1}(y)(d_3^2 - d_2^2) + [d_2^2 - \mathbf{1}(h_{hoc} - y)d_1^2]\}/4 = m_\kappa\ddot{y} + c(y + y_0), \end{cases} \quad (3)$$

where

$$c_0 = (1,356 - 1,162 \cdot 10^{13} e^{-0,07908 \rho_1} - 1,267 \cdot 10^{-31} e^{1,658 w_1}) \cdot 10^6;$$

$c_1 = (0,04669 - 5,198 \cdot 10^{11} e^{-0,07908 \rho_1}) \cdot 10^6$ ;  $\rho_1$  – density of the precompressed and dehydrated SHW, kg/m<sup>3</sup>;  $w_1$  – relative humidity of the precompressed and dehydrated SHW, % [10];  $W_{12} = W_1 + W_2$ .

The comparison of the results, obtained after application of the complete and simplified improved mathematical models of the vibrational compaction of SHW, using pressure pulse generator of relay differential action (PPGRDA), is shown in Fig. 1.

During the comparison of the results, in particular, the degree of SHW compaction, obtained using complete and simplified improved mathematical models of the vibrational hydraulic drive for SHW compaction, using PPGRDA, the error was 4.35%, it is acceptable for the realization of the

preliminary design calculations.

Analysis of the graph, shown in Fig. 1c, gives reason to believe, that the time of closing the gate of the pressure pulse generator of relay differential action (PPGRDA) approximately equals the time of opening. That is why, operation cycle of the improved vibration drive for SHW compaction consists of six basic phases:

1. Phase of the pressure increase of the working fluid (WF) in the pressure line of the hydraulic cylinder and in cavities, connected with it to the pressure  $p_{12} = (c_1 + c_0)S_{II1}/S_{II1}$ , at which compaction plate starts motion, can be described by the equation

$$Q_H = \sigma p_{12} + KW_{12} \dot{p}_{12} \tag{4}$$

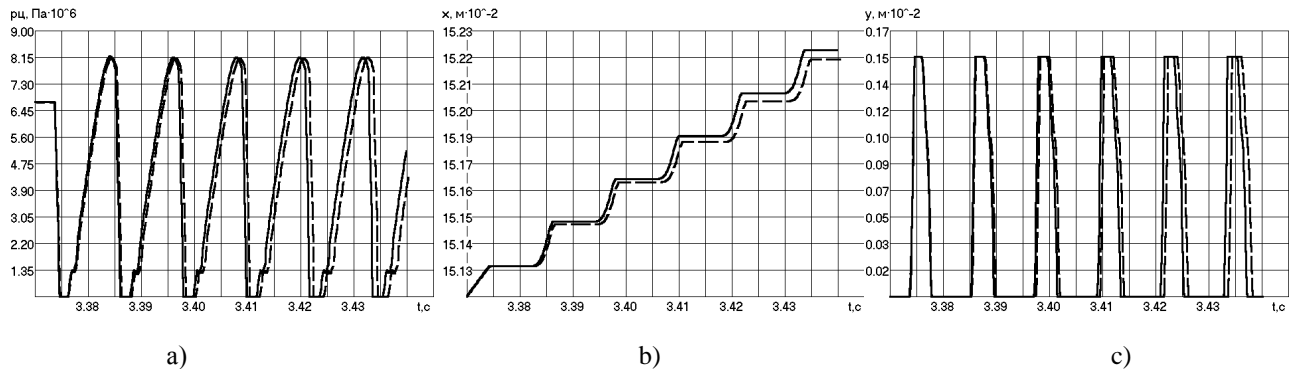


Fig. 1. Comparison of the results, obtained using complete (—) and simplified (---) improved mathematical model of the vibrational compaction of SHW: a) pressure change in the pressure cavity of the hydraulic cylinder pressing plate; b) displacement of the pressing plate; c) displacement of the gate element of the PPGRDA

Solving the equation (4) at the initial conditions  $p_{12}(0) = p_{3II}$ , we obtain

$$p_{12} = (1 - e^{-\sigma t / KW_{12}}) Q_H / \sigma + p_{3II} \tag{5}$$

where  $p_{3II}$  – drain pressure, Pa.

From the equation (5) we will find the duration of the first phase

$$t_1 = \frac{KW_{12}}{\sigma} \ln \frac{Q_H}{Q_H - (p_{12} - p_{3II})\sigma} = \frac{KW_{12}}{\sigma} \ln \frac{Q_H}{Q_H - [(c_1 + c_0)S_{II1}/S_{II1} - p_{3II}]\sigma} \tag{6}$$

2. Phase of the pressure increase in the pressure pipe of the hydraulic cylinder and cavities, connected with the pipe to the opening pressure of the pressure pulse generator of relay differential action (PPGRDA)  $p_{12} = p_{K11}$  and motion of the pressing plate can be described by the system of the equations

$$\begin{cases} Q_H = vS_{II1} + \sigma p_{12} + KW_{12} \dot{p}_{12}; \end{cases} \tag{7}$$

$$\begin{cases} p_{12}S_{II1} = m_p \dot{v} + (c_1 e^{6,094 Q_H t / (x_{max} S_{II1})} + c_0)S_{II1}, \end{cases} \tag{8}$$

where  $v = \dot{x}$  – speed of motion of SHW pressing plate, m/s.

After Laplace transformation [11] we obtain:

$$\begin{cases} Q_H / s = V(s)S_{II1} + P(s)\sigma + P(s)sKW_{12}; \end{cases} \tag{9}$$

$$\begin{cases} P(s)S_{II1} = V(s)sm_p + c_1 S_{II1} / [s - 6,094 Q_H / (x_{max} S_{II1})] + c_0 S_{II1} / s. \end{cases} \tag{10}$$

Substituting the equation (10) into the equation (9), we obtain

$$V(s) = \frac{-b_{22}s^2 + b_{12}s - b_{02}}{s(a_{32}s^3 + a_{22}s^2 + a_{12}s - a_{02})} \approx \frac{-b_{22}s^2 + b_{12}s - b_{02}}{s(a_{22}s^2 + a_{12}s - a_{02})}, \tag{11}$$

where  $a_{32} = KW_{12}m_p x_{\max} S_{II1}$ ;  $a_{22} = m_p(\sigma x_{\max} S_{II1} - 6,094Q_H KW_{12})$ ;  $a_{12} = x_{\max} S_{II1}^3 - 6,094Q_H \sigma m_p$ ;  
 $a_{02} = 6,094Q_H S_{II1}^2$ ;  $b_{22} = KW_{12} S_{II1} x_{\max} S_{II1} (c_1 + c_0)$ ;  $b_{02} = 6,094Q_H (Q_H S_{II1} - \sigma c_0 S_{II1})$   
 $b_{12} = Q_H x_{\max} S_{II1}^2 - \sigma S_{II1} x_{\max} S_{II1} (c_1 + c_0) + 6,094Q_H KW_{12} c_0 S_{II1}$ ; (12)

