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THE THEORY OF PLAFALES: QUADRUPLE ROLE OF THE BASIS FUNCTIONS OF SERENDIPITY FINITE ELEMENTS. REVIEW OF THE RESULTS

The paper presents review of the results obtained in constructing models of serendipity finite elements on the basis of the theory of plafales: clear understanding the role of basis functions of serendipity finite elements.

Keywords: *serendipity finite element, basis functions, plafal (-es,) information technology in FEM.*

Introduction

The history of the finite element method (FEM) started with the idea of R. Courant, an outstanding mathematician, which he made public in 1943 [1]. Initially, the researchers took no interest in the Courant's idea, as its realization required huge computational efforts. After the emergence of computers the method started to be actively developed by research engineers. And they, not mathematicians, occupied computers immediately in order to obtain answers to practical questions. Courant's procedure had become a new step in computational mathematics, though the influence of FDM (the method of finite differences) [2] was felt for a certain period of time (until the appearance of Turner's arbitrary triangulation). In 1954 Argyris [3] developed certain generalizations to the linear theory of structures and presented methods for investigation of discrete structures of complex configurations in a computer-friendly form. A year later he showed [4] that matrix equation of the system could be obtained by minimization of the system potential energy both for the stress and deflection methods. First formal FEM presentation along with the stiffness method belongs to Turner, Clough, Marting and Topp [5, 6], who for studying the problems of plane stress state used equations of the classical elasticity theory to describe the properties of triangular element. They applied matrix methods, intended for discrete structures, to continuous structures due to their division into finite number of elements. The term of "finite elements" was first introduced by Clough [7] in 1960. For approximation of two-argument functions traditional approach could be used, i.e. building approximation according to Lagrange (factorization of two one-dimensional Lagrange polynomials of the corresponding degree) and obtaining Lagrange finite elements (LFE) [8, 9]. Factorization leads to Lagrange elements having nodes in the middle of a finite element. Internal nodes increase the amount of computations and are not used for assembling finite elements. As to serendipity finite elements (SFE), they are deprived of these drawbacks. The primary aim of SFE creation is to provide the possibility of transforming an arbitrary quadrangle into a square and reducing the amount of computations by removing "extra" internal nodes. Such curvilinear element appeared in [10] for calculation of structures and was given the name of "serendipity finite element". Rapid development and popularization of FEM is explained by the professional background of its users. On the other hand, some believe (and not without reason) that lack of mathematical knowledge, characteristic of engineering-oriented professionals, was the main reason for emergence and spread of false hypotheses and inadequate models in FEM. Most errors were due to the construction of form functions (basis functions) of finite elements, in particular, the elements of serendipity family. A square with bilinear interpolation was first used as a computational template in 1964 [11]. This element is combined well with a triangular simplex, creating a simple and efficient FEM grid. Squares, as a rule, are efficient in the middle of the computational domain and triangles – in the boundary strip. In real two- and three-dimensional problems boundaries of the computational domain, boundaries between the elements as well as interfaces (in inhomogeneous environments) are often curvilinear [9, 11, 12]. Exactly this element was investigated by Ergatoudis, Irons and Zienkiewicz in 1968 [10]. It was an example of successful application of the isoparametric technique that consists in selecting piecewise polynomial functions in order to determine transformation of the coordinates [13]. The term "isoparametric" means that for coordinate transformation the same polynomials are selected as those interpolating a physical field, i.e. basis functions play a double role. In 1968 the authors did not take into account that basis functions play a triple role [10]. They are used in the problems of localization of the loads on a finite element. If there are internal nodes, transformation could be sensitive to displacements of these nodes. Probably, the authors [10] observed the feature and this was the reason for their abandoning the internal node of Lagrangian model. In the early 80-ies of the 20th century, when it became clear that the role of matrix algebra in FEM is exaggerated, geometrical approaches appeared [14] as well as stochastic procedures for constructing the bases [15, 16].

Analysis of the research

The paper is based on publications [17 – 26].

Research aim

The main aim of the research is to review the results of constructing the models of serendipity finite elements on the basis of the theory of plafales: clear understanding the quadruple role of the basis functions of serendipity finite elements and further application of the developed models (as algorithmic basis) for information technologies in FEM.

Current importance of the research

There is a possibility to create universal software-hardware complexes (SHC) as practical implementations of information technologies in FEM with artificial intelligence component for constructing form functions (basis functions) in automatic mode.

