## Yu. G. Vedmitskiy, Cand. Sc. (Eng.); V. V. Kukharchuk, Dc. Sc. (Eng.), Prof. TRANSIENT COMPLEX CIRCUITS, KIRCHHOFF'S LAWS AND THE COMPONENT RELATIONS IN COMPLEX-TEMPORAL FORM OF REPRESENTATION


#### Abstract

Notion of transient complex scheme of electric circuit is suggested and developed in the paper, its basic properties - Kirchhoff's laws and component relations in complex-temporal form of representation are determined. They in their unity are principle elements of theoretical basis of symbol-classic method of Cauchy problem solution, formulated in terms of instantaneous complex currents and voltages for calculation of transient processes in linear electric circuits of sinusoidal and periodic non- sinusoidal currents.


Key words: electric circuit, transient process, Cauchy problem, complex-symbolic method, instantaneous complex currents and voltages, Kirchhoff's laws, component relations, differential equations of motion, switching laws in complex form.

## Introduction

Development and improvement of the methods of calculation and analysis of transient processes in electric circuits of sinusoidal current still remains important and urgent problem while investigating electric engineering systems with periodic form of motion.

Such status of the problem is stipulated by needs of practical origin - these needs are various both by the depth of penetration and by the volume of scope[1]. This is explained, on one hand, by the increased and even critical sensitivity of operating parameters and characteristics of the abovementioned engineering systems, for instance, objects of electric power engineering or electromechanics, to transient process modes, that can be observed in them even during one production cycle, and on the other hand - by the volume of the region of such systems and their total impact to technogenic sphere on the whole.

At the same time purely theoretical component of the above-mentioned problem [2] remains important, as the system of trigonometric functions with even frequencies of the form $\cos \left(m \frac{2 \pi}{T} t\right)$ and $\sin \left(m \frac{2 \pi}{T} t\right)$, where $m=(1,2, \ldots)$, is one of the fundamental and most frequently used orthogonal normalized basis of Hilbert space of instantaneous voltages and currents, elements of which are able to determine evolution motion of each of the above-mentioned engineering systems not only in steady-state operation mode but also during transient process. In the latter case physical transient process in the system should be accompanied by simultaneous change of all the coefficients of trigonometric Fourier series in time, and on condition of period $T$ stability - even is able to be in relation of equivalence to such change. For linear or linearized electric engineering systems with periodic form of motion, this means, that Cauchy problem, that, as it is known, is mathematical interpretation of the problem of transient process analysis in physical and engineering systems with concentrated parameters, may be formulated and solved relatively each harmonic component from trigonometric series separately, then find solution, applying the principle of superposition. At the same time, as Fourier coefficients of each of transient harmonics, for instance, current, directly determine its amplitude and initial phase, as a result, the harmonic has the form

$$
\begin{equation*}
i^{(m)}(t)=I_{m}^{(m)}(t) \sin \left[m \omega t+\psi_{i}^{(m)}(t)\right] \tag{1}
\end{equation*}
$$

it is expedient to formulate Cauchey problem not in terms of instantaneous currents $i^{(m)}(t)$ (or voltages), but in terms of their complex images, i.e. instantaneous complex currents

$$
\begin{equation*}
I_{-m}^{(m)}(t)=I_{m}^{(m)}(t) e^{j y_{i}^{(m)}(t)}, \tag{2}
\end{equation*}
$$

where

$$
\begin{equation*}
i^{(m)}(t)=\operatorname{Im}\left\{I_{-m}^{(m)}(t) e^{j m o t}\right\} . \tag{3}
\end{equation*}
$$

Using such an approach, as it is shown in [3], both sets of functions $I_{m}^{(m)}(t)$ and $\psi_{i}^{(m)}(t)$, and in case of change during transient process period $T(t)$ - these are functions $m \omega(t)$, their first and higher derivatives, integrals, integral transformations both separately and in different combinations are able to describe and reveal not only visible, but also hidden and abnormal properties of transient processs in electric circuit both in qualitative form and analytically.

