O. Sofina, Cand. Sc. (Eng), Assist. Prof.; A. Lozun

DEVELOPMENT AND STUDY OF THE MODIFIED ALGORITHM OF FRACTAL CODING

The paper suggests modified algorithm of fractal coding using discrete cosine conversion, Haar wavelet – transform and parallel computations. The aim of the developed algorithm is to decrease computational complexity of fractal coding algorithm and increase the rate of image compression.

Key words: *image compression, fractal coding, discrete cosine conversion, wavelet – transform, parallel computations.*

Introduction

Nowadays as a result of wide spreading of electronic – computing equipment storage and processing of various types of data is realized in digital form.

Multimedia types of data – video – audiorecordings and digital images become more and more popular. One of the most actual problems of modern information technologies is the development of efficient methods of multimedia data compression, particularly – graphic information .Proceeding from the above- mentioned, one of the most urgent problems of modern information technologies is the development of efficient methods of multimedia data compression, particularly, graphic information.

Most frequency compression methods with losses are applied to multimedia information. Such approaches are valid as in case of multimedia objects it is possible to give up from saving their certain peculiarities (for instance, tiny elements on the image), that enables to increase the degree of compression. But there exist widely spread algorithms of the compression without losses, such as FLAC for audiofiles or PNG for digital images. Using these formats it is worth taking into account that they were designed as universal for certain type of data and, as a result they turned out to be useless for taking into consideration the peculiarities of the specific file to be compressed. As a consequence, considerable worsening of compression factor as compared with the analogous compression algorithms with losses, occurred.

For digital images, as the class of multimedia data, the most widely spread compression format with losses is JPEC. It became widely used as a result of digital cameras, scanner, etc. Taking into account the amount of images, presented in JPEG format, losses, connected with storage, transfer and processing of, probably, not optimally (by the quality and compression degree) compressed information, become evident.

Hence, studies, dealing with compression methods, become actual. They are based, on different image representation, namely, fractal coding, characteristic feature of this method is the feature of image self similarity. However, the existing methods of fractal compression of images require considerable development, taking into account numerous criteria (in particular, operation speed compression degree, decompression quality) to be considerable as real alternative to JPEG for numerous classes of images, being used in scientific, engineering and every day spheres of human activity.

A great deal of research was performed by native (D. C. Vatolin, V. V. Sergeev, V. V. Suifer, V. V. Alexsandrov, N. I. Gurskiy) and foreign (M. Barnsli, A. Jasquin, Y. Fisher, D. Zauper) scientists [1 – 3]. But, still there is much room for further research in this direction.

The aim of the paper is the construction of the modified algorithm of fractal coding , that will increase the speed of image compression process and achieve the acceptable compression factor, combining the already known DCT technologies, wavelet transform and parallel computations.

Main problems, to be solved for achieving this aim are development and study of optimal methods of image division into domain and ranking blocks, development of index and descriptor techniques of domain and ranking blocks matching and combining of the developed techniques for creation of the integral algorithm.

Division of the image into domain and ranking blocks

Image is divided into ranking blocks of the same size r and into all possible domain flock of 2r dimension. The given set of domains is called basic one. Further, additional set of domains is formed in the following manner: for each of “basic” domains seven transforms are built, namely:turns of the block at 90°, 180° and 270°, mapping of the block relatively the vertical axis of symmetry and turns of the obtained mapped block also at 90°, 180° and 270°,. As a result of combination of the basic and additional sets of domains the expanded set of domain blocks is formed. For each ranking block the search among expanded set of domains is carried out. For this purpose each ranking and domain block is given index and descriptor.

This method of image division into domain and ranking blocks is optimal because each block may be definitely identified by its consecutive number (in case of preliminary set fixed method of numeration): for saving the number of block, fewer number of bits is needed, for instance, than for the coordinates of its upper angle. One more advantage is that there is no need to indicate the size of each block in the compressed file : it is sufficient to indicate only once the size of ranking block, and also the coefficient of proportionality for determination of domain block size.

