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CONSIDERING THE POSSIBILITY TO USE MATHEMATICAL MODELS OF CUTTING-PACKING PROBLEMS FOR ARRANGING FLAT MUTUALLY-ORIENTED OBJECTS IN PRE-DEFINED AREAS WITH REGARD TO THE CONSTRUCTION AND ARCHITECTURE BRANCH

The paper aims at considering the possibility to use mathematical models of cutting-packing problems for arranging flat mutually-oriented objects in pre-defined areas in the construction and architecture field for creating architectural design solutions.

The most common cutting-packing problems and their mathematical models are considered in terms of relevance of the problem of arranging flat mutually-oriented objects in pre-defined areas to the branch of construction and architecture. Differences between the end tasks are indicated. The problem of arranging flat mutually oriented objects on pre-defined planes is formulated and its characteristic features are defined. Initial data and boundary conditions of arranging objects in pre-defined areas have been determined.

The necessity to create mathematical model of the problem of arranging flat mutually-oriented objects in pre-defined areas and to develop a corresponding information technology for the above problem simulation is substantiated.

Keywords: *cutting-packing problem, mathematical model, construction and architecture, mutually-oriented objects.*

Introduction

Innovations in different branches of economy determine the necessity to solve complex engineering problems and, accordingly, to create new models of technological processes. As an example of such problems we can distinguish processes of creating design solutions in the branch of construction and architecture and, particularly, with regard to performing finishing operations. These processes are inherently related to the optimization geometric modeling problem and must provide rational arrangement of flat mutually-oriented objects (taking into account engineering and technological requirements) in pre-defined location areas.

At present, it is important to develop information technologies that provide optimized technological operations, mentioned above, by modeling the processes of arranging flat mutually-oriented objects in the specified areas, elaboration of the methods for solving optimization problems in creation of design solutions and their implementation in the application software packages.

Recent research and publications

The problem of saving resources is relevant and timely for all spheres of human activities. One of the ways to address this challenge is to solve the problem of optimized arrangement of geometrical objects in pre-defined areas that have different ultimate goals depending on the direction of activities in the application field.

The geometric design task is to find optimal arrangement of the set of geometrical objects in the location area, taking into account technological limitations, in accordance with the qualitative arrangement criteria. Geometric design problems include lay-out of the equipment, control of complex technical systems, elaboration of master plans of industrial enterprises, designing electronic devices, optimal cutting of industrial materials, the problems of scheduling theory and project management, coating problems [1].

As an example, we will consider the problems of industrial cutting and architectural design.

In solving the industrial cutting (layout) problem it is important to create such cutting schemes, with which maximal set of workpieces could be obtained and, therefore, minimization of material waste is achieved.

In solving architectural design problems the ultimate goal of arranging flat mutually-oriented objects in pre-defined areas is full coverage of the location plane with geometric figures, arranged in definite sequence with definite location parameters, excluding negative areas of the given planes.

Problems of this type are termed “cutting-packing problems” and are referred to the class of NP-hard problems (NP - nondeterministic polynomial) [2]. For problems of this type polynomial algorithms have not been found, although it has not been proved that they do not exist. Due to the complexity of the mathematical description of the processes under study, specific results refer, as a rule, to the solutions of separate problems and are obtained under rather significant limitations concerning properties of the materials, constructive shapes, boundary and initial conditions.

The works of Stoyan Y. G. [3], Yemets O. A. [4], Grebennik I. V. [1], Chuprinka V. I. [5], Yaremchuk S. I. [6], Petrenko S. V. [7] and of other domestic and foreign scientists are concerned with solving various classes of arrangement optimization problems.

Those and other analogous studies are focused mainly on the problems of cutting (packing) the material in terms of waste minimization but do not give due attention to the problem of arranging flat mutually-oriented objects on pre-defined planes (particularly, in the field of construction and architecture).

Research aim

The aim of the research is considering the possibility to use mathematical models of cutting-packing problems for arranging flat mutually oriented objects in pre-defined areas in the field of construction and architecture for creation of architectural design solutions with regard to performing finishing operations. The paper also aims at substantiating the necessity to elaborate mathematical model of the problem of arranging flat mutually-oriented objects in pre-defined areas and to develop the corresponding information technology for modeling the above problem.

Presentation of the main material

Current research, concerned with the problems of optimized arrangement of geometrical objects in pre-defined areas, pay special attention to the problems of cutting-packing the material in various industries (mechanical engineering, light and wood processing industries, etc.) in terms of waste minimization.

For example, computer-aided design of cutting schemes in footwear and leather industries enables rational use of the material, when cutting it for workpieces, reducing the amount of waste polluting the environment, reducing the cost of goods, frequent changes of fashion for footwear and leather goods, using automatic complexes for cutting, which will improve the quality of goods by excluding human factor and satisfying the technological requirements to the process.

In the majority of applications of arrangement optimization problems it is necessary to organize packing the given set of objects within a certain area. In solving problems of this type, e. g. popular problems of dense arrangement, certain limitations are imposed on the arrangement of elements including conditions of non-intersection of the objects and their not going beyond the limits of the location area.