It is worth mentioning that theoretical electric engineering the problem of analysis of transient processes in electric circuits of sinusoidal current nowadays is able to solve systematically and efficiently solve practically in all its manifestations. For this purpose various calculation methods are proposed, namely--- classical, operator, spectral, method of Duhamel integral, etc. The essence of these methods is revealed in numerous scientific and educational sources, for instance [4-13]. At the same time, as research described in these sources, shows, fundamental Cauchy problem is formed in them mainly in terms of instantaneous currents and voltages. Such an approach prevails in cases when, in order to solve this problem, integral transformations, namely, integrals of Fourier, Laplace or Karson, are used. Unfortunately, the possibility of Cauchy problem formation relatively functions of instantaneous complex currents of the form (2) in the given reference is not revealed, and is not even mentioned. This, probably, explains the fact that methodical support of such an approach at present is not sufficiently studied.

The aim of the given research is development on the base of the function of the form (2), the notion of transient complex scheme of electric circuit, determination of characteristics features and fundamental properties of such circuit, in particular, Kirchhoff's laws and component relations in complex-temporal from of representation, development of basis principles and rules of its construction. The developed theoretical notions and relations in their total, must become important elements of theoretical basis of symbolic-classical method - one of information methods of calculation and analysis of transient processes in linear circuits of sinusoidal current [3].

## 1 Differential equation of transient process in complex form

It should be noted, that now and further the results of research will be given only regarding the first (that is, basic) harmonics of Fourier trigonometric series, however, relations, written for it, taking into account the order of multiplicity, will maintain its validity also for higher harmonics.

It is known, that transient process in linear electric circuit with several sources of sinusoidal voltage of the same frequency

$$
u_{1}=U_{m_{1}} \sin \left(\omega t+\psi_{u_{1}}\right), \ldots, u_{v}=U_{m_{v}} \sin \left(\omega t+\psi_{u_{v}}\right)
$$

can be described by ordinary linear differential equation of $n$-th order composed relatively certain instantaneous current $i(t)$ of the form (1), on the base of Kirchhoff's laws and component relations,

$$
\begin{equation*}
\sum_{k=0}^{n} a_{k} \frac{d^{k} i}{d t^{k}}=\sum_{q=1}^{v} \sum_{s=0}^{w} b_{s q} \frac{d^{s} u_{q}}{d t^{s}} \tag{4}
\end{equation*}
$$

with $n$ initial conditions: $i\left(0_{+}\right), \frac{d i\left(0_{+}\right)}{d t}, \ldots, \frac{d^{n-1} i\left(0_{+}\right)}{d t^{n-1}}$.

At the same time, taking into account the relation (3), Cauchy problem, determined in the method (4), is admissible, and in many cases - is expedient, to formulate in another way - in terms of instantaneous complex currents and voltages. In such interpretation transient process in the circuit will be presented by differential equations in complex form, added by $n$ initial conditions of the form: $\underline{I}_{m}\left(0_{+}\right), \frac{d \underline{I}_{m}\left(0_{+}\right)}{d t}, \ldots, \frac{d^{n-1} \underline{I}_{m}\left(0_{+}\right)}{d t^{n-1}}$. The equation itself will have the form:

$$
\sum_{k=0}^{n} A \frac{d^{k} I_{-m}(t)}{d t^{k}}=\sum_{q=1}^{v}\left(\begin{array}{ll}
B & \cdot U  \tag{5}\\
-0_{q} & -_{m_{q}}
\end{array}\right) .
$$

Complex coefficients in the equation (5) may be determined in different ways.
For instance, they can be calculated by means of coefficients of differential equation (4) but on condition that such equation for the given circuit would be obtained in advance. As it is shown in [3], for electric circuit of random order (without preset number of degrees of freedom) the law of transformation of equation coefficients (4) in corresponding coefficients of equation (5) will be in subordination to binomial theorem and will have the following form:
because, firstly, each $k$-th derivative of instantaneous current $i(t)$ in the left part (4) may be written through instantaneous complex current $\underline{I}_{m}(t)$ and its derivatives as

$$
\begin{equation*}
a_{k} \frac{d^{k} i}{d t^{k}}=\operatorname{Im}\left\{a_{k} \cdot \sum_{p=0}^{k}\left[\frac{k!}{p!(k-p)!} \cdot(j \omega)^{p} \cdot \frac{d^{k-p} I_{-m}(t)}{d t^{k-p}}\right] \cdot e^{j \omega t}\right\} \tag{7}
\end{equation*}
$$