Calculation of indexes for domain and ranking blocks

In order to calculate index for each block b, matrix B of its two – dimensional discrete cosine transform (DCT) is calculated. Proceeding from the analysis of [4] the following algorithm of index calculation using the obtained DCT matrices is suggested:

- “Upper left angle” of matrix B, that corresponds to current block b (submatrix $\widehat{\mathrm{B}} = \{\mathrm{B}_{ij}\}_{i,j=\overline{1,3}}$) is considered;
- In accordance with the order of elements selection for calculation of the index, given in Fig. 1, elements of these submatix from the 1^{st} to t^{th} where t – selected dimensionality of the index $t \in [1, 8]$) are evaluated. If current element is less than zero, then corresponding digit of the index in binary representation, starting from the left, value 0 is assigned, if greater or equal zero – value 1.

	1	5		
2	4	6		
3	7	8		

Fig. 1. Order of elements selection for index calculation

When turns or mapping change rule is applied to block b, DCT matrices B are set by such equations:

$$\begin{cases} F'_{ij} = (-1)^{i+1} F_{ij}, f'_{ij} = f_{N-i,j}, \\ F'_{ij} = (-1)^{j+1} F_{ij}, f'_{ij} = f_{i,N-j}, \\ F'_{ij} = F_{ij}, f'_{ij} = f_{i,j}, \end{cases} \quad (1)$$

where $i, j = \overline{1, N}$.

It follows that DCT-matrices (and, correspondingly, indices) of “additional” domains may be found rather simply, on the base of pre -calculated DCT – matrices of “basic” domains.

Calculation of the descriptors for domain and ranking blocks

For calculation of the descriptors two-dimensional Haar wavelet-transform is used [5]. The following algorithm of descriptor formation (for block b, of p×p pixcels dimension, where p=2q, q∈N) is suggested:

– coefficient e_1 is calculated;

$$e_1 = \frac{1}{p^2} \sum_{i=1}^{p} \sum_{j=1}^{p} b_{ij} . \tag{2}$$

– block is divided into four equals subblocks b_2, b_3, b_4 and b_5.

– we calculated the coefficients in succession

$$\begin{cases} e_2 = e_1 - \frac{1}{q^2} \sum_{i=1}^{q} \sum_{j=1}^{q} b_{ij} \\ e_3 = e_1 - \frac{1}{q^2} \sum_{i=1}^{q} \sum_{j=q+1}^{q} b_{ij} \\ e_4 = e_1 - \frac{1}{q^2} \sum_{i=q+1}^{q} \sum_{j=1}^{q} b_{ij} \\ e_5 = e_1 - \frac{1}{q^2} \sum_{i=q+1}^{q} \sum_{j=q+1}^{q} b_{ij} \end{cases} \tag{3}$$

that correspond to blocks b_2, b_3, b_4 and b_5.

As the index calculation, description formation for additional domain may be simplified, namely: instead of calculation, using the above-mentioned algorithm e = [$e_2 e_3 e_4 e_5$], simple rearrangement of descriptor’s components of the basic domain may be performed in accordance with the specific transformation of the block.

Generalizing algorithm of optimal domains search

1. The first ranking block r_1 is considered.

2. Subset D_{r_1} of the expanded set D of the domain is formed by means of sampling of all domains with completely or partially coinciding index value, applying the method of hierarchy search (domains, that belong to D_{r_1}, are called those, possessing “primary similarity” with ranking block, being considered or simply “primary”).

3. Descriptor of r_1 block is compared by turns, with the descriptors of each of domains, belonging to D_{r_1}.

4. Subset D_1 of the set D_{r_1}, is formed by means of selection of certain number of domains, the descriptors of which possess maximum degree approximation to ranking block r_1 in the sense of Euclidian distance (domains, that belong to D_1 are called those, possessing “secondary similarity” with ranking block, being considered, or simply “secondary”).

5. Among the found “secondary” domains domain d_1, is being searched and brightness and contrast coefficients s_1 and o_1, corresponding to the pair $r_1 - d_1$ proceeding from MSD minimization conditions: $d_1 = argmin(\text{СКВ}(r_1, \tilde{\tilde{d}}_1))$, where $\tilde{\tilde{d}}_1 = s\tilde{d}_1 + oE$, $\tilde{d}_1$ - block d_1, compressed to the size of block r_1, and E – single matrix of block r_1 size. Domain block, for which the given MSD minimization condition will be applied, is called optimal or the best.