Applying the method of the decomposition of the expression (11) into simpler fractions we obtain

$$V(s) = A_{v2} \frac{1}{s} + \frac{B_{v2}}{a_{22}} \frac{s + a_{12}/(2a_{22})}{[s + a_{12}/(2a_{22})]^2 - (4a_{02}a_{22} + a_{12}^2)/(4a_{22}^2)} +$$

$$+ \frac{2C_{v2} - B_{v2}a_{12}/a_{22}}{\sqrt{4a_{02}a_{22} + a_{12}^2}} \frac{\sqrt{4a_{02}a_{22} + a_{12}^2}/(2a_{22})}{[s + a_{12}/(2a_{22})]^2 - (4a_{02}a_{22} + a_{12}^2)/(4a_{22}^2)},$$
 (13)

where  $A_{v1} = b_{02}/a_{02}$ ;  $B_{v1} = -b_{22} - a_{22}b_{02}/a_{02}$ ;  $C_{v1} = b_{12} - a_{12}b_{02}/a_{02}$ ;  
 (14)

We find the original of the image (13)

$$v(t) = A_{v2} + \frac{B_{v2}}{a_{22}} e^{-\frac{a_{12}}{2a_{22}}t} ch\left(\frac{\sqrt{4a_{02}a_{22} + a_{12}^2}}{2a_{22}}t\right) + \frac{2C_{v2} - B_{v2}a_{12}/a_{22}}{\sqrt{4a_{02}a_{22} + a_{12}^2}} e^{-\frac{a_{12}}{2a_{22}}t} sh\left(\frac{\sqrt{4a_{02}a_{22} + a_{12}^2}}{2a_{22}}t\right). (15)$$

Excluding small coefficients of the expression (15) and taking into consideration the accepted designations according to (12, 14), at initial conditions  $v(0) = 0$  the velocity of the pressing plate during the second phase is found by the formula

$$v_2(t) \approx [1 - e^{-S_{\delta 1}^2 t / (m_D \sigma)}] KW_{12} S_{I1} (c_1 + c_0) / (m_D \sigma). (16)$$

Having integrated the expression (16), at initial conditions  $x(0) = 0$  we find the displacement of the pressing plate during the second phase

$$x_2(t) \approx KW_{12} S_{I1} (c_1 + c_0) \{t / (m_D \sigma) + [e^{-S_{\delta 1}^2 t / (m_D \sigma)} - 1] / S_{\delta 1}^2\}. (17)$$

Solving the system of equations (9 – 10) relatively  $P(s)$ , we obtain

$$P(s) = \frac{E_{p2} - A_{p2} S_{II1}}{KW_{12}} \frac{1}{s + \sigma / (KW_{12})} - \frac{B_{p2} S_{II1}}{a_{22}} \frac{s + a_{12}/(2a_{22})}{[s + a_{12}/(2a_{22})]^2 - (4a_{02}a_{22} + a_{12}^2)/(4a_{22}^2)} +$$

$$+ \frac{(2C_{p2} - B_{p2}a_{12}/a_{22}) S_{II1}}{\sqrt{4a_{02}a_{22} + a_{12}^2}} \frac{\sqrt{4a_{02}a_{22} + a_{12}^2}/(2a_{22})}{[s + a_{12}/(2a_{22})]^2 - (4a_{02}a_{22} + a_{12}^2)/(4a_{22}^2)} + D_{p2} \frac{1}{s},$$
 (18)

where  $A_{p2} = \frac{KW_{12}(B_{v2}\sigma - KW_{12}C_{v2})}{a_{12}KW_{12}\sigma - a_{22}\sigma^2 + a_{02}K^2W_{12}^2}$ ;  $B_{p2} = -\frac{a_{22}(B_{v2}\sigma - KW_{12}C_{v2})}{a_{12}KW_{12}\sigma - a_{22}\sigma^2 + a_{02}K^2W_{12}^2}$ ;

$C_{p2} = \frac{C_{v2}}{\sigma} + \frac{a_{02}A_{p2}}{\sigma}$ ;  $D_{p2} = (Q_H - A_{v2}S_{II1})/\sigma$ ;  $E_{p2} = -KW_{12}(Q_H - A_{v2}S_{II1})/\sigma$ .

(19)

Further we find the original of the image (18)

$$p_{12}(t) = -\frac{B_{p2} S_{II1}}{a_{22}} e^{-\frac{a_{12}}{2a_{22}}t} ch\left(\frac{\sqrt{4a_{02}a_{22} + a_{12}^2}}{2a_{22}}t\right) - \frac{(2C_{p2} - B_{p2}a_{12}/a_{22}) S_{II1}}{\sqrt{4a_{02}a_{22} + a_{12}^2}} e^{-\frac{a_{12}}{2a_{22}}t} sh\left(\frac{\sqrt{4a_{02}a_{22} + a_{12}^2}}{2a_{22}}t\right) +$$

$$+ D_{p2} + (E_{p2} - A_{p2} S_{II1}) e^{-\sigma t / (KW_{12})} / (KW_{12}). (20)$$

Neglecting the small coefficients of the expression (20) and taking into consideration the

accepted designations according to (12, 14, 19), at initial conditions  $p_{12}(0) = (c_1 + c_0)S_{II1}/S_{II1}$  the pressure in the pressure pipe of the hydraulic cylinder during the second phase is found by the formula

$$p_{12}(t) \approx \frac{(c_1 + c_0)S_{I1} + S_{I1}[(c_1 + c_0)\sigma x_{\max} S_{\dot{O}1} + 6,094Q_H KW_{12}c_0]}{S_{\dot{O}1}(6,094Q_H KW_{12} + \sigma x_{\max} S_{\dot{O}1})} \left(1 - e^{-\frac{\sigma}{KW_{12}}t}\right). \quad (21)$$

After the substitution  $p_{12} = p_{\kappa 1} = 4cy_0 / [\pi(d_2^2 - d_1^2)]$  we obtain the expression for finding the duration of the second phase

$$t_2 = \frac{KW_{12}}{\sigma} \ln \left\{ 1 - \frac{[4cy_0 S_{II1} - \pi(d_2^2 - d_1^2)(c_1 + c_0)S_{II1}](6,094Q_H KW_{12} + \sigma x_{\max} S_{II1})}{\pi(d_2^2 - d_1^2)S_{II1}[(c_1 + c_0)\sigma x_{\max} S_{II1} + 6,094Q_H KW_{12}c_0]} \right\}^{-1}. \quad (22)$$

