UDC 519.816

O. Kovalyuk, post-graduate student

MARKOV PROCESSES COMMUNICATION IN DECISION MAKING MODELS IN DISTRIBUTED SYSTEMS

The features of decision making in distributed systems have been analyzed. The decision making model, based on markov processes theory, for improvement of model parameters has been proposed.

Keywords: *decision making, distributed system, markov processes.*

Intensive development of computer technique and telecommunication technologies has opened new possibilities for the solving of problem of decision making in the distributed systems. Actuality of problem is conditioned by the necessity of increase of control efficiency in such distributed systems as city transport system , communication networks etc. However, the increase of computation power does not guarantee control with necessary accuracy in the real time, which is mainly explained by complicated system structure, the presence of local control criterion, different kinds of uncertainty influencing in majority of distributed systems.

In the simplest cases of decision making the methods of discrete mathematics [1], allowing to formalize connections between the elements of the system, are utilized in the distributed systems. However the methods of this group found the limited application due to maladjustment to consider the uncertainty of input information.

Another approach to decision making in the distributed systems is related to the game theory which has a powerful mathematical tools for the solution of the applied tasks [2]. Lately the theory of active systems, the basic idea of which consists in presentation of the system as interactive agents, attracted the attention of specialists [3].

However the main lack of game theory and theory of the active systems is complication of description of connections between elements that imposes limitations on their use in control systems.

Thus the task of development of decision making model which takes into consideration the communication between the elements of the system appears.

Let us use Markov process tools [4, 5] for the solution of the given task. We will consider the usage of this approach on the example of city transport network control.

It is shown in [6], that traffic junction control system can be represented as an aggregate of the single-channel queuing systems with the limited queues. Control in such system consists in the change of service rate of traffic flows μ by adjustment of duration of traffic-light signals.

A control criterion is average losses $\overline{G}$ which are calculated as the product of average sojourn time $\bar{t}$ and average length of queue $\bar{r}$.

$$\overline{G} = \bar{r} \cdot \bar{t} \quad (1)$$

Thus, decision making model of traffic junction is

$$\begin{cases} \sum_{i=1}^{k} \overline{G_i} \to \min \\ \sum_{i=1}^{k} \mu_i \le T \end{cases}, \quad (2)$$

where $\overline{G_i}$ – average losses of i^{th} flow; μ_i – service rate of i^{th} flow; k - quantity of flows (queuing systems); T - period of control.

It is shown in queuing theory that parameters $\bar{t}$ and $\bar{r}$ has been calculated using ultimate probabilities of the states.

Average queue length:

$$\bar{r} = 1 \cdot \tilde{p}(2) + 2 \cdot \tilde{p}(3) + ... + (q-1) \cdot p(q) = \sum_{q=1}^{Q} (q-1) \cdot p(q), \tag{3}$$

where $\tilde{p}(q)$ – probabilitiy of q vehicles being at the traffic junction; Q – maximum vehicles quantity in a queue.

Average sojourn time in a queue

$$\bar{t} = \tilde{p}(1)\frac{1}{\mu} + \tilde{p}(2)\frac{2}{\mu} + ... + \tilde{p}(q)\frac{q}{\mu} = \sum_{q=1}^{Q} \tilde{p}(q)\frac{q}{\mu} \tag{4}$$

Use Markov process tools for modelling of traffic flows behavior, because traffic flow state usually depends on the state in a pervious moment of time. Consider as an example traffic network shown below.

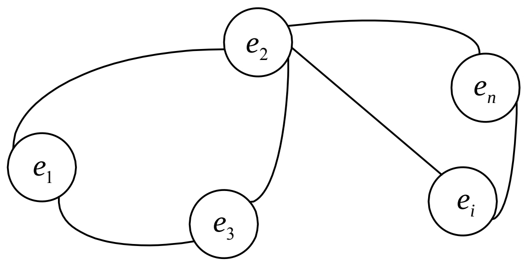

Fig. Traffic net part

Let n – number elements (traffic queues) that determine the general network state; m – maximum states quantity in the system. Denote by p_{ij} probability of element transition from state S_i to S_j. The element state S shows the quantity of traffic vehicles at the junction. Thus, transition probability of element is represented by the matrix

$$P = \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1m} \\ p_{21} & p_{22} & \cdots & p_{2m} \\ ... & ... & ... & ... \\ p_{m1} & p_{m2} & .... & p_{mm} \end{pmatrix}, \tag{5}$$

For the element of transport network Markov chain is ergodic, because an element can pass to any of the states for the finite number of steps. Another feature of this Markov chain is heterogeneity, conditioned by different intensity of traffic flow over a day, and therefore by different values of p_{ij}.