that creates the possibility for the next rearrangement and reduction of the obtained expression to the form of the left part of the formula (5):

$$
\sum_{k=0}^{n}\left\{a_{k} \cdot \sum_{p=0}^{k}\left[\frac{k!}{p!(k-p)!} \cdot(j \omega)^{p} \cdot \frac{d^{k-p} I_{-m}(t)}{d t^{k-p}}\right]\right\}=\sum_{k=0}^{n}\left\{\sum_{p=0}^{n-k}\left[\frac{(k+p)!}{k!p!} \cdot(j \omega)^{p} \cdot a_{k+p}\right] \cdot \frac{d^{k} I_{-m}(t)}{d t^{k}}\right\}
$$

and, secondly, each $s$-th derivative in the right part of the equation (4) also may be written in the similar manner and on this base, taking into account the invariability in time of the amplitudes, initial phases and frequency of external sources of energy may be rewritten in complex form, reduced to the form of the right part of the formula (5):

$$
\sum_{q=1}^{v}\left(\sum_{s=0}^{n} b_{s q} \frac{d^{s} u_{q}}{d t^{s}}\right)=\operatorname{Im}\left\{\sum_{q=1}^{v}\left[\sum_{s=0}^{n}\left((j \omega)^{s} \cdot b_{s q}\right) \cdot U_{-m_{q}}\right] \cdot e^{j \omega t}\right\}=\operatorname{Im}\left\{\sum_{q=1}^{v}\left(\begin{array}{cc}
B_{-0_{q}} & U \\
-m_{q}
\end{array}\right) \cdot e^{j \omega t}\right\} .
$$

At the same time, it is worth mentioning, that besides the given indirect method, the coefficients of the equation (5), as the equation itself, may be obtained directly by means of transient complex scheme of the investigated circuit and Kirchhoff's laws in complex - temporal form of representation, that enables to avoid the necessity of anterior composing of equation (4) and determination of its coefficients with further recalculation by the formulas (6).

## 2 Transient complex circuits, Kirchhoff's laws and component relations in complex temporal form of representation

Instantaneous complex currents and voltages are independent analytical objects, that is why, differential equation of transient process of the form (5) may be built by single rules in convenient and generally accepted in theoretical engineering method, that is, by means of topologically structured and imperatively subordinate objects.

Let us introduce the notion of transient complex scheme, we will use this notion to denote topologically structured by two terminal elements object with instantaneous complex currents in the branches and voltages at its sections, mathematically interconnected by Kirchhoff's laws and component relations, presented in complex - temporal form of representation.

Kirchhoff's laws in complex-temporal are the base for formation of the system of integraldifferential equations, composed relatively instantaneous complex currents in branches or voltages on them, that further could be used for construction of differential equations of the form (5) or (4), and in independent manner - for more profound and accurate study of transient processes in electric circuits.

Let us formulate these laws. First Kirchhoff's law in complex - temporal form states, that algebraic sum of instantaneous complex currents of branches, that coincide in the nodes of transient complex scheme at any moment of time equals zero:

$$
\begin{equation*}
\sum_{l=1}^{h} I_{-m_{l}}(t)=0 . \tag{8}
\end{equation*}
$$

Second Kirchhoff's law in complex-temporal form, in its turn, states, that algebraic sum of instantaneous complex voltages at separate elements of random closed contour of transient complex scheme always and constantly equals zero:

$$
\begin{equation*}
\sum_{l=1}^{c} U_{-m_{l}}(t)=0 . \tag{9}
\end{equation*}
$$

The technique of composition of equation system by Kirchhoff's laws in complex - temporal form does not differ from the fixed one.

It is worth mentioning that these Kirchhoff's laws in complex - temporal form are postulates: as these laws are characterized by greater degree of generalization, it is impossible neither to introduce them nor substantiate by means of Kirchhoff's laws, written relatively instantaneous voltages and currents (that is - in classic form). At the same time such generalization connects both forms and makes impossible the emergence of contradictions between them. Validity of Kirchhoff's laws in complex - temporal form not only remain valid but provides capacity to function and ability of their classical analogues. Such character of relation between known and inductivity introduced forms of Kirchhoff's laws is very important because it serves necessary condition of truth of the latter. Simultaneously, confirm and approve such status, or on the contrary - refute it, this is to be done by practical verification, strict, durable and detailed.

