O. B. Mokin, Dr. Sc(Eng.) , Professor., B. I. Mokin, Dr. Sc(Eng.)., Professor. Acad. of NAPS of Ukraine,V. A. Lobatiuk

OPTIMIZATION OF HYBRID AUTOMOBILE MOTION WITH NONRUNING ELECTRIC DRIVE SYSTEM

Optimization problem of motion of transport vehicle with combined drive from internal combustion engine and d.c. electric motor along the road, that besides horizontal sections contains uphills and downhills and the motion is realized on condition that electric drive system is disconnected.

Key words: *motion optimization, hybrid automobile, internal combustion engine, d.c. motor.*

Initial preconditions and problem set-up

In [1] we performed transformation of mathematical models of transport vehicles with combined drive both form internal combustion engine and d.c. electric current to the problem of their motion optimization along the road, that besides horizontal sections contains uphills and downhills, optimization criteria and limitations are chosen, the scheme of problem decomposition for the cases, when one of the drives, on some reason, does not function and when they create traction force on the shaft simultaneously.

In the given paper we will show how this optimization problem is solved, when as a result of storage battery discharge or as a result of the fault in electric system of hybrid automobile its electric drive is switched off and automobile moves only by means of internal combustion engine.

In this case, as it is shown in [1], mathematical model of automobile dynamics will have the form:

$$\frac{d\nu}{d\tau} = \frac{q}{\nu} - f_0 - f_1\nu - f_2\nu^2 \tag{1}$$

- in case, if automobile moves along horizontal section of the road;

$$\frac{d\nu}{d\tau} = \frac{q}{\nu} + f_0^*\sin\beta - f_0\cos\beta - f_1\nu - f_2\nu^2 \tag{2}$$

- in case, if automobile moves downhills;

$$\frac{d\nu}{d\tau} = \frac{q}{\nu} - f_0^*\sin\beta - f_0\cos\beta - f_1\nu - f_2\nu^2 \tag{3}$$

- in case, if automobile moves uphills.

As optimization criterion we will have the functional

$$e_q = \int_0^{\tau_q} q\,d\tau, \tag{4}$$

and as isoperimetric limitation we will have the functional

$$l_q = \int_0^{\tau_q} \nu\,d\tau \tag{5}$$

It should be noted that in the expression (1)-(5) ν, τ, q, e_q, l_q correspondingly – relative motion speed of automobile, relative time, relative fuel consumption, relative energy consumption and relative route; $f_0, f_0^*, f_1, f_2, \tau_q$ – relative parameters, and β - angle of slope of road bed to horizontal plane – all these relative values are determined by means of corresponding named units in [1], acquaintance with which is obligatory, prior to the acquaintance with the results, obtained in the given paper, that is why, in the given paper we will not commit to their definition.

In this research we will synthesize the dependences $\nu(\tau), q(\tau)$, that provide minimum of criterion (4), simultaneously, satisfying constrains (5) and one of constrains (1),(2) or (3), i.e., we synthesize the laws of optimal motion of hybrid automobile, when it moves only by means of internal combustion engine in case of disconnected electric drive system. This problem in [1] is defined the first in the suggested decomposition.

Solution of the set problem

The set problem we will solve, firstly, for the case, when hybrid automobile moves on the road, laid on the horizontal plane.

We will start the solution from the definition of Langrangian function, that, according to recommendations, given, for instance, in [2], [3], for our optimization problem, will have the form:

$$H^{(q)}(\nu,\nu',q,q',\psi,\psi',\tau)=q+\lambda_1(\nu'-\frac{q}{\nu}+f_0+f_1\nu+f_2\nu^2)+\lambda_2(\psi'-\nu) \tag{6}$$

As it is seen, the components of Langrangian function is integrand expression q of optimization criterion (4), multiplied by undetermined Langrangian multiplier λ_1 dynamics equation (1) and multiplied by undetermined Langrangian multiplier λ_2 that we will obtain from the functional (5), lowering upper boundary, introducing new symbol ψ for denoting of this functional after lowering of its upper boundary and differentiating this functional.