3. Phase of opening of the gate element of the PPGRDA to the value of the upper overlap  $y = h_6$  and displacement of the pressing plate can be described by the system of equations:

$$\begin{cases} Q_H = vS_{II1} + \sigma p_{12} + \dot{y}\pi(d_3^2 - d_1^2)/4 + \mu\pi d_{II}^2 \sqrt{2p_{12}/\rho_{pp}}/4 + KW_{12}\dot{p}_{12}; \end{cases} \quad (23)$$

$$\begin{cases} p_{12}S_{II1} = m_p \dot{v} + (c_1 e^{6,094Q_H t/(x_{\max} S_{II1})} + c_0)S_{II1}; \end{cases} \quad (24)$$

$$\begin{cases} p_{12}\pi(d_3^2 - d_1^2)/4 = m_{\kappa} \dot{y} + c(y + y_0). \end{cases} \quad (25)$$

Linearization of the expenditures across the throttle will be written in the following way

$$\sqrt{p_{12}} \approx p_{12} / \sqrt{\bar{p}_{12}}, \quad (26)$$

where  $\bar{p}_{12} = (p_{\dot{e}e1} + \delta_{\dot{c}e})/2$  – average value of the pressure PP in the pressure pipe of the hydraulic cylinder.

After Laplace transform and linearization we obtain:

$$\begin{cases} Q_H / s = V(s)S_{II1} + P(s)\{KW_{12}s + [\sigma + \mu\pi d_{II}^2 \sqrt{2/(\rho_{pp}\bar{p}_{12})}]/4\} + Y(s)s\pi(d_3^2 - d_1^2)/4; \end{cases} \quad (27)$$

$$\begin{cases} P(s)S_{II1} = V(s)sm_p + c_1 S_{II1} / [s - 6,094Q_H / (x_{\max} S_{II1})] + c_0 S_{II1} / s; \end{cases} \quad (28)$$

$$\begin{cases} P(s)\pi(d_3^2 - d_1^2)/4 = Y(s)(m_{\kappa}s^2 + c) + cy_0 / s. \end{cases} \quad (29)$$

Serially substituting the equations (28, 29) into the equation (27), we obtain

$$V(s) = \frac{b_{43}s^3 - b_{33}s^3 + b_{23}s^2 + b_{13}s - b_{03}}{s(-a_{53}s^5 - a_{43}s^4 + a_{33}s^3 - a_{23}s^2 - a_{13}s + a_{03})} \approx \frac{b_{43}s^3 - b_{33}s^3 + b_{23}s^2 + b_{13}s - b_{03}}{s(a_{33}s^3 - a_{13}s + a_{03})}. \quad (30)$$

After the transformations by Cardano formulas [12] of the equation (30) has the form

$$V(s) \approx \frac{b_{43}s^3 - b_{33}s^3 + b_{23}s^2 + b_{13}s - b_{03}}{a_{33}s(s - 2\gamma_1)(s^2 + 2\gamma_1s + \gamma_1^2 + \gamma_2^2)}, \quad (31)$$

where  $A_K = \sqrt[3]{-a_{03}/(2a_{33}) + \sqrt{Q_K}}; B_K = \sqrt[3]{-a_{03}/(2a_{33}) - \sqrt{Q_K}}; Q_K = -a_{13}^3/(27a_{33}^3) + a_{03}^2/(4a_{33}^2);$   
 $a_{53} = 16KW_{12}m_{\kappa}m_p x_{\max} S_{II1}; a_{43} = 4m_{\kappa}m_p \{x_{\max} S_{II1}[4\sigma + \mu\pi d_{II}^2 \sqrt{2/(\rho_{pp}\bar{p}_{12})}] - 6,094KW_{12}Q_H\};$   
 $a_{33} = 24,38Q_H m_p m_{\kappa} - x_{\max} S_{II1}[16S_{II1}m_{\kappa} + \pi^2(d_3^2 - d_1^2)^2 m_p + 16KW_{12}m_p c]; \gamma_1 = (A_K + B_K)/2;$   
 $\gamma_2 = \sqrt{3}(A_K - B_K)/2; a_{03} = 97,5Q_H S_{II1}^2 c; b_{03} = 24,38Q_H c\{c_0 S_{II1}[4\sigma + \mu\pi d_{II}^2 \sqrt{2/(\rho_{pp}\bar{p}_{12})}] - 4Q_H S_{II1}\};$

$$\begin{aligned}
 a_{23} &= 4m_p c x_{\max} S_{II1} [4\sigma + \mu \pi d_{II}^2 \sqrt{2/(\rho_{pp} \bar{p}_{12})}] - 6,094 Q_H [16 S_{II1} m_{\kappa} + \pi^2 (d_3^2 - d_1^2)^2 m_p + 16 K W_{12} m_p c]; \\
 a_{13} &= 4c \{ 4 x_{\max} S_{II1}^3 - 6,094 Q_H m_p [4\sigma + \mu \pi d_{II}^2 \sqrt{2/(\rho_{pp} \bar{p}_{12})}] \}; \quad b_{43} = 16 K W_{12} S_{II1} m_{\kappa} x_{\max} S_{II1} (c_1 + c_0); \\
 b_{23} &= 97,5 Q_H^2 m_{\kappa} S_{II1} + \pi (d_3^2 - d_1^2) x_{\max} S_{II1} [\pi (d_3^2 - d_1^2) S_{II1} (c_1 + c_0) - 4 S_{II1} c y_0] + 16 K W_{12} S_{II1} c x_{\max} S_{II1} (c_1 + \\
 &+ c_0) - 24,38 Q_H c_0 S_{II1} m_{\kappa} [4\sigma + \mu \pi d_{II}^2 \sqrt{2/(\rho_{pp} \bar{p}_{12})}]; \quad b_{13} = 4 S_{II1} c x_{\max} S_{II1} (c_1 + c_0) [4\sigma + \sqrt{2/(\rho_{pp} \bar{p}_{12})}] \times \\
 &\times \mu \pi d_{II}^2] - 16 Q_H x_{\max} S_{II1}^2 c - 6,094 \pi (d_3^2 - d_1^2) Q_H [\pi (d_3^2 - d_1^2) c_0 S_{II1} - 4 S_{II1} c y_0] - 97,5 K W_{12} Q_H c_0 S_{II1} c; \\
 b_{33} &= 4 m_{\kappa} \{ 4 Q_H x_{\max} S_{II1}^2 - x_{\max} S_{II1} S_{II1} [4\sigma + \mu \pi d_{II}^2 \sqrt{2/(\rho_{pp} \bar{p}_{12})}] + 24,38 K W_{12} Q_H c_0 S_{II1} \}; \quad (32)
 \end{aligned}$$