6. If the domain d_1 found at the previous step belongs to “basic” set of domains, then its consecutive number v_1, is fixed, if this domain to the “additional” set – number of transformation τ_1, is fixed, by means of which it was obtained from the “initial” domain and the number of the given “initial” domain $\tilde{v}_1$.

7. The second ranking block $r_2$is considered for this block steps 2 - 6 are repeated, then ranking

block r_3 is considered and so on , until optimal domain d_N is found for the last ranking block r_{N_r}, where N_r – number of ranking blocks.

Fig. 2 shows the scheme of the suggested algorithm of optimal domains search, that takes into account paralleling of computations on the level of ranking blocks, as for each of them a certain amount of independent operations must be performed.

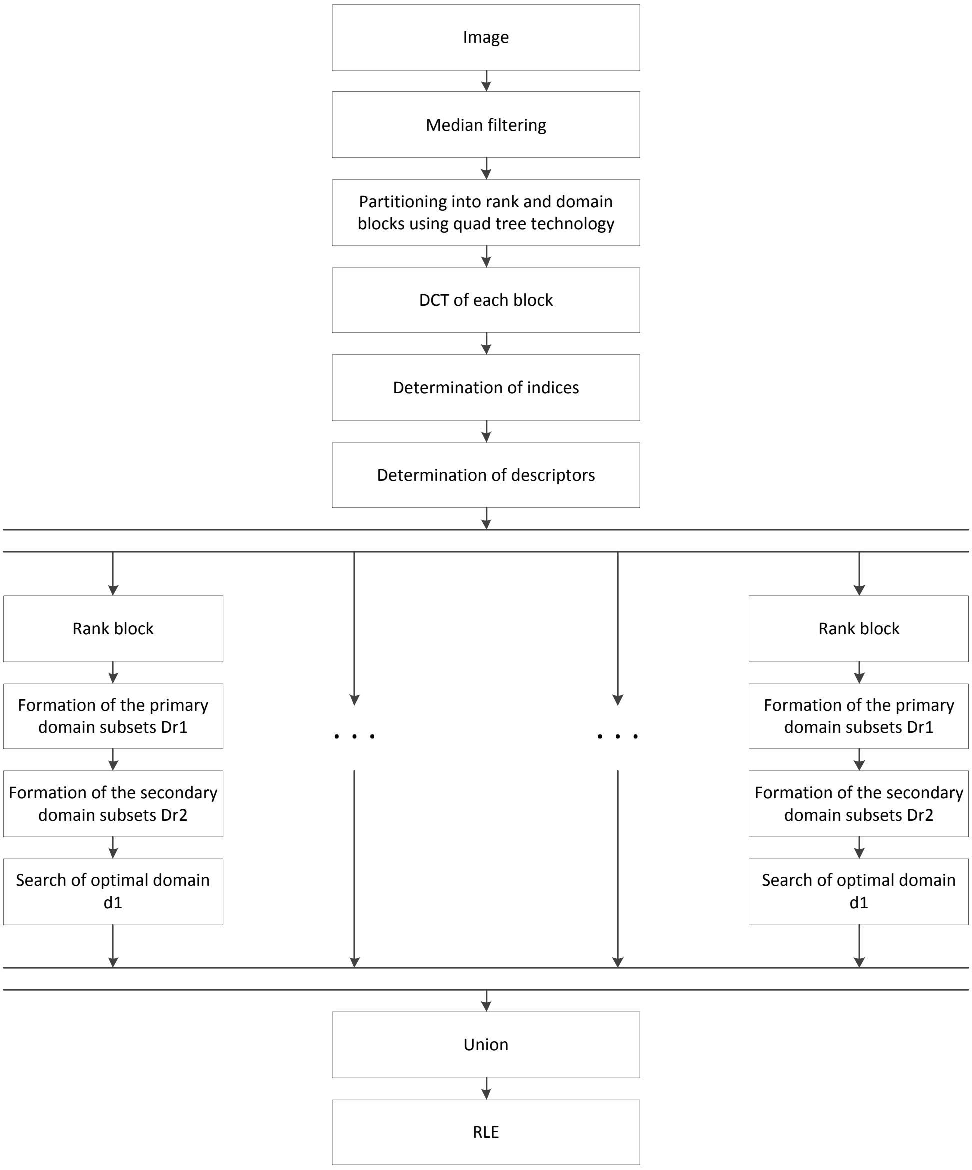

Fig. 2. Algorithm of optimal domains search

Modification of standard deviation formula calculation

Modification of standard deviation formula calculation is carried out to decrease computation load on the system.