One of important consequences of the above - mentioned is that Kirchhoff's laws in complextemporal form due to its generalization subordinate not only steady-state modes of electric circuits of sinusoidal current operation, but also the course of transient processes unlike Kirchhoff's laws in complex form, which, as it is known are built on the base of classic laws for theoretical support of widely-spread method of complex amplitudes (symbolic method) in order to calculate steady-state modes in these electric circuits.

For calculation and analysis of transient processes besides Kirchhoff's laws in complex temporal form of representation (8) and (9) it is necessary to use special mathematical equations, which in theoretical electric engineering are called component relations. Component relations establish mathematical connections between instantaneous voltages and currents at separate
idealized passive elements of electric circuit.
It is natural, that in our case these connections must be rewritten in complex temporal form. It is important to underline, that in such connections between instantaneous voltages and currents of separately taken main passive two-terminal elements of electric circuit important laws of electric engineering, namely, Ohm's law and Farraday's law of induction, find their reflection. That is why, component relations, represented by complex-temporal form also must be direct manifastation of these laws.

It follows from the above-mentioned that transient complex schemes must include the elements with such properties, at which instantaneous complex currents and voltages will correspond to instantaneous currents and voltages at separate elements of real electric circuits. Such mutual single -valuedness must become the element that will be able to provide necessary adequacy of the obtained results of the research to the realities of the output problem during the analysis of dynamic modes on the base of transient complex schemes.

Thus, for main passive two-terminal elements of electric circuit, namely, - active resistance, inductance and capacitance, we form the component relations in complex-temporal form on the base of classic relations, written for instantaneous currents and voltages.

Resistive element. For the given two-terminal element mathematical connection between instantaneous voltage and current is determined by Ohm's law [8]:

$$
\begin{equation*}
u(t)=R i(t) \tag{10}
\end{equation*}
$$

where $u(t)=U_{m}(t) \sin \left[\omega t+\psi_{u}(t)\right], i(t)=I_{m}(t) \sin \left[\omega t+\psi_{i}(t)\right]$, that, taking into account (1) (3) allows to obtain relation for their complex images in the form of Ohm's law in complex temporal form:

$$
\begin{equation*}
\underline{U}_{m}(t)=R \underline{I}_{m}(t) . \tag{11}
\end{equation*}
$$

Inductive element. Mathematical connection between instantaneous voltage and current at this two-terminal element is stipulated by Faraday's law of induction, on the base of this law corresponding component relation is introduced [8]

$$
\begin{equation*}
u(t)=L \frac{d}{d t} i(t) \tag{12}
\end{equation*}
$$

Taking into account (7), where for $k=1$ we have $L \frac{d}{d t} i(t)=\operatorname{Im}\left\{\left[L \frac{d}{d t} I_{-m}(t)+j \omega L I_{-m}(t)\right] \cdot e^{j \omega t}\right\}$, the given component relation allows to determine the character of mathematic connections between instantaneous complex voltages and current on the inductance, namely:

$$
\begin{equation*}
U_{-m}(t)=L \frac{d}{d t} I_{-m}(t)+j \omega L_{-m}^{I}(t) \tag{13}
\end{equation*}
$$

Capacitance element. As it is known [8], on this element instantaneous voltage and current are interconnected by component relation:

$$
\begin{equation*}
i(t)=C \frac{d}{d t} u(t) \tag{14}
\end{equation*}
$$

as a result, mathematical connection between corresponding instantaneous complex voltage and current will have the form:

$$
\begin{equation*}
\underline{I}_{m}(t)=C \frac{d}{d t} \underline{U}_{m}(t)+j \omega C \underline{U}_{m}(t) \tag{15}
\end{equation*}
$$

since according to (7) $C \frac{d}{d t} u(t)=\operatorname{Im}\left\{\left[C \frac{d}{d t} U_{-m}(t)+j \omega C U_{-m}(t)\right] \cdot e^{j \omega t}\right\}$.
Component relations in complex-temporal form, in particular (11), (13), (15) not only reveal regularities in connections between instantaneous complex voltages and currents on separately taken elements of electric circuit, they allow to establish mutual and single-valued correspondence among these elements and elements of transient complex scheme, having determined important principles and rules of construction the latter.