As it is known from the theory of variational calculus [2], [3] in order that the dependences $\nu(\tau),q(\tau)$ afforded minimum criterion (4), they must be found by means of the solution of equations system:

$$\begin{cases} H_{\nu}^{(q)}-\dfrac{d}{d\tau}H_{\nu'}^{(q)}=0, \\ H_{q}^{(q)}-\dfrac{d}{d\tau}H_{q'}^{(q)}=0, \\ H_{\psi}^{(q)}-\dfrac{d}{d\tau}H_{\psi'}^{(q)}=0, \end{cases} \tag{7}$$

where:

$$\begin{aligned} H_{\nu}^{(q)}&=\frac{\partial H^{(q)}}{\partial \nu}, H_{\nu'}^{(q)}=\frac{\partial H^{(q)}}{\partial \nu'}, \\ H_{q}^{(q)}&=\frac{\partial H^{(q)}}{\partial q}, H_{q'}^{(q)}=\frac{\partial H^{(q)}}{\partial q'}, \\ H_{\psi}^{(q)}&=\frac{\partial H^{(q)}}{\partial \psi}, H_{\psi'}^{(q)}=\frac{\partial H^{(q)}}{\partial \psi'} \end{aligned} \tag{8}$$

Substituting partial derivatives (8), taken from the expression (6), in the system of equations (7), we obtain the system of equations

$$\begin{cases} \lambda_1(\dfrac{q}{\nu^2}+f_1+2f_2\nu)-\lambda_2-\dfrac{d\lambda_1}{d\tau}=0, \\ 1-\dfrac{\lambda_1}{\nu}=0, \\ -\dfrac{d\lambda_2}{d\tau}=0 \end{cases} \tag{9}$$

From the third equation of the system (9) we obtain –

$$\lambda_2=-C_2, \tag{10}$$

where C_2 - unknown constant.

From the second equation of the system (9) we obtain –

$$\lambda_1 = \nu . \tag{11}$$

Substituting expressions (10), (11) in the first equation of the system (9), we obtain the equation –

$$\nu(\frac{q}{\nu^2} + f_1 + 2f_2\nu) + C_2 - \frac{d\nu}{d\tau} = 0, \tag{12}$$

that is easily reduced to the form, suitable for integration -

$$\frac{\nu}{2f_2\nu^3 + f_1\nu^2 + C_2\nu + q} d\nu = d\tau, \tag{13}$$

or

$$\varphi(\nu)d\nu = d\tau, \tag{14}$$

where

$$\varphi(\nu) = \frac{\nu}{2f_2\nu^3 + f_1\nu^2 + C_2\nu + q} \tag{15}$$

Integrating the expression (14) we will have –

$$\int \varphi(\nu)d\nu = \tau + C_1, \tag{16}$$

where C_1 - unknown constant.

The obtained integral (16) does not have exact analytical expression, that is why, we will take it approximately, making use of function (15) decomposition into Taylor power series in the neighborhood of the point $\nu = 0$, from which (to take into account non-linear character of this function) we use first three terms, i.e.,we present this function in the form -

$$\varphi(\nu) \approx \varphi(0) + \varphi'(0)\nu + \frac{\varphi''(0)}{2}\nu^2 \tag{17}$$