General formula of values transformation of domain block pixels has the following form:

$$D_i^* = sD_i + o, \tag{4}$$

where D_i^* and D_i transformed and initial i-th domain block, correspondingly ; s – coefficient of contrast change; o – coefficient of brightness shift.

For evaluation of the discrepancy (distance) between the transformed domain and the given ranking blocks it is necessary to introduce the corresponding metric. As a rule, function of standard deviation is used:

$$Q=\sum_{i=1}^{N}(D_i^*-R_i)^2=\sum_{i=1}^{N}\big((sD_i+o)-R_i\big)^2, \quad (5)$$

where R_i – i-th ranking block; D_i^* and D_i – transformed and initial i-th domain block, correspondingly; N – number of pixels in ranking block.

It is obvious, that the less is the distance between blocks, the more they are similar.

Coefficients s and o can be found from the formula (5), taking partial derivatives by these variables. Let us remove the brackets in the expression (5):

$$Q=\sum_{i=1}^{N}s^2D_i^2+2soD_i+o^2-2sR_iD_i-2oR_i+R_i^2. \quad (6)$$

We have the following condition:

$$\begin{cases}\frac{\partial Q}{\partial s}=\sum_{i=1}^{N}(2sD_i^2+2D_io-2R_iD_i)=s\sum_{i=1}^{N}D_i^2+o\sum_{i=1}^{N}D-\sum_{i=1}^{N}R_iD_i=0\\ \frac{\partial Q}{\partial o}=\frac{1}{N}\sum_{i=1}^{N}R_i-\frac{1}{N}s\sum_{i=1}^{N}D_i\end{cases} \quad (7)$$

We express the shift by brightness:

$$o=\frac{1}{N}\sum_{i=1}^{N}R_i-\frac{1}{N}s\sum_{i=1}^{N}D_i \quad (8)$$

Let us substitute (8) in the equation of partial derivative by s (7) and we obtain the following formulas to find the coefficients:

$$\begin{cases}s=\frac{N\sum_{i=1}^{N}R_iD_i-\sum_{i=1}^{N}R_i\sum_{i=1}^{N}D_i}{\sum_{i=1}^{N}D_i^2-\left(\sum_{i=1}^{N}D_i\right)^2}\\ o=\frac{1}{N}\left(\sum_{i=1}^{N}R_i-s\sum_{i=1}^{N}D_i\right)\end{cases} \quad (9)$$

Having transformed the formula (3), we obtain the expression to find the distance:

$$Q=s^2\sum_{i=1}^{N}D_i^2+No^2+\sum_{i=1}^{N}R_i^2-2s\sum_{i=1}^{N}R_iD_i+2so\sum_{i=1}^{N}D_i-2o\sum_{i=1}^{N}R_i \quad (10)$$

Formulas (6, 7) allow to simplify the computation loading, because the sums $\sum_{i=1}^{N}R_i$, $\sum_{i=1}^{N}R_i^2$, $\sum_{i=1}^{N}D_i$, $\sum_{i=1}^{N}D_i^2$ can be calculated still before the exhaustive search, when the sets of ranking and domain blocks are formed. Then, on the stage of comparison it is necessary to calculate only the sum $\sum_{i=1}^{N}R_iD_i$ and find the coefficients.

Experimental results of developed method testing

First we perform the study of classical algorithm of fractal compression. For this purpose the testing sample was formed, it comprises the following, different by content and structure, types of images, met most frequently:

- land images of the scenes of artificial objects;
- land images of natural objects;
- aerospace images;
- astronomic images;
- images of non-structured objects, such as clouds or smoke.

Fig. 3 contains the examples of these classes of images that further will be used for investigation.