Applying the method of the expression decomposition (31) into simpler fractions, we obtain

$$V(s) = A_{v3} \frac{1}{s} + B_{v3} \frac{1}{s - 2\gamma_1} + \frac{C_{v3}}{a_{33}} \frac{s + \gamma_1}{(s + \gamma_1)^2 + \gamma_2^2} + \frac{D_{v3} - C_{v3} \gamma_1}{a_{33} \gamma_2} \frac{\gamma_2}{(s + \gamma_1)^2 + \gamma_2^2}, \quad (33)$$

$$\begin{aligned}
 \text{where } A_{v3} &= \frac{b_{03}}{2\gamma_1 (\gamma_1^2 + \gamma_2^2)}; \quad B_{v3} = \frac{2\gamma_1 (b_{13} + 2b_{23}\gamma_1 - 4b_{33}\gamma_1^2)(\gamma_1^2 + \gamma_2^2) - a_{33} b_{03} (\gamma_2 - \gamma_1)}{2a_{33}\gamma_1 (\gamma_1^2 + \gamma_2^2)(9\gamma_1^2 + \gamma_2^2)}; \\
 C_{v3} &= -b_{33} - a_{33} b_{03} / [2\gamma_1 (\gamma_1^2 + \gamma_2^2)] - a_{33} B_{v3}; \quad D_{v3} = b_{23} - 4a_{33}\gamma_1 B_{v3} - 2b_{33}\gamma_1 - a_{33} b_{03} / (\gamma_1^2 + \gamma_2^2). \quad (34)
 \end{aligned}$$

We find the original of the image (33)

$$v(t) = A_{v3} + B_{v3} e^{2\gamma_1 t} + (C_{v3} / a_{33}) e^{-\gamma_1 t} \cos(\gamma_2 t) + [(D_{v3} - C_{v3} \gamma_1) / (a_{33} \gamma_2)] e^{-\gamma_1 t} \sin(\gamma_2 t). \quad (35)$$

As a result of neglecting small coefficients in the expression (35) and taking into consideration the accepted designations according to (32, 34) the velocity of the pressing plate motion during the third phase is found by the formula

$$v_3(t) \approx [3,469 Q_f m_{\delta} m_{\epsilon} \tilde{n}_0 S_{I1} \pi d_{II}^2 / (S_{\delta 1}^2 \sqrt{\rho_{pp} \bar{p}_{12}})] [\sqrt{m_{\delta} m_{\epsilon} / (S_{\delta 1}^2 \tilde{n})} + 1,64 \mu] \sin[1,0913 \sqrt{S_{\delta 1}^2 \tilde{n} / (m_{\delta} m_{\epsilon})} t]. \quad (36)$$

Having integrated the expression (36) at initial conditions  $x(0) = 0$  we find the displacement of the pressing plate during the third phase

$$x_3(t) \approx \frac{3,18 Q_f m_{\delta} m_{\epsilon} \tilde{n}_0 S_{I1} \pi d_{II}^2}{S_{\delta 1}^2 \sqrt{\rho_{pp} \bar{p}_{12}}} \sqrt[3]{\frac{m_{\delta} m_{\epsilon}}{S_{\delta 1}^2 \tilde{n}}} \left( \sqrt[3]{\frac{m_{\delta} m_{\epsilon}}{S_{\delta 1}^2 \tilde{n}}} + 1,64 \mu \right) \left[ 1 - \cos \left( 1,0913 \sqrt{\frac{S_{\delta 1}^2 \tilde{n}}{m_{\delta} m_{\epsilon}}} t \right) \right]. \quad (37)$$

Solving the system of the equations (27 – 29) relatively  $P(s)$ , we obtain

$$P(s) \approx \frac{m_{\delta}}{a_{33} S_{\delta 1}} \frac{b_{43} s^3 - b_{33} s^3 + b_{23} s^2 + b_{13} s - b_{03}}{(s - 2\gamma_1)(s^2 + 2\gamma_1 s + \gamma_1^2 + \gamma_2^2)} + \frac{\tilde{n}_1 S_{I1}}{S_{\delta 1} [s - 6,094 Q_f / (x_{\max} S_{\delta 1})]} + \frac{\tilde{n}_0 S_{I1}}{S_{\delta 1} s}, \quad (38)$$

Applying the method of the expression decomposition (38) into simpler fractions we obtain

$$\begin{aligned}
 P(s) &= \frac{c_0 S_{II1}}{S_{II1}} \frac{1}{s} + \frac{c_1 S_{II1}}{S_{II1}} \frac{1}{s - 6,094 Q_H / (x_{\max} S_{II1})} + \frac{A_{p3} m_p}{a_{33} S_{II1}} \frac{1}{s - 2\gamma_1} + \\
 &+ \frac{B_{p3} m_p}{a_{33} S_{II1}} \frac{s + \gamma_1}{(s + \gamma_1)^2 + \gamma_2^2} + \frac{m_p (C_{p3} - B_{p3} \gamma_1)}{a_{33} S_{II1} \gamma_2} \frac{\gamma_2}{(s + \gamma_1)^2 + \gamma_2^2}, \quad (39)
 \end{aligned}$$

$$\text{where } A_{p3} = (-b_{03} + 2b_{13}\gamma_1 + 4b_{23}\gamma_1^2) / (9\gamma_1^2 + \gamma_2^2); \quad B_{p3} = b_{23} - A_{p3}; \quad C_{p3} = [b_{03} + A_{p3}(\gamma_1^2 + \gamma_2^2)] / (2\gamma_1). \quad (40)$$

We find the original of the image (39)

$$\begin{aligned}
 p_{12}(t) &= c_0 S_{II1} / S_{II1} + (c_1 S_{II1} / S_{II1}) e^{6,094 Q_H t / (x_{\max} S_{II1})} + [A_{p3} m_p / (a_{33} S_{II1})] e^{2\gamma_1 t} + \\
 &+ [B_{p3} m_p / (a_{33} S_{II1})] e^{-\gamma_1 t} \cos(\gamma_2 t) + [m_p (C_{p3} - B_{p3} \gamma_1) / (a_{33} S_{II1} \gamma_2)] e^{-\gamma_1 t} \sin(\gamma_2 t). \quad (41)
 \end{aligned}$$

As a result of neglecting small coefficients in the expression (41) and taking into account the accepted designations according to (32, 34), at initial conditions  $p_{12}(0) = 4cy_0 / [\pi(d_2^2 - d_1^2)]$  pressure in the pressure cavity of the hydraulic cylinder of the pressing plate for the phase three is found by the formula

$$p_{12_3}(t) \approx 4cy_0 / [\pi(d_2^2 - d_1^2)] + (\tilde{n}_1 S_{\dot{I}1} / S_{\dot{O}1}) [e^{6,094 Q_f t / (x_{\max} S_{\dot{O}1})} - 1] \quad (42)$$

