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SUBSTANTION OF THE EXTREMUM OCCURRENCE ON THE GRAPH OF THE MATHEMATICAL MODEL OF STATICS OF THE INSTRUMENT CURRENT TRANSFORMER

Substantiation of the extremum occurrence on the graph of mathematical model of statics of the instrument current transformer is carried out, it is shown that after reaching the extremum the output characteristic of such transformer is of falling character, as a result, different values of the current in the primary winding correspond to the same values of the current in its secondary winding.

Key words: *instrument current transformer, mathematical model, output characteristic, extremum, measurement ambiguity, substantiation.*

Problem set up and preconditions

In [1] the characteristics of the processes in instrument current transformers and their mathematical models, given in [2, 3, 4], were specified, the presence of falling section in output characteristic of such a transformer, that did not proceed from basic research devoted to the study of current transformer, was demonstrated heuristically. That is, in [1] we, practically, suggested only hypothesis that the output characteristic of instrument current transformer has the form, shown in Fig. 1 and proposed logic concept of proving this hypothesis.

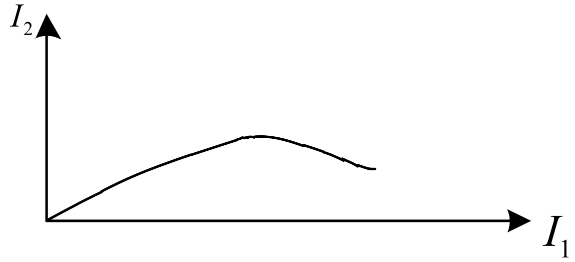

Fig. 1. Indicative graph of the real output static characteristic $I_2 = f(I_1)$ of the instrument current transformer

The problem of mathematical substantiation of this hypothesis will be set.

For this purpose two output preconditions will be used – first, the fact that as in studies [1, 2, 3, 4], the real value I_2 of the current $i_2(t)$ in the secondary winding of the instrument current transformer during the period T will be determined by means of the known relation

$$I_2 = \sqrt{\frac{1}{T}\int_0^T (i_2(t))^2 dt}\ , \qquad (1)$$

secondly, the graph of the current $i_2(t)$ in the secondary winding of the instrument current transformer during the period T has the same from as in [1], but with the reference to the period, i. e., it has the form, demonstrated in Fig. 2, where the graph of the square $(i_2(t))^2$ of this current is shown.