As $\varphi(0) = 0$ then the expression (17) is simplified to the expression-

$$\varphi(\nu) \approx \varphi'(0)\nu + \frac{\varphi''(0)}{2}\nu^2 , \tag{18}$$

or (taking into consideration the values of first and second derivatives from the function (15) into zeroes) to the expression –

$$\varphi(\nu) \approx \frac{1}{q}\nu - \frac{C_2}{q^2}\nu^2 \tag{19}$$

Substituting the expression (19) into (16) and taking integrals from power functions, we obtain -

$$\frac{1}{2q}\nu^2 - \frac{C_2}{3q^2}\nu^3 = \tau + C_1 , \tag{20}$$

or –

$$\nu^3 - \frac{3q}{2C_2}\nu^2 + \frac{3q^2}{C_2}(\tau + C_1) = 0 \tag{21}$$

This equation, by means of substitution

$$y = \nu - \frac{q}{2C_2} \tag{22}$$

is reduced to the form–

$$y^3 + 3py + 2a = 0, \tag{23}$$

where:

$$3p = -\frac{3q^2}{4C_2^2},$$
$$2a = -\frac{q^3}{4C_2^3} + \frac{3q^2}{C_2}(\tau + C_1) \qquad (24)$$

Equation (23), as it is shown in [4], refers to the class of equations, real positive root of which is found from the expression –

$$y_+ = (-a + (a^2 + p^3)^{\frac{1}{2}})^{\frac{1}{3}} + (-a - (a^2 + p^3)^{\frac{1}{2}})^{\frac{1}{3}} \qquad (25)$$

Substituting the expressions (24) into (25), and the results of this substitution into the expression (22), we obtain –

$$\begin{aligned} v = {} & (\frac{q^3}{8C_2^3} - \frac{3q^2}{2C_2}(\tau + C_1) + ((-\frac{q^3}{8C_2^3} + \frac{3q^2}{2C_2}(\tau + C_1))^2 + (-\frac{q^2}{4C_2^2})^3)^{\frac{1}{2}})^{\frac{1}{3}} + \\ & + (\frac{q^3}{8C_2^3} - \frac{3q^2}{2C_2}(\tau + C_1) - ((-\frac{q^3}{8C_2^3} + \frac{3q^2}{2C_2}(\tau + C_1))^2 + (-\frac{q^2}{4C_2^2})^3)^{\frac{1}{2}})^{\frac{1}{3}} + \frac{q}{2C_2} \end{aligned}, \qquad (26)$$

this will be mathematical model of motion speed of hybrid automobile with switched off system of electric drive, optimal at the criterion (4) of minimum fuel consumption of internal combustion engine.

This model, in general form may be presented as

$$v = F_v(q, C_1, C_2, \tau), \qquad (27)$$

remembering that fuel consumption q should be substituted, proceeding from the equation (1) of automobile motion dynamics, i.e., in the from –

$$q = v\frac{dv}{d\tau} + f_0 v + f_1 v^2 + f_2 v^3, \qquad (28)$$

In its turn, the expression (28) in general form may be presented as

$$q = F_q(v, v', \tau) \qquad (29)$$

Substituting the expression (29) into the expression (27), we obtain –

$$v = F_v(F_q(v, v', \tau), C_1, C_2, \tau), \qquad (30)$$

Parametric by τ expression (30) is parametric equation with one unknown v, for solution of which it is necessary to define, in advance, constants C_1, C_2. To do this, it is necessary, making use of the left (l) and right (r) boundary conditions –

$$v\big|_{\tau=0} = v_л,$$
$$v\big|_{\tau=\tau_q} = v_n \qquad (31)$$

and expression (30), compose two equations:

$$v_л = F_v(F_q(v_л, v'_л, 0), C_1, C_2, 0), \qquad (32)$$

$$v_n = F_v(F_q(v_n, v'_n, \tau_q), C_1, C_2, \tau_q), \qquad (33)$$

Having solved them relatively C_1, C_2, we find numerical values C_1^*, C_2^* of these constants. Substituting these numerical values in the expression (30), we obtain parametric equation-

$$v = F_v(F_q(v, v', \tau), C_1^*, C_2^*, \tau), \qquad (34)$$

that is the mathematical model of hybrid automobile optimal speed of motion at switched off system of electric drive during time τ_q, in the process of its motion only at the expense of internal combustion engine within the limits from the left to the right boundary of horizontal section of the road.