Solving the system of equations (27 – 29) relatively  $Y(s)$ , we obtain

$$Y(s) \approx \frac{\pi(d_3^2 - d_1^2)m_p}{4S_{II1}} \left[ \frac{A_{y3}}{m_\kappa s^2 + c} + \frac{B_{y3}s}{(s - 2\gamma_1)(m_\kappa s^2 + c)} + \frac{C_{y3}s^2 + D_{y3}s}{a_{33}(m_\kappa s^2 + c)(s^2 + 2\gamma_1 s + \gamma_1^2 + \gamma_2^2)} \right] + \frac{\pi(d_3^2 - d_1^2)c_0 S_{III}}{4S_{II1}(m_\kappa s^2 + c)[s - 6,094 Q_H / (x_{\max} S_{II1})]} + \frac{\pi(d_3^2 - d_1^2)c_0 S_{III}}{4S_{II1}s(m_\kappa s^2 + c)} - \frac{cy_0}{s(m_\kappa s^2 + c)} \quad (43)$$

Applying the method of the decomposition of the expression (43) into simpler fractions we obtain

$$Y(s) = \left[ \frac{\pi(d_3^2 - d_1^2)c_0 S_{III}}{4S_{II1}c} - y_0 \right] \frac{1}{s} + \frac{\pi(d_3^2 - d_1^2)}{4S_{II1}} \left[ \frac{m_p A_{y3}}{s - 2\gamma_1} + \frac{K_{y3}}{s - 6,094 Q_H / (x_{\max} S_{II1})} \right] + \frac{s}{s^2 + c / m_\kappa} \times \left\{ \frac{\pi(d_3^2 - d_1^2)[m_p(B_{y3} + D_{y3} / a_{33}) + L_{y3}]}{4S_{II1}m_\kappa} + y_0 \left[ 1 - \frac{\pi(d_3^2 - d_1^2)}{4S_{II1}} \right] \right\} + \frac{\pi(d_3^2 - d_1^2)}{4S_{II1}} \left[ \frac{\sqrt{c / m_\kappa}}{s^2 + c / m_\kappa} \times \left. \frac{m_p(A_{y3} + C_{y3} + E_{y3} / a_{33}) + M_{y3}}{\sqrt{m_\kappa c}} + \frac{m_p F_{y3}}{a_{33}} \frac{s + \gamma_1}{(s + \gamma_1)^2 + \gamma_2^2} + \frac{m_p(G_{y3} - F_{y3}\gamma_1)}{a_{33}\gamma_2} \frac{\gamma_2}{(s + \gamma_1)^2 + \gamma_2^2} \right] \right\} \quad (44)$$

where  $A_{y3} = 2\gamma_1 B_{y3} / (c + 4m_\kappa \gamma_1^2)$ ;  $B_{y3} = -m_\kappa A_{y3}$ ;  $C_{y3} = B_{y3} - 2m_\kappa \gamma_1 A_{y3}$ ;  $G_{y3} = -E_{y3}(\gamma_1^2 + \gamma_2^2) / c$ ;  $F_{y3} = -D_{y3} / m_\kappa$ ;  $D_{y3} = C_{y3} / (2\gamma_1) - E_{y3}[c - m_\kappa(\gamma_1^2 + \gamma_2^2)] / (2\gamma_1 c)$ ;  $M_{y3} = 6,094 Q_H L_{y3} / (x_{\max} S_{II1})$ ;  
 $E_{y3} = \frac{c\{2\gamma_1 m_\kappa D_{y3} + [c - m_\kappa(\gamma_1^2 + \gamma_2^2)]C_{y3}\}}{[c - m_\kappa(\gamma_1^2 + \gamma_2^2)]^2 + 4\gamma_1^2 m_\kappa c}$ ;  $K_{y3} = \frac{\tilde{n}_1 S_{\dot{I}1} x_{\max}^2 S_{\dot{O}1}^2}{\tilde{n} x_{\max}^2 S_{\dot{O}1}^2 + 37,14 Q_f^2 m_\kappa}$ ;  $L_{y3} = -m_\kappa K_{y3}$ . (45)

We find the original of the image (44)

$$y(t) = \pi(d_3^2 - d_1^2)\tilde{n}_0 S_{\dot{I}1} / (4S_{\dot{O}1}c) - y_0 + \pi(d_3^2 - d_1^2)[m_\delta A_{y3} e^{2\gamma_1 t} + K_{y3} e^{6,094 Q_f t / (x_{\max} S_{\dot{O}1})}] / (4S_{\dot{O}1}) + \{ \pi(d_3^2 - d_1^2)[m_\delta(B_{y3} + D_{y3} / a_{33}) + L_{y3}] / (4S_{\dot{O}1}m_\kappa) + y_0[1 - \pi(d_3^2 - d_1^2) / (4S_{\dot{O}1})] \} \cos(\sqrt{c / m_\kappa} t) + [ \pi(d_3^2 - d_1^2) / (4S_{\dot{O}1}) ] \{ [m_\delta(A_{y3} + C_{y3} + E_{y3} / a_{33}) + M_{y3}] \cos(\sqrt{c / m_\kappa} t) / \sqrt{m_\kappa c} + (m_\delta F_{y3} / a_{33}) e^{-\gamma_1 t} \cos(\gamma_2 t) + [m_\delta(G_{y3} - F_{y3}\gamma_1) / (a_{33}\gamma_2)] e^{-\gamma_1 t} \sin(\gamma_2 t) \}. \quad (46)$$

As a result of neglecting small coefficients in the expression (46) and taking into account the accepted designations according to (32, 34, 45), at initial conditions  $y(0) = 0$  the displacement of the gate element of the PPGRDA during the third phase is found by the formula

$$y_3(t) \approx [1,748\pi(d_3^2 - d_1^2)Q_f^2 m_\delta^2 \sqrt{m_\kappa} (3,487 S_{\dot{O}1}^{2/3} \sqrt{c} - \sqrt[3]{m_\delta m_\kappa}) / (S_{\dot{O}1}^{8/3} c^{5/6})] \sin(\sqrt{c / m_\kappa} t) \quad (47)$$

Comparing the displacement of the gate element of the pressure pulse generator to  $h_\delta$ , the expression for finding the duration of the third phase is obtained

$$t_3 \approx \sqrt{m_\kappa / c} \arcsin \{ S_{\dot{O}1}^{8/3} c^{5/6} h_\delta / [1,748\pi(d_3^2 - d_1^2)Q_f^2 m_\delta^2 \sqrt{m_\kappa} (3,487 S_{\dot{O}1}^{2/3} \sqrt{c} - \sqrt[3]{m_\delta m_\kappa})] \} \quad (48)$$

4. Opening phase of the gate element of the PPGRD to the value of the bottom overlap  $y = h_{нж}$  and the displacement of the pressing plate.