For solving cutting tasks a number of methods for arranging flat geometrical objects are used. Among them the methods, based on sequential placement of geometrical objects [7], are the most common.

Also, cutting tasks can be adequately formalized using Euclidean combinatorial optimization apparatus. In the simplest case it means the following: into semi-infinite strips, having equal widths, rectangles of the same width are placed without overlapping in order to minimize the maximum length of the occupied parts of the strips [4].

One of the ways to solve cutting problems is decomposition of the set of permissible solutions into convex subsets, replacing the initial problem solution with solution of the sequence of obtained subproblems, organizing a targeted enumeration of subsets and applying a

g-projection method for solving optimization problems [6].

In order to solve the problems of optimal arrangement of geometrical objects, the method of sequential single placement (a modification of Gauss-Zeidel method) is used. In the sequential single method of arranging geometrical objects all the objects are placed sequentially one by one. Pre-arranged objects are considered to be immovable. Each object is placed so that from all of its possible positions the one is chosen, with which target value of the function reaches the lowest value only for the variables that are parameters of the object to be placed [3].

Mathematical models of building dense packing layouts and lattice schemes for cutting roll materials are also used. For this the following structural components are determined and formalized: analytical description of the parts for which dense lattice packing is designed; parameters that uniquely define the component position on the plane; analytical representation of the mutual non-intersection conditions when the components are overlapped; mathematical representation of feasible solutions; analytical representation of the target function.

For solving cutting-packing problems a method, based on coverage of the zone of feasible solutions, is also proposed for optimization problems with linear target function and linear constraints on composite images of combinatorial sets. The method is based on the properties of composite images of combinatorial sets and functions, defined on them [1].

The method consists in covering the region with sets, which either do not contain points of the set or contain only the points, known beforehand. As a result of this coverage, such points are excluded, which do not belong to the region and, therefore, are not solution of the problem. Search for the problem solution is reduced to the analysis of finite and limited set of points found during the construction of sets, covering the region.

After considering the goals of the existing methods and ways for performing (realizing) the tasks of cutting - packing (on planes), we can make a conclusion that they are aimed, mainly, at rational arrangement of the elements of the given shape on a definite location plane in order to provide its efficient usage (in terms of waste minimization).

The problem of rational arrangement of flat mutually oriented objects on the given planes in solving architectural design problems (the branch of construction and architecture) consists in optimal orderly arrangement of the elements of definite shape with pre-definedd parameters of their location on the given plane, taking into account the forbidden planes, in order to provide efficient use of the objects to be located (waste minimization).

Below, by a simplified example, we will consider the possibility to solve the problem of arranging rectangular elements of various shapes in a strict order and providing full coverage of the location area, using:

a) cutting (packing) problem – the method of sequential single placement of geometrical objects [3] (Fig. 1a);

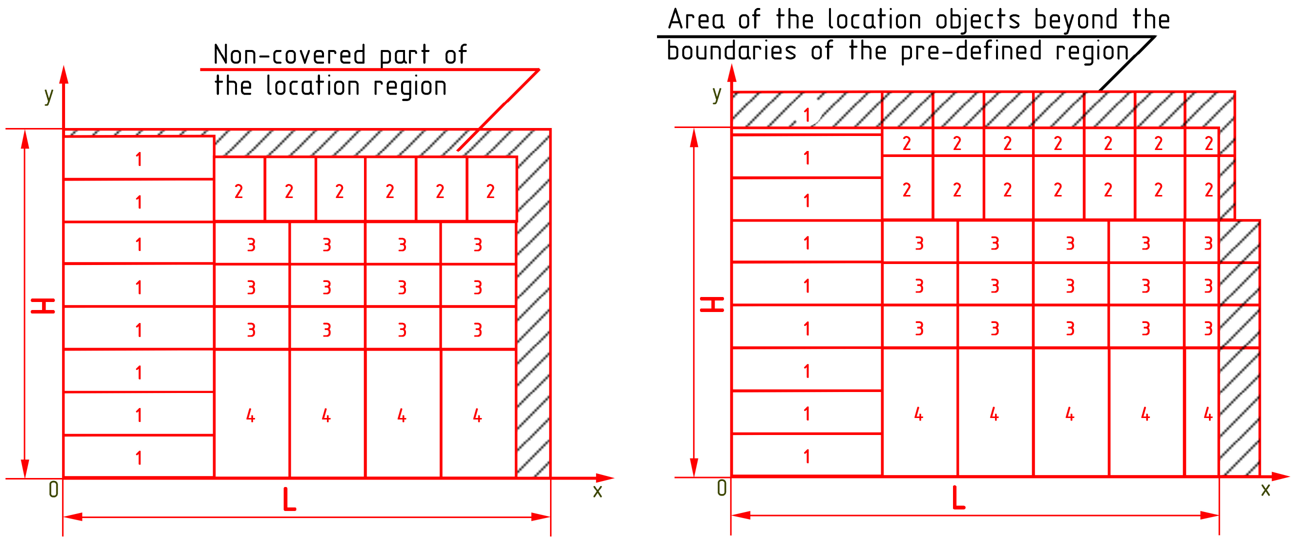
b) architectural design problem – arrangement of flat mutually oriented objects on definite planes (Fig. 1b).