Main part

Serendipity models are an example of simultaneous interpolation and approximation: they interpolate a function at the boundaries of an element and approximate inside it. Main drawback of the standard SFE bases [8 – 11, 27 – 30] is unnatural per node distribution of the load from the unit bulk force: in angular nodes loads are negative (Zienkewicz paradox) [9]. Standard form functions (Zienkewicz bases) play a double role: they are used in isoparametric technique. Standard model has no degrees of freedom because it is constructed according to "rigid" [31] recipes of matrix algebra under Lagrange interpolation hypothesis. The number of additional monomes in SFE interpolant depends on the order of the corresponding LFE basis. First alternative SFE models appeared in 1982 [15, 16] due to the impossibility to find rational understanding of the unnatural per node distribution of the bulk force. At present there are several methods for building alternative models (32). SFE with negative loads in the nodes are not suitable for computer testing. Appearance of alternative serendipity models (which realize adequate distribution of uniform bulk force) is associated with probabilistic-geometrical method of the basis function construction, developed by Khomchenko [15, 16, 33 – 40]. In fact, A. N. Homchenko initiated and his followers further developed constructive (in the spirit of Bernstein [41]) theory of serendipity approximations, the results of which prove constructively the triple role of the basis SFE functions.

Quadruple role of basis functions

In publications [17 – 22] the following key aim was set: to prove constructively the quadruple role of SFE basis functions. The fourth role characteristic is t (time). A priori, software complexes, known in SFE, such as Nastran, Штуцер, Ansys, etc., as well as computer-aided design (CAD) systems, e.g. Solid Works, contain sets of bases in their algorithmic base, which were previously found by the researchers. At the same time, none of the modern software complexes and CAD systems contain alternative SFE bases, as only one information technology (in Turbo Pascal) was created by the students of A. N. Khomchenko for computer diagnostics of stationary physical fields [42]. So, there arises interest in creation of a new generation of universal software-hardware complexes (HSC), which solve the following classes of practical tasks:

1. Automatic mode of constructing optimal form functions of SFE (bases, which realize a theoretically substantiated and physically adequate distribution of nodal loads), using known computational templates.
2. Automatic mode of constructing optimal SFE bases with the use of computational templates, at which form functions have not been found yet, e. g. for regular n-triangles of $n = 2^{2^k} + 1, k \ge 2$ type [43].
3. Automatic mode of constructing optimal SFE bases, which satisfy Laplace differential harmonicity criterion [44], integral harmonicity criteria of Koebe and Privalov [45, 46].

Definitely, the above SHC is a practical implementation of information technology in FEM, which performs collection, processing, storing and displaying digital information for a user. This information technology and the results of the constructive theory of serendipity approximations could be used as a qualitative tool for further development of software complexes and CAD systems in FEM.

For the first class of problems an algebraic-geometrical method could be used [47] as an algorithmic basis of SHC. For ensuring realization of SHC line, which solves problems of the second and third classes, it is necessary to develop qualitative mathematical models and to employ artificial intelligence [48]. Among the infinite quantity of optimal SFE bases, which realize one and the same load spectrum, searching for the basis, satisfying differential and (or) integral harmonicity criteria, is an NP-hard problem (an exhaustive search problem) [49].

For successful solution of the second- and third-class problems the above-mentioned hardware-software complexes must perform comprehensive analysis of $L = L(x, y, t)$ of the given configuration, forming the surface of basis function $N(x, y) = L(x, y, T)$, where $T-$ time moment of surface $N(x, y)$ formation. An indispensible component of the analysis is investigation of intermediate surfaces $M(x, y) = L(x, y, T)$ (t = fix (fixed value), which are formed (could be obtained) within a certain time interval $t \in [0, T]$. A priori, having analytical form function $N(x, y)$, we can perform visualization (to obtain illustrative 3D images in space *x, y, z*) of non-stationary surface $L(x, y, t) = N(x, y) \circ T(t)$ ($\circ -$symbol of functions composition); in a separate case – $L(x, y, t) = N(x, y) \bullet T(t)$, *T(t)* – normalizing factor.