Similarly to phase three the approximate dependences for the phase four are found:

$$v_4(t) \approx [3,469Q_f m_\delta m_\epsilon \tilde{n}_0 S_f \pi (d_A^2 + d_a^2) / (S_{\delta 1}^2 \sqrt{\rho_{pp} \bar{p}_{12}})] [\sqrt{m_\delta m_\epsilon} / (S_{\delta 1}^2 \tilde{n}) + 1,64\mu] \sin[1,0913 \sqrt{S_{\delta 1}^2 \tilde{n} / (m_\delta m_\epsilon)} t]; \quad (49)$$

$$x_4(t) \approx \frac{3,18Q_f m_\delta m_\epsilon \tilde{n}_0 S_f \pi (d_A^2 + d_a^2)}{S_{\delta 1}^2 \sqrt{\rho_{pp} \bar{p}_{12}}} \sqrt{\frac{m_\delta m_\epsilon}{S_{\delta 1}^2 \tilde{n}}} \left( \sqrt{\frac{m_\delta m_\epsilon}{S_{\delta 1}^2 \tilde{n}}} + 1,64\mu \right) \left[ 1 - \cos \left( 1,0913 \sqrt{\frac{S_{\delta 1}^2 \tilde{n}}{m_\delta m_\epsilon}} t \right) \right]; \quad (50)$$

$$t_4 \approx \sqrt{m_\epsilon / c} \arcsin \{ S_{\delta 1}^{8/3} c^{5/6} (h_{ie} - h_a) / [1,748\pi (d_3^2 - d_1^2) Q_f^2 m_\delta^2 \sqrt{m_\epsilon} (3,487 S_{\delta 1}^{2/33} \sqrt{c} - \sqrt{m_\delta m_\epsilon})] \}. \quad (51)$$

Displacement of the gate element of the PPGRDA during phase four is determined by the dependence (47).

5. Opening phase of the gate element of the PPGRDA to the value of the positive overlap  $y = h_n$  and displacement of the pressure plate.

Similarly to phase three and phase four we find approximate dependences for the phase five:

$$y_5(t) \approx [1,748\pi d_3^2 Q_f^2 m_\delta^2 \sqrt{m_\epsilon} (3,487 S_{\delta 1}^{2/33} \sqrt{c} - \sqrt{m_\delta m_\epsilon}) / (S_{\delta 1}^{8/3} c^{5/6})] \sin(\sqrt{c / m_\epsilon} t); \quad (52)$$

$$t_5 \approx \sqrt{m_\epsilon / c} \arcsin \{ S_{\delta 1}^{8/3} c^{5/6} (h_i - h_{ie}) / [1,748\pi d_3^2 Q_f^2 m_\delta^2 \sqrt{m_\epsilon} (3,487 S_{\delta 1}^{2/33} \sqrt{c} - \sqrt{m_\delta m_\epsilon})] \}. \quad (53)$$

Velocity and displacement of the pressing plate during phase five is determined by the dependences (49) and (50), correspondingly. Pressure in the pressure cavity of the hydraulic cylinder of the pressing plate during phases four and five is found by the formula (42).

6. Phase of opening of the gate element of the pressure pulse generator to the value of the complete overlap  $y = h_n + h_h$  and pressure reduction to the closing pressure of PPGRDA  $p_{12} = p_{\kappa 12}$  can be described by the system of equations:

$$\begin{cases} Q_H = \sigma p_{12} + \dot{y} \pi d_3^2 / 4 + \mu \pi d_3 (y - h_n) \sqrt{2 p_{12} / \rho_{pp}} + \mu \pi d_{\kappa}^2 \sqrt{2 p_{12} / \rho_{pp}} / 4 + K W_{12} \dot{p}_{12}; \\ p_{12} \pi d_3^2 / 4 = m_\kappa \ddot{y} + c(y + y_0). \end{cases} \quad (54)$$

$$\begin{cases} Q_H = \sigma p_{12} + \dot{y} \pi d_3^2 / 4 + \mu \pi d_3 (y - h_n) \sqrt{2 p_{12} / \rho_{pp}} + \mu \pi d_{\kappa}^2 \sqrt{2 p_{12} / \rho_{pp}} / 4 + K W_{12} \dot{p}_{12}; \\ p_{12} \pi d_3^2 / 4 = m_\kappa \ddot{y} + c(y + y_0). \end{cases} \quad (55)$$

We will perform the linearization of the expenses across the throttle by means of the expression (26) and across the slot, formed as a result of the gate element passage of the positive overlapping

$$\sqrt{p_{12}} \approx \sqrt{\bar{p}_{12}}. \quad (56)$$

After Laplace transform and linearization we obtain:

$$\begin{cases} (Q_H + \mu \pi d_3 h_n \sqrt{2 \bar{p}_{12} / \rho_{pp}}) / s = P(s) \{ K W_{12} s + [\sigma + \mu \pi d_{\kappa}^2 \sqrt{2 / (\rho_{pp} \bar{p}_{12})} / 4] \} + \\ + Y(s) (\pi d_3^2 / 4 + \mu \pi d_3 \sqrt{2 \bar{p}_{12} / \rho_{pp}}); \end{cases} \quad (57)$$

$$\begin{cases} P(s) \pi d_3^2 / 4 = Y(s) (m_\kappa s^2 + c) + c y_0 / s. \end{cases} \quad (58)$$

Substituting the equation (58) into the equation (57), we obtain

$$P(s) = \frac{b_{26} s^2 - b_{16} s - b_{06}}{s(a_{36} s^3 + a_{26} s^2 + a_{16} s + a_{06})} \approx \frac{b_{26} s^2 - b_{16} s - b_{06}}{s(a_{26} s^2 + a_{16} s + a_{06})}. \quad (59)$$

$$\begin{aligned} \text{where } a_{26} &= 4m_\epsilon [4\sigma + \mu \pi (d_A^2 + d_a^2) \sqrt{2 / (\rho_{pp} \bar{p}_{12})}]; & b_{06} &= 16c [\mu \pi d_3 (y_0 - h_i) \sqrt{2 \bar{p}_{12} / \rho_{pp}} - Q_H]; \\ a_{36} &= 16K W_{12} m_\epsilon; & b_{16} &= 4\pi d_3^2 c y_0; & a_{06} &= 4\{c[4\sigma + \mu \pi (d_A^2 + d_a^2) \sqrt{2 / (\rho_{pp} \bar{p}_{12})}] + \mu \pi^2 d_3^3 \sqrt{2 \bar{p}_{12} / \rho_{pp}}\}; \\ a_{16} &= 16K W_{12} c + \pi^2 d_3^4; & b_{26} &= 16m_\epsilon (Q_H + \mu \pi d_3 h_i \sqrt{2 \bar{p}_{12} / \rho_{pp}}). \end{aligned} \quad (60)$$