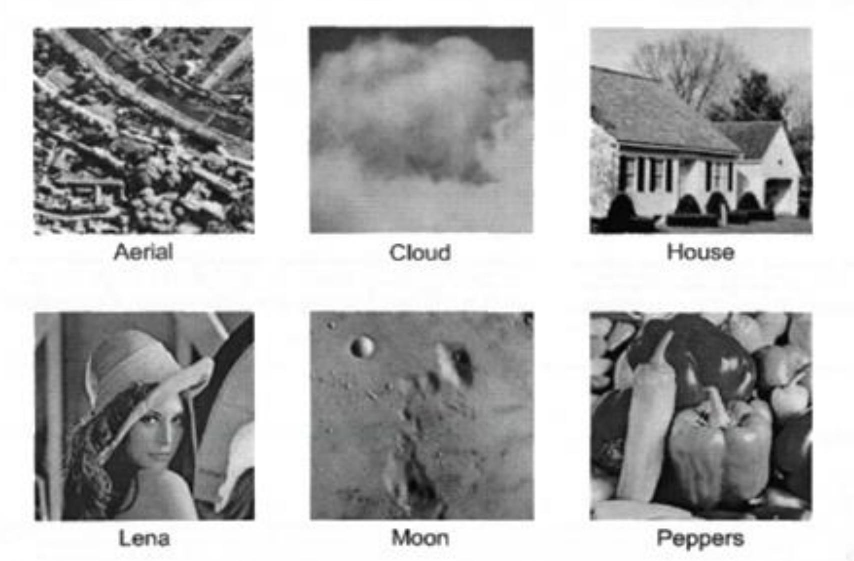

Fig. 3. Test images

Let us consider the indices of efficiency, obtained by means of classis algorithm of fractal compression. Table 1 contains the values of coder operation time, standard deviation of the restored image after decompression from initial image, signal/noise ratio and also the components of the criterion of minimum length of description (MLD) and its resulting value for images of various size - 60, 120, 180 and 240 pixels, compressed at the fixed size of ranking block, that is 6 pixels. We will define MLD criterion.

Principle of minimal length of description will be formulated in the in the following way: among the set of models it is necessary to select the model, that enables to minimize the sum of model description length (in bits) and the length of data, described, applying this model (in bits). The term “model” implies certain information structure, representing coded image. As the length of model description we will consider volume L_{img} of the compressed image in bits.

Length of data, described by means of applying the model implies that part of data, that didn’t enter in the model itself. In other words, this is the volume of the losses L_{loss} as a result of compression. Hence, MLD criterion can be calculated by the formula:

$$L = L_{loss} + L_{img} \quad (11)$$

We will consider designations, used in Table 1. Here N – size of the image, t – time of computation in seconds, σ – standard deviation of the restorted image from the original, PSNR – signal – noise ratio, L_{loss} – evaluation of the volume of information losses during compression in bits, L_{img} – evaluation of the length of the compressed file in bits, L – sum of L_{loss} values and L_{img} – minimal length of compressed image description. Minimization of this length is realized at all possible correspondences of domain and ranking blocks at their fixed dimensions.

Table 1

Results of testing of classic algorithm for images of different size

N	t	σ	PSNR	L_{img}	L_{loss}	L
Aerial						
60	0,82	32,365	16,685	3022	25233	28255
120	16,04	21,336	21,547	13014	92574	105588
180	91,10	13,840	25,307	30421	187285	217706
240	287,56	12,329	26,311	55485	322719	378204
Cloud						
60	0,82	3,343	35,133	3023	12913	15936

120	15,97	1,720	40,903	13014	38951	51965
180	87,24	1,146	44,429	30421	69578	99999
240	286,70	1,121	44,665	55485	124105	179590
House						
60	0,82	21,403	21,520	3023	21725	24748
120	16,03	13,743	25,367	13014	78847	91861
180	86,64	9,088	28,959	30421	159497	189918
240	301,86	8,765	29,138	55485	281237	336722
Lena						
60	0,82	14,308	24,121	3023	20531	23554
120	15,94	8,918	28,561	13014	69354	82368
180	86,44	6,216	31,731	30421	135884	166305
240	288,86	6,086	31,916	55485	238511	293996
Moon						
60	0,82	8,141	29,916	3023	16859	19882
120	16,04	5,685	32,652	13014	62723	75737
180	86,43	4,100	35,806	30421	127659	158080
240	298,76	4,414	35,196	55485	235437	290922
Peppers						
60	0,82	16,944	22,345	3023	21298	24321
120	15,97	9,946	27,757	13014	71099	84113
180	86,34	6,019	32,538	30421	134361	164782
240	284,15	5,334	32,767	55485	228856	284341

Let us consider how high is the compression quality, using classic fractal algorithm. Rather good quality of the restored image is considered to be PSNR~30, then Table 3 contains evaluations of compression coefficients in case of image restoration with such quality by means of division of output number of bits in the image by the value of $L_{img.}$ The given value corresponds to the closest to 30 value of PSNR in Table 1. Besides, Table 3 also contains compression coefficients for JPEG format in case of regulations , corresponding to PSNR~30.