As the expression (34) is mathematical model of optimal speed of hybrid automobile motion, then the expression (29) derivative from it, will be mathematical model of optimal consumption of the fuel in the internal combustion engine while its motion along horizontal section of road.

Now we will find out, what will change if hybrid automobile at the same conditions moves uphills or downhills.

First, we should pay the attention to the fact, that in the equations (1), (2), (3) of automobile motion dynamics when this transport vehicle moves along horizontal section, the parameter f_0 takes place, and in case of uphills or downhills motion parameters $f_0 \cos\beta, f_0^* \sin\beta$ with corresponding sign take place. Then, it should be noted that in the equation (12), from which we got mathematical model (26) of motor vehicle optimal motion speed along horizontal section of the road, parameter f_0 in explicit form is missing. This means that in explicit form in the equation, from which we will obtain mathematical models of optimal speed of automobile motion on uphills and downhills and parameters $f_0 \cos\beta, f_0^* \sin\beta$, will be missing i.e., the structure of mathematical model of optimal speed of automobile motion does not depend on whether it moves on horizontal section of the road or it moves uphills or downhills.

But mathematical model of optimal consumption of fuel in internal combustion engine, that for the motion along horizontal section of the road had the form (28), for downhill motion will have the form

$$q = v\frac{dv}{d\tau} - f_0^* v \sin\beta + f_0 v \cos\beta + f_1 v^2 + f_2 v^3,, \quad (35)$$

that follows from the equation (2), for uphill motion it will have the form –

$$q = v\frac{dv}{d\tau} + f_0^* v \sin\beta + f_0 v \cos\beta + f_1 v^2 + f_2 v^3,, \quad (36)$$

that follows from the expression (3).

From the expressions (35), (36), it is seen the specific feature of automobile motion (on horizontal section, on uphills or downhills) is taken into account in the models of fuel consumption, and due to these models that are multiplicative components of speed models, it is taken into account in mathematical models of optimal speed of automobile motion.

Material, considering computation methods of synthesized models realization and to the account of time section τ_q variation, during which the automobile covers the section l_q of the road, we will present in one of the following publications.

Conclusions

1. Mathematical models of hybrid automobile optimal motion in case of switched off system of electric drive during its motion by means of internal combustion engine along the section of the road, laid on horizontal plane are synthesized.
2. It is shown how synthesized models are transformed to the motion conditions of hybrid automobile by means of internal combustion engine on uphills and downhills.

REFERENCES

1. Декомпозиція задачі оптимізації руху транспортного засобу з комбінованим приводом [Електронний ресурс] / О. Б. Мокин, Б. І. Мокин, В. А. Лобатюк, О. П. Кубрак // Наукові праці Вінницького національного технічного університету. – 2015. – № 3. – Режим доступу до журн.: http://praci.vntu.edu.ua/index.php/praci/article/view/9/9.

2. Петров Ю. П. Вариационные методы теории оптимального управления / Ю. П. Петров // Москва-Ленинград: Энергия, 1965. – 220 с.

3. Мокін Б. І. Теорія автоматичного керування, методологія та практика оптимізації / Б. І. Мокін, О. Б. Мокін // Вінниця: ВНТУ, 2013. – 210 с.

4. Бронштейн И. Н. Справочник по математике / И. Н. Бронштейн, К. А. Семендяев // Москва: Наука, 1967. – 608 с.

Mokin Olexsandr – Doctor of Science(Eng.), Professor, Head of the ***Department of Renewable Power Engineering and Transport Electric Complexes (RPETEC)***., e-mail: abmokin@gmail.com.

Mokin Borys – Acad. of NAPS of Ukraine, Doctor of Science(Eng.), Professor, ***Department of Renewable Power Engineering and Transport Electric Complexes (RPETEC***, e-mail: borys.mokin@gmail.com.

Lobatiuk Vitaliy -- Post graduate, Department of Renewable Power Engineering and Transport Electric Complexes (RPETEC).
Vinnytsia National Technical University.