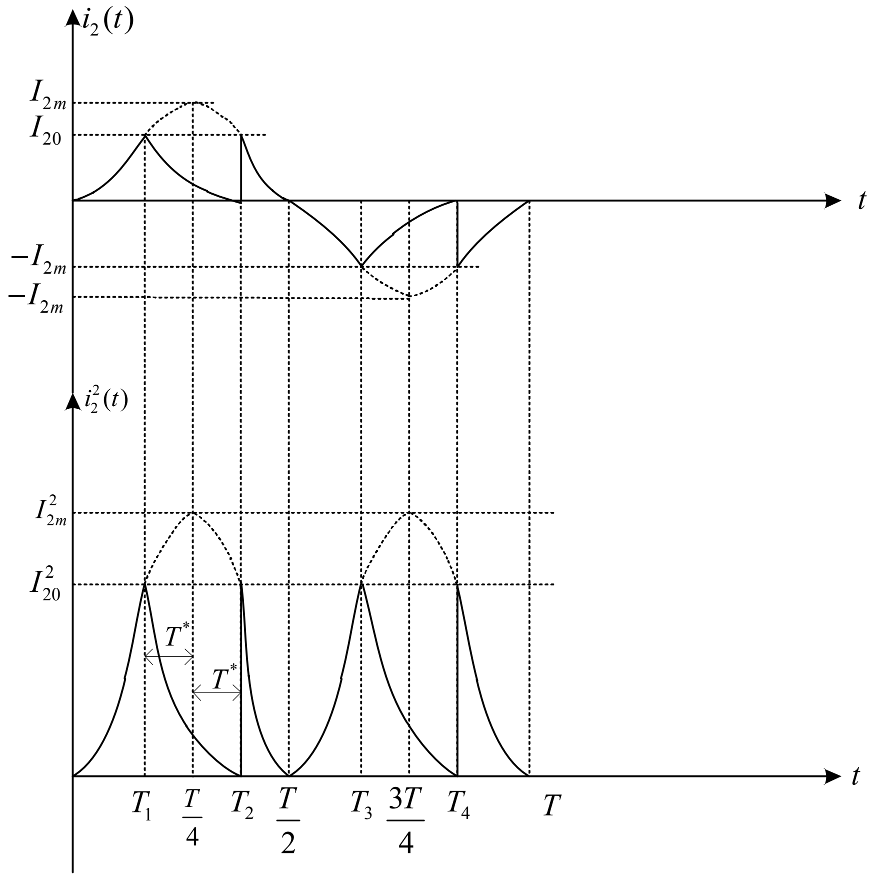

Fig. 2. Graph of the current $i_2(t)$ and its square $(i_2(t))^2$ during the period T

Solution of the set problem

Analyzing the graph, shown in Fig. 2, it is easy to note that the graphs of $(i_2(t))^2$ current squares at each half period $\frac{T}{2}$ coincide and as at half period $\frac{T}{2}$ for the current $i_2(t)$ the expression

$$i_2(t) = I_{2m} \sin \omega t[1(t) - 1(t - T_1)] + I_{20} e^{-\alpha\|t - T_1\|}[1(t - T_1) - 1(t - T_2)] + \\ + I_{2m} \sin \omega t[1(t - T_2) - 1(t - \frac{T}{2})], \tag{2}$$

is valid , then the relation (1) can be rewritten as:

$$I_2 = \sqrt{\frac{1}{T}\int_0^T (i_2(t))^2 dt} = \sqrt{\frac{2}{T}\int_0^{\frac{T}{2}} (i_2(t))^2 dt} = \\ = \sqrt{\frac{2}{T}[\int_0^{T_1} I_{2m}^2 \sin^2 \omega t\, dt + \int_{T_1}^{T_2} I_{20}^2 (e^{-\alpha(t - T_1)})^2 dt + \int_{T_2}^{\frac{T}{2}} I_{2m}^2 \sin^2 \omega t\, dt]}, \tag{3}$$

where $\omega = \frac{2\pi}{T}$ – is the circular current frequency, and $1(t)$ – is unit function, for which it is valid

$$\begin{cases} 1(t) = \begin{cases} 1 & for\ \forall t \in [0, \infty), \\ 0 & for\ \forall t < 0, \end{cases} \\ 1(t-\tau) = \begin{cases} 1 & for\ \forall t \in [\tau, \infty). \\ 0 & for\ \forall t < \tau \end{cases} \end{cases} \quad (4)$$

Taking into account, that

$$sin^2\, \omega t = \frac{1 - cos\, 2\omega t}{2}, \quad (5)$$

$$\left(e^{-\alpha(t-T_1)}\right)^2 = e^{-2\alpha(t-T_1)}, \quad (6)$$

substituting the expressions (5), (6) into the expression (3) and taking after this substitution the integers in the expression (3) and simplifying the results of the integration, we obtain

$$I_2 = \sqrt{\frac{I_{2m}^2}{T}\left(T_1 - \frac{sin\, 2\omega T_1}{2\omega}\right) - \frac{I_{20}^2}{\alpha T}\left(e^{-2\alpha(T_2 - T_1)} - 1\right) + \frac{I_{2m}^2}{T}\left(\frac{T}{2} - T_2 + \frac{sin\, 2\omega T_2}{2\omega}\right)}. \quad (7)$$