Table 2

Compression coefficients at PSNR~30

Image	Compression coefficient	
	Fractal algorithm	JPEG
Aerial	3,6	4,4
Cloud	59,4	47,3
House	8,2	8,0
Lena	14,7	15,8
Moon	33,3	24,2
Peppers	14,8	15,1

It is worth mentioning, that when standard programming facilities are used, JPEG compression coefficient may be for worse as a result of nonoptimal regulation recording of large volume of additional information in the file.

As it is seen from the Table compression coefficients differ greatly for various images. Now we will test the developed modified algorithm of the fractal coding and compare the results. As the main drawback of classic algorithm is rapid growth of operation time while increase of image size, its practical usage is inexpedient. The developed methods of domain and ranking blocks correspondences search in fractal coding were described in the given paper. We will perform experimental testing of the developed algorithm and determine losses of image quality. As the growth of operation rate does not influence the size of the compressed file, it will be sufficient to consider the value of standard deviation and PSNR for the restored image.

Table 3 contains characteristics of the quality of the restored image, obtained after compression by the modified algorithm and also its operation time on the images of various size.

Table 3

The results of modified algorithm testing

N	*t*	σ	PSNR	L_{img}	L_{loss}	*L*
Aerial						
60	0,09	35,010	16,120	3022	25626	28648
120	0,48	24,425	20,373	13014	95273	108287
180	1,68	16,261	23,906	30421	194581	225002
240	3,93	14,431	24,943	55485	335408	390893
Cloud						
60	0,09	4,021	33,532	3023	13659	16682
120	0,50	1,992	39,628	13014	41788	54802
180	1,74	1,312	43,251	30421	75539	105960
240	4,01	1,235	43,780	55485	131567	187052
House						
60	0,09	25,520	19,992	3023	22599	25622
120	0,47	17,643	23,198	13014	82894	95908
180	1,60	11,260	27,098	30421	167840	198261
240	4,14	15,148	24,522	55485	300273	355758
Lena						
60	0,09	17,702	22,158	3023	21563	24586
120	0,53	10,83	27,429	13014	73525	86539
180	1,66	7,389	30,374	30421	143374	173795
240	4,10	7,076	30,497	55485	248954	304439
Moon						
60	0,09	9,981	27,097	3023	17621	20644
120	0,45	6,564	31,785	13014	65029	78043
180	1,62	4,755	34,342	30421	133436	163857
240	4,05	5,000	33,909	55485	244204	299689
Peppers						
60	0,09	22,30	20,072	3023	22770	25793
120	0,48	12,073	26,181	13014	75027	88041
180	1,60	7,388	30,303	30421	142956	173377
240	4,04	6,416	31,671	55485	241339	296824

As it is seen form Table 3, operation rate is not only higher, as compared with classic algorithm (30 times on the images of 120 pixels of size) but also has other dependence on image size. As a result on the images of 60 pixels of size, we obtain 8 multiple gain, and on the images of 240 pixels of sixe – we obtain 70 – 75 multiple gain.

Comparing Tables 1 and 3 it may be noted that quality criterion of minimal length of description (MLD) as a result of application of optimized algorithm worsens approximately by 4 %. For part of images, constant worsening (for instance, from 1.5% to 3.5% for Aerial) is observed at the increase of image size, where as for other images the reduction of worsening may take place. Thus, gain in operation rate as compared with classic algorithm grows when image size increases, where as the loss in compression coefficient does not almost depend on the size or does not depend at all.

In the given work the application of parallel computation is suggested. It should be noted that paralleling of computations was already applied for optimized algorithm of fractal coding, that was described above. In Fig. 3, there is a graph, that represents the results of comparison of serial and parallel algorithm. As the paralleling of computations does not depend on the structure of the image and also does not influence the quality of the restored image, four images of different size were chosen for testing. Hence, we will consider the dependence of compression time on the size of the image. Operation of parallel algorithm in the given case is realized for 16-processor system.