Applying the method of the expression (59) decomposition into the simpler fractions we obtain



$$P(s) = A_{p6} \frac{1}{s} + \frac{B_{p6}}{a_{26}} \frac{s + a_{16} / (2a_{26})}{[s + a_{16} / (2a_{26})]^2 - (a_{16}^2 - 4a_{06}a_{26}) / (4a_{26}^2)} + \frac{2C_{p6} - B_{p6}a_{16} / a_{26}}{\sqrt{a_{16}^2 - 4a_{06}a_{26}}} \frac{\sqrt{a_{16}^2 - 4a_{06}a_{26}} / (2a_{26})}{[s + a_{16} / (2a_{26})]^2 - (a_{16}^2 - 4a_{06}a_{26}) / (4a_{26}^2)}, \quad (61)$$

where  $A_{p6} = -b_{06} / a_{06}$ ;  $B_{p6} = b_{26} + a_{26}b_{06} / a_{06}$ ;  $C_{p6} = -b_{16} + a_{16}b_{06} / a_{06}$ . (62)

We find the original of the image (61)

$$p_{12}(t) = A_{p6} + \frac{B_{p6}}{a_{26}} e^{-\frac{a_{16}-t}{2a_{26}}} ch\left(\frac{\sqrt{a_{16}^2 - 4a_{06}a_{26}}}{2a_{26}} t\right) + \frac{2C_{p6} - B_{p6}a_{16} / a_{26}}{\sqrt{a_{16}^2 - 4a_{06}a_{26}}} e^{-\frac{a_{16}-t}{2a_{26}}} sh\left(\frac{\sqrt{a_{16}^2 - 4a_{06}a_{26}}}{2a_{26}} t\right). \quad (63)$$

As a result of neglecting the small coefficients of the expression (63) and taking into account the accepted designations according to (61, 62), at initial conditions  $p_{12}(0) = 4c(y_0 + h_i) / (\pi d_3^2)$  the pressure in the pressure line of the hydraulic cylinder during the sixth phase is found by the formula

$$p_{12_6}(t) \approx \frac{4c(y_0 + h_i)}{\pi d_3^2} - \frac{4d_3^2 h_i \bar{p}_{12}}{d_{\bar{A}}^2 + d_{\bar{a}}^2} \left\langle 1 - e^{-\frac{\pi d_3^4}{8m_{\bar{e}} \mu (d_{\bar{A}}^2 + d_{\bar{a}}^2)} \sqrt{\frac{\rho_{pp} \bar{p}_{12}}{2}} t} \left[ 1 - sh\left[ \frac{\pi d_3^4}{8m_{\bar{e}} \mu (d_{\bar{A}}^2 + d_{\bar{a}}^2)} \sqrt{\frac{\rho_{pp} \bar{p}_{12}}{2}} t \right] \right] \right\rangle. \quad (64)$$

Solving the system of the equations (57 – 58) relatively  $Y(s)$ , we obtain

$$Y(s) = \frac{\pi d_3^2}{4} \left[ \frac{A_{p6}}{s(m_{\kappa} s^2 + c)} + \frac{B_{p6} s + C_{p6}}{(a_{26} s^2 + a_{16} s + a_{06})(m_{\kappa} s^2 + c)} \right] + \frac{cy_0}{s(m_{\kappa} s^2 + c)}. \quad (65)$$

Applying the method of the expression (67) decomposition into simpler fractions we obtain

$$Y(s) = \left( \frac{\pi d_3^2}{4} A_{y6} + y_0 \right) \frac{1}{s} + \left[ \frac{\pi d_3^2 (B_{y6} + F_{y6})}{4m_{\kappa}} - y_0 \right] \frac{s}{s^2 + c/m_{\kappa}} + \frac{\pi d_3^2}{4} \left\{ \frac{G_{y6}}{\sqrt{m_{\kappa} c}} \frac{\sqrt{c/m_{\kappa}}}{s^2 + c/m_{\kappa}} + \frac{2E_{y6} - D_{y6}a_{16} / a_{26}}{\sqrt{a_{16}^2 - 4a_{06}a_{26}}} \times \right. \\ \left. \times \frac{\sqrt{a_{16}^2 - 4a_{06}a_{26}} / (2a_{26})}{[s + a_{16} / (2a_{26})]^2 - (a_{16}^2 - 4a_{06}a_{26}) / (4a_{26}^2)} + \frac{D_{y6}}{a_{26}} \frac{s + a_{16} / (2a_{26})}{[s + a_{16} / (2a_{26})]^2 - (a_{16}^2 - 4a_{06}a_{26}) / (4a_{26}^2)} \right\}, \quad (66)$$

where  $A_{y6} = A_{p6} / c$ ;  $B_{y6} = -m_{\kappa} A_{p6} / c$ ;  $D_{y6} = -a_{26} F_{y6} / m_{\kappa}$ ;  $G_{y6} = (C_{p6} - cE_{y6}) / a_{06}$ ;

$$E_{y6} = \frac{B_{p6} m_{\bar{e}} a_{06} a_{16} + C_{p6} (m_{\bar{e}} a_{06} a_{26} - ca_{26}^2 - m_{\bar{e}} a_{16}^2)}{m_{\bar{e}} (2\bar{n} a_{06} a_{26} - m_{\bar{e}} a_{06}^2 - ca_{26}^2) - c^2 a_{26}^2}; \quad F_{y6} = \left( \frac{ca_{26}}{a_{06}} - m_{\kappa} \right) \frac{E_{y6}}{a_{16}} - \frac{C_{p6} a_{26}}{a_{06} a_{16}}. \quad (67)$$

We find the original of the image (66)

$$y(t) = \frac{\pi d_3^2}{4} A_{y6} + y_0 + \left[ \frac{\pi d_3^2 (B_{y6} + F_{y6})}{4m_{\bar{e}}} - y_0 \right] \cos\left(\sqrt{\frac{c}{m_{\bar{e}}}} t\right) + \frac{\pi d_3^2}{4} \left[ \frac{G_{y6}}{\sqrt{m_{\bar{e}} c}} \sin\left(\sqrt{\frac{c}{m_{\bar{e}}}} t\right) + \right. \\ \left. + \frac{D_{y6}}{a_{26}} e^{-\frac{a_{16}-t}{2a_{26}}} ch\left(\frac{\sqrt{a_{16}^2 - 4a_{06}a_{26}}}{2a_{26}} t\right) + \frac{2E_{y6} - D_{y6}a_{16} / a_{26}}{\sqrt{a_{16}^2 - 4a_{06}a_{26}}} e^{-\frac{a_{16}-t}{2a_{26}}} sh\left(\frac{\sqrt{a_{16}^2 - 4a_{06}a_{26}}}{2a_{26}} t\right) \right]. \quad (68)$$

As a result of neglecting small coefficients in the expression (68) and taking into account the accepted designations according to (60, 67), at initial conditions  $y(0) = h_n$ , the displacement of the gate element during the phase six is found by the formula

$$y_6(t) \approx h_i \{1 + \pi d_3^2 \sqrt{\rho_{pp} \bar{p}_{12}} / 2t / [24m_{\bar{e}} \mu (d_{\bar{A}}^2 + d_{\bar{a}}^2)]\}. \quad (69)$$

Comparing the displacement of the gating element of the pressure pulse generator with  $h_n + h_n$ ,

we obtain the expression for finding the duration of the phase six

$$t_6 \approx 24m_e \mu (d_A^2 + d_a^2) h_i \sqrt{2/(\rho_{pp} \bar{P}_{12})} / (\pi d_3^4 h_i) \quad (70)$$

Verification of the correctness of the simplified model is performed by means of comparison of the frequency and amplitude of the vibration compaction of SHW.