Main conditions and differences between these problems are presented in comparative Table 1.

Table 1

Conditions and differences between the cutting (packing) problem and architectural design problem

Cutting (packing) problem	Architectural design problem
Pre-defined location area Ω with dimensions H, L.	*Pre-defined location area Ω with dimensions H, L*
Location objects are not permitted to go beyond the outer contour	. Location objects are permitted to go beyond the outer contour
Area of region Ω, covered with location objects, should be maximal.	The entire area of region Ω should be covered
The located objects should satisfy the mutual non-intersection condition and be at a predefined distance from each other	*The located objects should satisfy the mutual non-intersection condition and be at a predefined distance from each other*
A pre-defined region of possible locations is considered	A pre-defined region of possible locations is considered with the possibility of going beyond the limits of the given region
Sequence of the objects placement is determined by the cutting-packing problem	A strictly-defined sequence of elements placement is specified, depending on a design solution
Waste is a non-covered part of the location region Ω, which is impermissible in solving cutting (packing) problems	Waste is location objects area outside the limits of the specified area Ω, which is permissible in solving architectural design problems



a) Cutting (packing) problem b) Architechtural design problem

Fig. 1. Arrangement of the flat rectangular objects in the pre-defined location region

After consideration of conditions and differences between cutting (packing) problems and architectural design problems it could be stated that cutting (packing) problems do not provide the specified sequence of the elements location depending on the design solution and objects going beyond the outer contour (if necessary) for full coverage of the location region.

From the above it could be concluded that in spite of similar direction of realizing arrangement optimization problems, methods for solving cutting problems cannot be fully used for solving the problems of arranging flat mutually-oriented objects in the given areas (on the given planes) as the final tasks are completely different. Therefore, there is a necessity to create a mathematical model for the problem of arranging flat mutually oriented objects in the given areas and to develop a respective information technology of modeling the indicated problem.

In order to create the above mathematical model and to develop the corresponding information technology it is necessary to solve the following tasks:

– to determine initial data and boundary conditions;
– to formalize optimization problem of arranging flat mutually-oriented objects on the given planes;
– to adopt basic method for arranging geometric objects on a plane;
– to use this basic method for elaboration of the mathematical model and procedures for solving this problem;
– to develop algorithms of solving this problem;
– to create a corresponding information technology.

Conclusions

After reviewing the most common cutting and packing problems and their mathematical models in terms of relevance of the problem of arranging flat mutually-oriented objects in pre-defined areas to the branch of construction and architecture, indicating differences in the final tasks, it has been determined that the considered mathematical models of the cutting and packing problems cannot be used for arranging flat mutually-oriented objects in pre-defined areas in the branch of construction and architecture for creating architectural design solutions.

It has been recognized that there is a necessity to create mathematical model of the problem of arranging flat mutually-oriented objects in pre-defined areas and to develop a corresponding information technology for modeling the indicated problem.

REFERENCES

1. Гребеннік І. В. Математичні моделі та методи комбінаторної оптимізації в геометричному проектуванні : автореф. дис. на здобуття наук. ступеня докт. техн. наук : спец. 01.05.02 "Математичне моделювання та обчислювальні методи" / І. В. Гребеннік. – Харків, 2006. – 49 с.

2. Основные методы решения задачи фигурной нерегулярной укладки плоских деталей [Електронний ресурс] / Р. Т. Мурзакаев, В. С. Шилов, А. В. Буркова // Электронный научный журнал : Инженерный вестник Дона. – 2013. – Режим доступу до ресурсу: http://www.ivdon.ru/magazine/archive/n4y2013/2043.

3. Стоян Ю. Г. Математические модели и оптимизационные методы геометрического проектирования / Ю. Г. Стоян, С. В. Яковлев. – Киев: Наукова думка, 1986. – 259 с.

4. Емец О. А. О задачах оптимизации взаимного расположения прямоугольников в условиях стохастической, интервальной или нечеткой неопределенности / О. А. Емец, Т. Н. Барболина // Математичне та комп'ютерне моделювання. Серія : Фізико-математичні науки. – 2015. – Вип. 12. – С. 83 – 100.

5. Чупринка В. І. Розвиток наукових основ автоматизованого проектування схем розкрою деталей взуття та шкіргалантереї : автореф. дис. на здобуття наук. ступеня докт. техн. наук : спец. 05.18.18 "Технологія взуття, шкіряних виробів і хутра" / В. І. Чупринка. – Київ, 2009. – 35 с.

6. Яремчук С. І. Збіжність методу G-проекції / С. І. Яремчук, Л. В. Рудюк // Радиоэлектроника и інформатика. – 2004. – Выпуск № 4 (29). – С. 69 – 73.

7. Петренко С. В. Оптимизация размещения двумерных геометрических объектов на анизотропном материале с использованием методов математического программирования: дис. канд. техн. наук : 05.13.18 / Петренко Семен Васильевич. – Уфа, 2005. – 107 с.

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