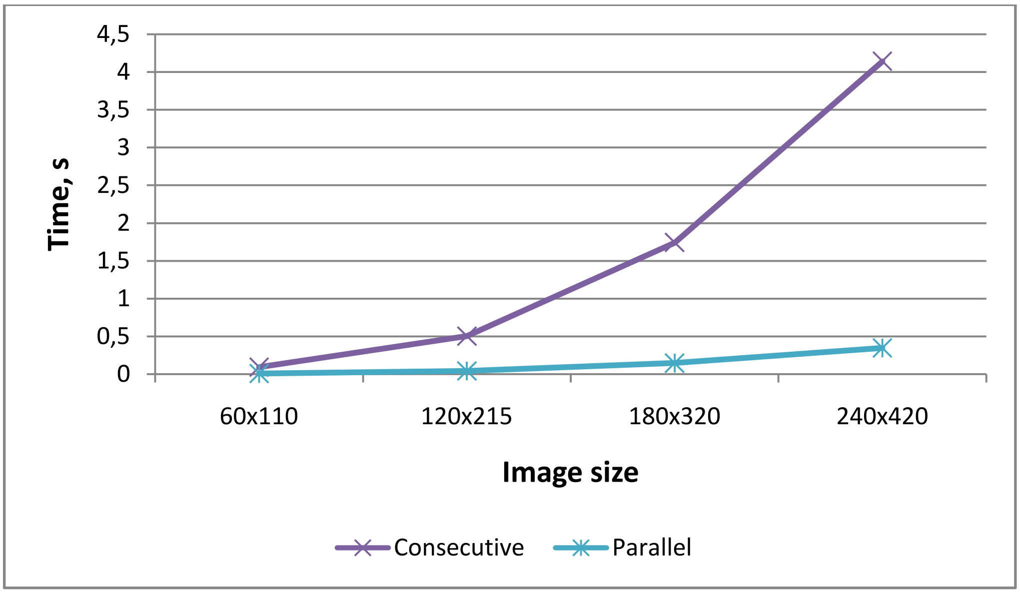

Fig. 4. Dependence of execution time on the size of the image

As it is seen, the rate of images processing applying parallel algorithm is far higher, than operation time of serial algorithm. It was fund out that the usage of each additional processor will increase the operation rate by 60 – 75%.

Conclusions

The developed algorithm of optimal domains search, that is modified algorithm of fractal coding, enables to increase image processing rate due to the usage of DCT, Haar wavelet transform and parallel computations and gives the possibility to decrease computation loading on the system.

As fractal coding refers to NP – complete problems, acceptable solution of such problems is achieved only when efficient subject-dependent heuristics are used.

Greater part of studies, dealing with fractal compression are dedicated to the development of similar types of heuristics.

For suboptimal realization of fractal compression it is necessary to limit exhaustive search of domain blocks, ranking blocks and corresponds between them and this was proposed to do in the given paper. Eurisitcs must cut off unpromising ways of image division into ranking blocks without performing further compression steps, and in case of fixed division of the image into ranking blocks – cut off unpromising subsets of domain blocks for certain ranking blocks.

In the given paper the formation of subjects – specific heuristics, was realized by means of the most widely used methods, namely DCT coefficients and wavelet –decomposition of the corresponding blocks. These transformations give the possibility to construct features, that insignificantly distort pixel- wise degree of blocks similarity.

Decrease of computational complexity is achieved by means of reducing pixel-wise comparisons, due to the usage of indices and descriptors. Pixel – wise comparisons are realized only in case of evaluation of correspondence of secondary domains to ranking one. Also, modified formula of standard deviation calculation influences the decrease computational load, because the value of sums and sums square of domain and ranking blocks may be calculated prior to the start of exhaustive search.

Thus, modified algorithm of fractal compression is suggested, its efficiency is confirmed experimentally.

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Sofina Olga – Cand. Sc. (Eng.), Assistant Professor with the Department of Automation and Instrumentation Technologies.

Lozun Alla – Master with the Department of Automation and Instrumentation Technologies.
Vinnytsia National Technical University.