And now we will return again to Fig. 2 and we will see that

$$T_1 = \frac{T}{4} - T^*, \qquad T_2 = \frac{T}{4} + T^*, \quad (8)$$

where

$$0 \le T^* < \frac{T}{4}, \quad (9)$$

Substituting the expression (8) in the expression (7) and simplifying the results of the substitution, we obtain

$$I_2 = \sqrt{\frac{2I_{2m}^2}{T}\left(\frac{T}{4} - T^* - \frac{sin\, 2\omega T^*}{2\omega}\right) - \frac{I_{20}^2}{\alpha T}\left(e^{-2\alpha T^*} - 1\right)} = f_1\left(T^*\right). \quad (10)$$

The correctness of the obtained expression can be verified by the suitability for the analysis of the processes in the secondary winding of instrument current transformer with non-saturated state of the core, for which, as it is seen from Fig. 2,

$$T^* = 0. \quad (11)$$

Substituting the expression (11) into the expression (10), we obtain

$$I_2 = \frac{I_{2m}}{\sqrt{2}}, \quad (12)$$

that corresponds to the known from the theoretical fundamentals of electric engineering, real value of sinusoidal current in the winding of the transformer with non-saturated core and proves the correctness of the obtained model (10).

Further standard procedure of studying $f_1\left(T^*\right)$ function for the extremum will be applied, the procedure includes taking the derivative from this function with the respect of its argument T^*, setting of the obtained expression to zero and solution of the obtained equation.

Thus, differentiating the expression (10), we will have

$$\frac{dI_2}{dT^*}=\frac{\frac{2I_{2m}^2}{T}\left(-1-cos\,2\omega T^*\right)+\frac{2I_{20}^2}{T}e^{-2\alpha T^*}}{2\sqrt{\frac{2I_{2m}^2}{T}\left(\frac{T}{4}-T^*-\frac{sin\,2\omega T^*}{2\omega}\right)-\frac{I_{20}^2}{\alpha T}\left(e^{-2\alpha T^*}-1\right)}}. \quad (13)$$

Since the denominator of the expression (13) in the range of changes T^* within the frame of the condition (9) at any of the points will not be equal zero then the setting of this expression to zero will lead to the expression

$$\frac{2I_{2m}^2}{T}\left(-1-cos\,2\omega T^*\right)+\frac{2I_{20}^2}{T}e^{-2\alpha T^*}=0, \quad (14)$$

or

$$I_{2m}^2\left(1+cos\,2\omega T^*\right)=I_{20}^2e^{-2\alpha T^*}, \quad (15)$$

or

$$f_2\left(T^*\right)=f_3\left(T^*\right), \quad (16)$$

where

$$f_2\left(T^*\right)=I_{2m}\sqrt{1+cos\,2\omega T^*}, \quad (17)$$

$$f_3\left(T^*\right)=I_{20}e^{-2\alpha T^*}. \quad (18)$$

In the limits to the boundary between the saturated and non-saturated states of the instrument current transformer core the equation is performed

$$kI_{2m}=I_{1m}, \quad (19)$$

where I_{1m} – is the amplitude of the current in the primary winding of instrument current transformer i. e., the current, being measured, and k – is transformation ratio of this transformer when it operates in the range in the boundary between non-saturated and saturated states of the core, that is why the, the expression (17)can also be presented in the following form

$$f_2\left(T^*\right)=\frac{I_{1m}}{k}\sqrt{1+cos\,2\omega T^*}, \quad (20)$$

It should be remembered that

$$\alpha=\frac{r}{L}, \quad (21)$$

where r, L – are active resistance and the inductance of the closed circuit of the secondary winding of instrument current transformer, on their ratio the graph of the exponential curve in the expression (18), by which the function $f_3(T^*)$, is set, will have different paths, shown in Fig. 3 for two ratios . In the same Figure3 two graphs of $f_2(T^*)$, function are shown, this function is set be the expression (20), for two values of sinusoidal current amplitude in the primary winding of the instrument current transformer.