For heterogeneous chain the matrix (5) at fixed moments of time takes different values $P^{(1)}$, $P^{(2)}$,..., $P^{(k)}$. In this case elements of matrix (5) are the functions of the states of other system elements. For the system, presented in the Figure, transition probabilities of element e_1 will depend on the states of elements e_2 and e_3:

$$p_{1ij} = f\left(S_{2\tau_{21}}, S_{3\tau_{31}}\right), \tag{6}$$

where p_{1ij} – transition probability of 1^{th} element from state i to state j; $S_{i\tau_{i1}}$ – state of i^{th} element remote to τ_{i1} back from current moment of time; τ_{i1} –. time of influence pass from i^{th} to 1^{th} element, which measured in system work steps.

Convert equation (6) using transition probability of the states

$$p_{1ij} = \varphi\left(\widetilde{p}_{2\tau_{21}}, \widetilde{p}_{3\tau_{31}}\right). \qquad (7)$$

Thus transition probability is represented by the function

$$p_{vij} = \psi\left(\widetilde{P}^{(k)}, \widetilde{P}^{(k-1)}, ..., \widetilde{P}^{(0)}, C_v, \mathrm{T}\right), \quad v = 1...n, \quad i, j = 1...m, \qquad (8)$$

where $\widetilde{P}^{(k)}$ – matrix of state probabilities of the elements at step k. The size of matrix is $n \times m$; C_v – 4-demantion weight matrix which size is $[n, m, n+1, m+1]$; T – matrix of delays of influence transfer. The size of matrix is $n \times n$.

Let us analyze the sense of these matrixes.

The element $\widetilde{p}_{ij} \in \widetilde{P}$ determines probability of state j of element i. If maximum quantity of the states is $m_i < m$ then $\widetilde{p}_{ij} = 0$, $j \in [m_i, m]$.

The element $c_{vij}^{lh} \in C_v$ determines the influence of state h of element l on the vector of transition probabilities of v^{th} element. c_{vij}^{00} is transition probability of element v from i^{th} state to j^{th} state without taking into account the influence of other elements.

The elements of matrix $\tau_{ij} \in T$ are integers which show in how many steps state of i^{th} element will influence the state of j^{th} element.

Taking into account the dependence(7) in linearized form [7] let us represent transition probability of v^{th} element

$$p_{vij}^{(k)} = c_{vij}^{00} + \sum_{l=1}^{n} \sum_{h=1}^{m} \left(c_{vij}^{lh} \cdot \widetilde{p}_{lh}^{(k-\tau_{lh})}\right). \qquad (9)$$

Thus for heterogeneous Markov chain the probability that v^{th} element after k steps will be in j^{th} state is calculated by the formula

$$\widetilde{p}_{vj}^{(k)} = \sum_{i=1}^{m} \widetilde{p}_{vi}^{(k-1)} p_{vij}^{(k)} \qquad (10)$$

or

$$\widetilde{p}_{vj}^{(k)} = \sum_{i=1}^{m} \left\{ \widetilde{p}_{vi}^{(k-1)} \cdot \left[c_{vij}^{00} + \sum_{l=1}^{n} \sum_{h=1}^{m} \left(c_{vij}^{lh} \cdot \widetilde{p}_{lh}^{(k-\tau_{ij})} \right) \right] \right\}. \qquad (11)$$

Let us write down equations similar to (10) for the states of each element of the system. New system will contain $n \times m$ equations

$$\begin{cases} \widetilde{p}_{11}^{(k)} = \sum_{i=1}^{m} \widetilde{p}_{1i}^{(k-1)} p_{1i1}^{(k)} \\ \widetilde{p}_{vj}^{(k)} = \sum_{i=1}^{m} \widetilde{p}_{vi}^{(k-1)} p_{vij}^{(k)} \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ \widetilde{p}_{nm}^{(k)} = \sum_{i=1}^{m} \widetilde{p}_{ni}^{(k-1)} p_{nim}^{(k)} \end{cases} \qquad (12)$$

If transition probability of element to the next state at step k depends only on states at previous steps, equations of the system are independent and are solved separately. In the case when element state at step k will depend on the states of other elements at step k, right part of equations system (11) will contain probabilities from the left part of other equations.

Let us define item which depends on k in (11)

$$\begin{aligned}\widetilde{p}_{vj}^{(k)} &= \sum_{i=1}^{m}\left(\widetilde{p}_{vi}^{(k-1)}\cdot\left[c_{vij}^{00}+\sum_{\substack{l=1\\ \tau_{lv}>0}}^{n}\sum_{h=1}^{m}\left(c_{vij}^{lh}\cdot\widetilde{p}_{lh}^{(k-\tau_{lv})}\right)\right]\right)+\\ &+\sum_{i=1}^{m}\left(\widetilde{p}_{vi}^{(k-1)}\cdot\left[c_{vij}^{00}+\sum_{\substack{l=1\\ \tau_{lv}=0}}^{n}\sum_{h=1}^{m}\left(c_{vij}^{lh}\cdot\widetilde{p}_{lh}^{(k)}\right)\right]\right)\end{aligned}. \tag{13}$$