In the case, when basis function is viewed as a time function in an explicit form, e.g. for first-order SFE ($N_i(x, y, t) = \mu_1^{(i)}(t) + \mu_2^{(i)}(t)\, x + \mu_3^{(i)}(t)\, y + \mu_4^{(i)}(t) xy$, where i – node number), standard form functions could be obtained with the application of matrix algebra apparatus and taking interpolation hypothesis into account [8, 9, 29]. As a result, for bilinear interpolation basis the following identity is valid:

$$\begin{cases} N_i(x, y) \equiv N_i(x, y, T_i) = \mu_1^{(i)}(T_i) + \mu_2^{(i)}(T_i)\, x + \mu_3^{(i)}(T_i)\, y + \mu_4^{(i)}(T_i) xy, \\ \mu_1^{(i)}(T_i) = \frac{1}{4}, \mu_2^{(i)}(T_i) = \frac{1}{4} x_i, \mu_3^{(i)}(T_i) = \frac{1}{4} y_i, \mu_4^{(i)}(T_i) = \frac{1}{4} x_i y_i, \; x_i, \; y_i = \pm 1, i = 1,2,3,4. \end{cases} \quad \text{a}$$

In fact, with the application of time component, a new approach to constructing SFE bases appears, namely: the sought-for SFE form functions are logical consequence of comprehensive analysis of models $L = L(x, y, t)$.

In the strict sense and in a general form, $u = u(x, y, t)$ – three-dimensional topological manifold M^3 [50, 51] in four-dimensional space, $M(x, y)$ – projection (two-dimensional manifold M^2) of manifold M^3 on three-dimensional space. Thus, model of mappings ($Hom_{Top}(E^m, M^n)$ [52]) (Top – the category of topological spaces [52], E^m – m-dimensional Euclidean space [53]) is as follows:

1. $u : E^3 \to E^4, u = u(x, y, t)$ – monomorphism (in a general form), *x, y, t* – dimensions of E^3. Three-dimensional manifold M^3 is obtained as a result of mapping.

2. $f : M^3 \to E^3, f$ – monomorphism (in a general form), *x, y, z* – dimensions of E^3. Two-dimensional manifold M^2 (the perspective) is a result of mapping action.

With the application of the "theory of plafales" apparatus [24, 25], the procedure of obtaining surface $M(x, y)$ (projections of three-dimensional manifold M^3 on three-dimensional space) is as follows:

1. $u : PF_k^{U^{SP}} \cong E^2 \to E^3, u = u(x, y, t)$ – monomorphism (in a general form), *x, y, z* – dimensions of E^3. First-order surface E^2 (plane) is homeomorphic to the object of the theory of plafales – the static canvas of plafal $PF_k^{U^{SP}}$ [25, P. 16]. In terms of algorithmic complexity, the above operation is more optimal than the model consisting of two successive mappings, as sought-for manifold M^2 is obtained as a result of single mapping. The above mathematical component was incorporated (as an algorithmic component) into the newly-created information technology in C# for real-time rendering. Practical implementation of this technology is software complex "Testing non-stationary temperature fields with dynamic thermoelements" [23].

The developed mathematical models of SFE [17 – 22], based on the apparatus of the theory of plafales [24, 25], include configurations $L = L(x, y, t)$ on square and triangular templates and, consequently, simulate formation of non-stationary surfaces of field functions $U(x, y, t) = \sum_{i=1}^{m} N_i(x, y) \bullet U_i(t)$. Search for solution of all three classes of problems by SHC involves computer time and its power resources [54]. Time is a complex tool: it serves as a qualitative indicator of SHC and computer operation for processing the results of constructing basis and field functions. **Quadruple role of basis SFE functions has the following significance:** 1. They are used in isoparametric technique and in the problems of the distribution of loads on the finite element. 2. On 2D computational templates (square, triangle, etc.) basis function is a time function in the implicit form, namely, $N_i(x, y) = L_i(x, y, T_i)$. Qualitative properties and the requirements to SFE form functions result from the analysis of $L = L(x, y, t)$ models by SHC.

Conclusions

Using the apparatus of mathematics (the theory of categories), the paper shows the advantage of applying "the theory of plafales" apparatus for comprehensive analysis of $L = L(x, y, t)$ models as algorithmic bases of SHC of the second- and third-class problems. For second-class problems SHC develop a constructive (in the framework of the constructive theory of functions [41]) mathematical model (if necessary) on the basis of publications [17 – 22]. Quadruple role of basis functions has the following significance: 1. They are used in isoparametric technique and in the problems of distribution of the loads on a finite element. 2. On 2D computational templates (square, triangle, etc.) basis function is function of time in the implicit form, namely: $N_i(x, y) = L_i(x, y, T_i)$. Qualitative characteristics and the requirements to SFE form function result from the analysis of $L = L(x, y, t)$ models by SHC. Followers of the constructive theory of serendipity approximations (the school of A. N. Khomchenko) [23, 42] developed information technologies for testing stationary and non-stationary physical fields respectively.

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