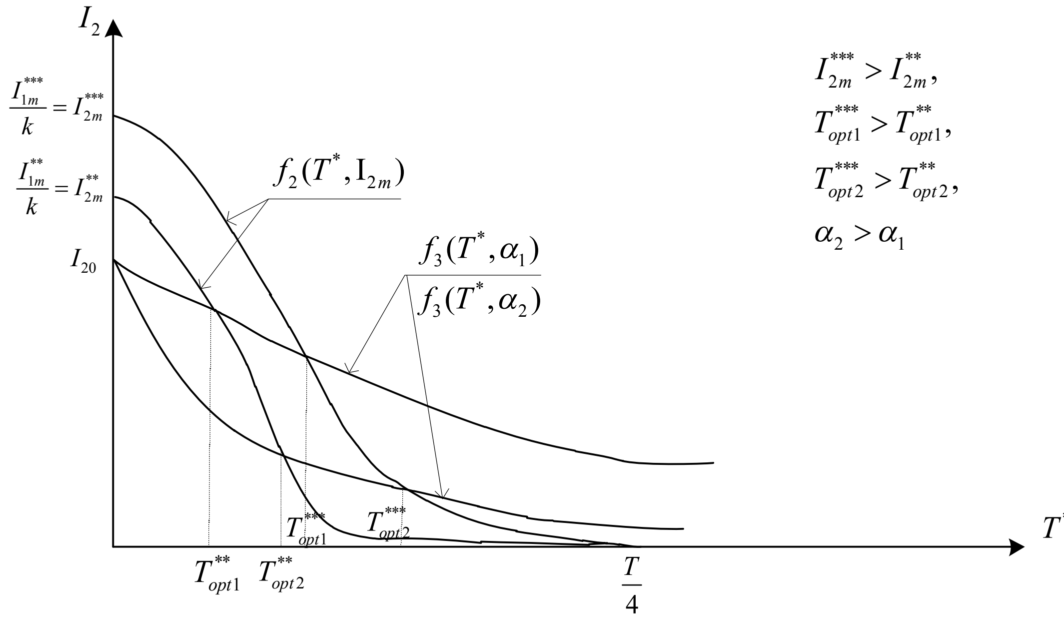

Fig. 3. Graphic interpretation of the procedure of function $I_2 = f_1(T^*)$ extremum points revealing

It follows from the equation (16) that the absciss of the intersection point of functions $f_2(T^*), f_3(T^*)$ graphs shown in Fig. 3, determines the value of T^* parameter, we denote it as T^*_{opt}, at this parameter function $f_1(T^*)$ has the extremum, that in our case is maximum, as the second derivative $\frac{d^2 I_2}{d(T^*)^2}$ from this function in this point is less than zero, this can be easily proved, taking the derivative from the expression (13) and substituting it in the obtained result T^*_{opt} instead of T^*.

In Fig. 3 it is seen that with the growth of the amplitude I_{1m} of the sinusoidal current, being measured, and, correspondingly, with the growth of its real value I_1,connected with the amplitude by the expression, analogous to the expression, written for the secondary current in the form (12), the coordinate T^*_{opt} which is the function

$$T^* = f_4(I_{1m}) = f_4(\sqrt{2}I_1) = f_5(I_1), \tag{22}$$

also increases, starting from the zero value at the boundary between non-saturated and saturated states of the core of the instrument current transformer and asimptotically approaching the value $\frac{T}{4}$, as it is shown in Fig. 4.

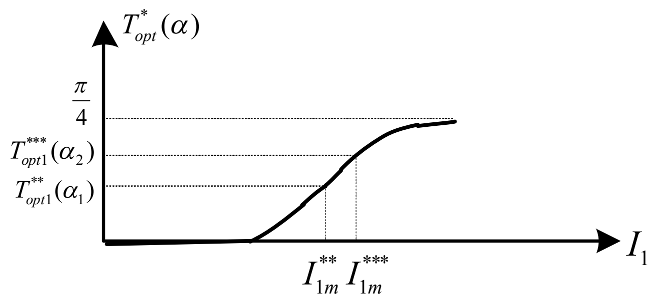

Fig. 4. Dependence graph $T^{*}_{opt} = f_4(I_{1m}) = f_4(\sqrt{2}I_1) = f_5(I_1)$