Let us transform the second item

$$\widetilde{p}_{vj}^{(k)} = \sum_{i=1}^{m}\left(\widetilde{p}_{vi}^{(k-1)}\cdot\left[c_{vij}^{00}+\sum_{\substack{l=1\\ \tau_{lv}>0}}^{n}\sum_{h=1}^{m}\left(c_{vij}^{lh}\cdot\widetilde{p}_{lh}^{(k-\tau_{lv})}\right)\right]\right)+\sum_{i=1}^{m}\left(\widetilde{p}_{vj}^{(k-1)}\cdot c_{vij}^{00}\right)+\sum_{i=1}^{m}\widetilde{p}_{vi}^{(k-1)}\cdot\sum_{\substack{l=1\\ \tau_{lv}=0}}^{n}\sum_{h=1}^{m}\left(c_{vij}^{lh}\cdot\widetilde{p}_{lh}^{(k)}\right).$$

Let us introduce the following designations

$$\sum_{i=1}^{m}\left\{\widetilde{p}_{vj}^{(k-1)}\cdot\left[c_{0ij}^{(v)}+\sum_{i=1,k_{ij}\neq\tau_{ij}}^{n}\sum_{j=1}^{m}\left(c_{ij}^{(v)}\cdot\widetilde{p}_{ij}^{(\tau_{ij})}\right)\right]\right\}+\sum_{i=1}^{m}\left(\widetilde{p}_{vj}^{(k-1)}\cdot c_{vij}^{00}\right)=b_{vj}\ .$$

Thus equation (13) will take the form

$$\widetilde{p}_{vj}^{(k)}-\sum_{i=1}^{m}\widetilde{p}_{vi}^{(k-1)}\cdot\sum_{\substack{l=1\\ \tau_{lv}=0}}^{n}\sum_{h=1}^{m}\left(c_{ij}^{lh}\cdot\widetilde{p}_{lh}^{(k)}\right)=b_{vj} \tag{14}$$

and system (11) will be written

$$\begin{cases}\widetilde{p}_{1j}^{(k)}-\sum\limits_{i=1}^{m}\widetilde{p}_{1i}^{(k-1)}\cdot\sum\limits_{\substack{l=1\\ \tau_{lv}=0}}^{n}\sum\limits_{h=1}^{m}\left(c_{1ij}^{lh}\cdot\widetilde{p}_{lh}^{(k)}\right)=b_{1j}\\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\ \widetilde{p}_{vj}^{(k)}-\sum\limits_{i=1}^{m}\widetilde{p}_{vi}^{(k-1)}\cdot\sum\limits_{\substack{l=1\\ \tau_{lv}=0}}^{n}\sum\limits_{h=1}^{m}\left(c_{vij}^{lh}\cdot\widetilde{p}_{lh}^{(k)}\right)=b_{vj}\\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\ \widetilde{p}_{mn}^{(k)}-\sum\limits_{i=1}^{m}\widetilde{p}_{mi}^{(k-1)}\cdot\sum\limits_{\substack{l=1\\ \tau_{lv}=0}}^{n}\sum\limits_{h=1}^{m}\left(c_{mij}^{lh}\cdot\widetilde{p}_{lh}^{(k)}\right)=b_{mn}\end{cases}. \tag{15}$$

The solution of the system (15) are probabilities of the states of elements at k^{th} step, which can be found by the known methods of solving of linear equations system. According to Kramer's rule

$$\widetilde{p}_{ij}^{(k)}=\frac{\Delta}{\Delta_j}, \tag{16}$$

where Δ – main determinant of the system; Δ_j – determinant, created by replacement of column j by column of absolute terms.

Conclusions

Thus the model of Markov processes communication in the distributed system has been developed, which allows to take into account communication between the elements of the system. The use of the offered model provides the accuracy of calculation of the states probabilities and increases the efficiency of system functioning. The model could be widely used for calculation of parameters of telecommunication networks, web sites and other complex objects.

REFERENCES

1. Зыков А. А. Основы теории графов. – М.: Наука ,1987. – 384с.

2. Дюбин Г. Н., Суздаль В. Г. Введение в прикладную теорию игр / Под ред. Н.Н. Воробьева. – М.: Наука, 1981. – 336с.

3. Новиков Д. А., Петраков С. Н. Курс теории активных систем. – М.: СИНТЕГ, 1999. – 104 с.

4. H. S. Chang, M. C. Fu, J. Hu, and S. I. Marcus A Survey of Some Simulation-based Methods in Markov Decision Processes // Communications in Information and System, 2007. – №7. – P. 59 – 92.

5. Дубовой В. М., Ковалюк О. О. Стійкість процесу керування транспортною мережею міста // Вісник Вінницького політехнічного інституту. – Вінниця, 2006.–№6.–С. 106 – 111

6. Дынкин Е. Б., Юшкевич А. А. Управляемые марковские процессы и их приложения. – М.: Наука, 1975. – 341с

7. Дубовой В. М., Ковалюк О. О. Оцінювання параметрів транспортних потоків міста // Вісник Хмельницького національного університету. – Хмельницький, 2005. – №4 .– С. 197 – 200

Oleh Kovalyuk — post-graduate student of Computer control systems Department.

Vinnitca National State University