We find the oscillation frequency of the vibration compaction of SHW:

$$\nu = 1/[t_1 + t_2 + 2(t_3 + t_4 + t_5 + t_6)] = 1/[0,01084 + 2,414 \cdot 10^{-4} + 2 \cdot (6,686 \cdot 10^{-5} + 6,685 \cdot 10^{-6} + 2,166 \cdot 10^{-5} + 2,488 \cdot 10^{-4})] = 84,98 \text{ (Гц)}. \quad (71)$$

Then we find the amplitude of the SHW compaction plate displacement:

$$A = x_2 + x_3 + x_4 + x_5 = 1,915 \cdot 10^{-4} + 2,762 \cdot 10^{-9} + 2,914 \cdot 10^{-11} + 3,059 \cdot 10^{-10} = 1,915 \cdot 10^{-4} \text{ (м)} = 0,1915 \text{ (мм)}. \quad (72)$$

The values of the oscillations amplitude, obtained by means of the simplified mathematical model differ from the corresponding results of the complete mathematical model [9] not more than by 5.34%, it is acceptable for the realization of the preliminary design calculations.

The considered technique of the preliminary assessment and selection of the output parameters of the vibrational drive for SHW compaction has the approximate character, it is recommended for the application in the process of the draft design for the revealing of the general technical characteristic. Final calculation of the real parameters of the vibration drive for SHW compaction should be carried out, using more complete mathematical model [9], taking into account the characteristic features of the specific design scheme.

### Conclusions

Simplified improved mathematical model of the vibrational drive for SHW compaction, using pressure pulse generator of the relay differential action is suggested, the given model enabled to obtain analytical interdependences of its basic parameters and dependences of the frequency and amplitude on the basic parameters of the given drive, that can be used for the realization of the preliminary design calculations of its parameters, as one of the components for the solution of the problem of the creation of scientific-engineering fundamentals for the design of the efficient tools of the machines for the collection and processing of the solid household waste.

### REFERENCES

1. Qdais Abu H. Kinetics of solid waste biodegradation in laboratory lysimeters / H. Abu Qdais, A. Alsheraideh // Jordan Journal of Civil Engineering. – 2008. – Vol. 2, № 1. – P. 45 – 52.
2. Popovych V. V. Efficiency of the waste disposal vehicle operation in the environment "city-landfill site" / V. V. Popovych, O. V. Prydatko, M. I. Sychevskiy // Scientific Bulletin of NLTU of Ukraine. – 2017. – Vol. 27, № 10. – P. 111 – 116. (Ukr).
3. Cabinet of Ministers of Ukraine. Government Decree № 265 "On approval of the Program of solid household waste management" [Electronic resource] March 4, 2004. Access mode : <http://zakon1.rada.gov.ua/laws/show/265-2004-%D0%BF>. (Ukr).
4. Berezyuk O. V. Approximated mathematical model of hydraulic drive of container upturning during loading of solid domestic wastes into a dustcart / O. V. Berezyuk, V. I. Savulyak // Technical Sciences. – University of Warmia and Mazury in Olsztyn, Poland. – 2017. – № 20 (3). – P. 259 – 273.
5. Berezyuk O. V. Dynamics of hydraulic drive of hanging sweeping equipment of dust-cart with extended functional possibilities / O. V. Berezyuk, V. I. Savulyak // TEHNOMUS Journal. New Technologies and Products in Machine Manufacturing Technologies. – Suceava, Romania. – 2015. – № 22. – P. 345 – 351.
6. Bereziuk O. V. Analytical study of the mathematical model of the vibrational hydraulic drive for solid household waste compaction / O. V. Bereziuk // Bulletin of National Technical University "Kharkiv Polytechnical Institute". – 2008. – № 38. – P. 96 – 102. (Ukr).
7. Patent of Ukraine № 92720 U, IPC (2014.01) F15B 21/00. Pressure pulse generator of the relay differential action / Bereziuk O. V.; claimer and owner of patent Bereziuk O. V. – №u201404118 ; claimed 16.04.2014. received Scientific Works of VNTU, 2020, № 1

26.08.2014, Bulletin № 16. (Ukr).

8. Kotz I. V. Development and study of the valves-pulsators for the hydraulic drives of the vibrational mining machines: Dissertation... for the Degree of the Candidate of Science (Engineering) : 05.02.03 / Kotz Ivan Vasyliovych. – Vinnytsia, 1994. – 227 p. (Rus).

9. Bereziuk O. V. Numerical study of the improved mathematical model of the vibrational hydraulic drive for the compaction of the solid household waste in the waste disposal vehicle on the base of information technologies / O. V. Bereziuk // Materials of the 2<sup>nd</sup> All-Ukrainian Scientific-Practical Internet Conference of the Students, Post-Graduates and Young Scientific on the subject «Application of modern computer-based systems and networks in the sphere of control». – Kherson : Self employed person Vyshemyrskyi V. S., – 2019. – P. 290 – 293. (Ukr).

10. Bereziuk O. V. Experimental studies of the solid household waste dehydration process using the screw extruder / O. V. Bereziuk // Bulletin of Vinnytsia Polytechnic Institute. – 2018. – № 5. – P. 18 – 24. – DOI: 10.31649/1997-9266-2018-140-5-18-24. (Ukr).

11. Sveshnikov A. G. Theory of complex variable functions / A. G. Sveshnikov, A. N. Tikhonov. – [6<sup>th</sup> Edition, Stereotyped Edition]. – M. : PHIZMATLIT, 2005. – 336 p. (Rus).

12. Gusak A. A. Reference book on higher mathematics / A. A. Gusak, G. M. Gusak, E. A. Brychikova. – Minsk : TetraSystems, 1999. – 640 p. (Rus).

Editorial office received the paper 16.02.2020.

The paper was reviewed 26.02.2020.

**Bereziuk Oleg** – Cand. Sc. (Eng.), Assistant Professor with the Department of Health and Safety, Pedagogy of Safety.

Vinnytsia national technical university.