Substituting the expression (19) into (10), we will obtain

$$I_2 = \sqrt{\frac{I_1^2}{k^2T}\left(\frac{T}{4} - T^* - \frac{sin\,2\omega T^*}{2\omega}\right) - \frac{I_{20}^2}{\alpha T}\left(e^{-2\alpha T^*} - 1\right)} = f_1(I_1). \tag{23}$$

It follows from the expression (23) that until the core of instrument current transformer is non-saturated and the equality (11) is performed, the real value I_1 of the current in the secondary winding of this transformer is increase proportionally with the growth of the real value I_2 of the measured current. With the advent of the core saturation, accompanied with the growth of T^* value, the proportionality of I_2 increase with the growth of I_1 is violated and this growth becomes non-linear, reaching in the point with the abscissa T^*_{opt} maximum of I_{2opt}, that, according to the expression (23) will be set by the expression

$$I_{2opt} = \sqrt{\frac{I_1^2}{k^2T}\left(\frac{T}{4} - T^*_{opt} - \frac{sin\,2\omega T^*_{opt}}{2\omega}\right) - \frac{I_{20}^2}{\alpha T}\left(e^{-2\alpha T^*_{opt}} - 1\right)}. \tag{24}$$

It follows from the expression (24) that the numerical value of I_{2opt} maximum parametrically depends on, the numerical value of which, according to the expression (21) in its turn, depends on the ratio of the active resistance and the inductance of the circuit of the secondary winding of instrument current transformer. And as the hyperbola relatively α in the second member of radical expression in the ratio(24) with the decrease of α increases faster than the exponential curve with index, that comprises the same α drops, then with the decrease of α, the value of maximum I_{2opt} will increase, that is why the instrument current transformer will have better metrological indices, the less active resistance of its secondary winding circuit will be.

It after passing the maximum point of the current I_2 the current I_1 will continue to increase, causing the increase of T^* value, as it follows from the expression (23), the current I_2 will start to decrease, approaching at the value $T^* = \frac{T}{4}$ to the value

$$I_{2(\frac{T}{4})} = \sqrt{\frac{I_{20}^2}{\alpha T}(1 - e^{-\frac{\alpha T}{2}})}. \tag{25}$$

Graphic interpretation of the interconnection between the expressions (22) – (25) is performed in Fig. 5.

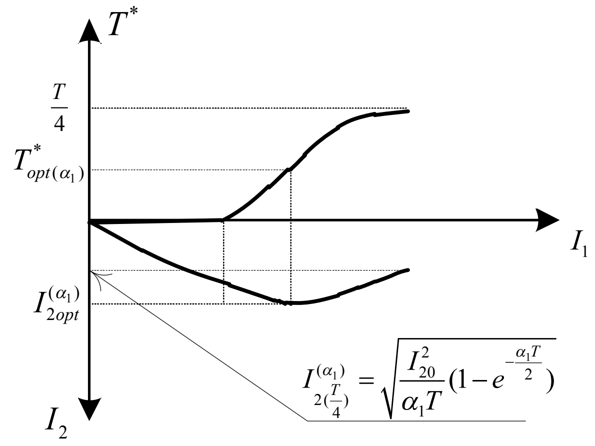

Fig. 5. Graphic interpretation of the interconnection between the expressions (22) – (25)

Having presented this graphic interpretation we complete the substantiation of the extremum occurrence on the graph of the mathematical model of instrument current transformer statics and falling section in this graph, presence of which leads to the fact that the same values of the current in the secondary winding of the instrument current transformer may correspond different values of the current, measured by this transformer.

Conclusions

1. Substantiation of the extremum occurrence on the graph of the mathematical model of instrument current transformer statics was performed, applying mathematical tools.
2. It is shown, that the output characteristic of the instrument current transformer after maximum point has falling section, the presence of this section results in the fact that to the same values of the current in the secondary winding of the instrument current transformer may correspond different values of current in its primary winding, i. e., the current, measured by the transformer.
3. It was mathematically proved that the instrument current transformer will have much better characteristics, the less active resistance of its secondary winding circuit will be.

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