For basic passive elements of electric circuit these rules are shown in Table 1, where in each row, located on the left, the element of output circuit is presented and on the right - corresponding fragment of transient complex scheme, which, on condition of Kirchhoff's laws realization, is able to provide between instantaneous complex voltages and currents necessary component relation in complex-temporal form.

Table 1
Table of correspondence between elements of electric circuit and transient complex scheme

| Resistive element |  | Inductive element |  | Capacitance element |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\xrightarrow[u(t)]{\stackrel{i(t)}{\longrightarrow}}$ | $\xrightarrow[{\underset{U}{n}(t)}_{I_{n}(t)}^{\rightarrow}]{\stackrel{R}{\square}}$ | $\xrightarrow[u(t)]{\stackrel{i(t)}{\longrightarrow}}$ | $\xrightarrow[\underline{U}_{n}(t)]{\stackrel{I_{m}(t)}{\sim}} \stackrel{\text { joL }}{\sim}$ | $\xrightarrow[u(t)]{\stackrel{i(t)}{\longrightarrow}} \\|^{C}$ |  |
| $u=R i$ | $\underline{U}_{m}=R \underline{I}_{m}$ | $u=L \frac{d i}{d t}$ | $\underline{U}_{m}=L \frac{d \underline{I}_{m}}{d t}+j \omega L \underline{L}_{m}$ | $i=C \frac{d u}{d t}$ | $\underline{I}_{m}=C \frac{d \underline{U}_{m}}{d t}+j \omega C \underline{U}_{m}$ |

## 3 The example of Cauchy problem setting on the base of transient complex scheme, Kirchhoff's laws and component relations in complex-temporal form

Let us consider the example of Cauchy problem setting in terms of instantaneous complex currents and voltages by means of suggested transient complex scheme, Kirchhoff's laws and component relations in complex-temporal form of representation.

Simultaneously we will perform partial test on adequacy of the basic elements to physical course of transient process in the circuit of sinusoidal current, described by instantaneous currents or voltages. For this purpose we will make use of one of topologically widely used in theoretical engineering examples of electric circuit of the 2-nd order, diagram of which is shown in Fig. 1, a. Near, in Fig. 1, b, according to the rules, defined above (see Table 1) transient complex scheme of this circuit is built.


Fig. 1. Electric circuit of the 2-nd order and its transient complex scheme

System of equations, that in generally accepted method is composed for transient complex scheme on the base of Kirchhoff's laws (8), (9) and component relations (11), (13), (15) in complex-temporal form relatively instantaneous complex currents in branches and voltage in capacitance element, has the form:

$$
\left\{\begin{array}{l}
\underset{-m_{1}}{I}(t)-\underset{-m_{2}}{I}(t)-\underset{-m_{3}}{I}(t)=0 ;  \tag{16}\\
I_{-m_{2}}(t)-C \frac{d}{d t} U \quad(t)-j \omega C{ }_{-m_{C}}^{U} \quad(t)=0 ; \\
L \frac{d}{d t} I_{-m_{1}}(t)+\left(R_{1}+j \omega L\right){\underset{-m}{m_{1}}}(t)+\underset{-m_{C}}{U}(t)=\underset{-m}{U} ; \\
R_{2}{\underset{-m}{3}}^{U}(t)-\underset{-m_{C}}{U}(t)=0,
\end{array}\right.
$$

where $\underline{-m}_{U}^{U}=U_{m} e^{j \psi_{u}}=\underline{\text { const }}$.
It is not difficult to obtain the equation of transient process if we rewrite the system (16) relatively one of the functions to be found, for instance, instantaneous complex current $\underline{I}_{m_{1}}(t)$. Then differential equation will have the form of the equations (5) on condition that $n=2$, namely:

$$
\begin{equation*}
\underline{A}_{2} \frac{d^{2}}{d t^{2}} \underline{I}_{m_{1}}(t)+\underline{A}_{1} \frac{d}{d t} \underline{I}_{m_{1}}(t)+\underline{A}_{0} \underline{I}_{m_{1}}(t)=\underline{B}_{0} \underline{U}_{m} \tag{17}
\end{equation*}
$$

where coefficients

$$
\underline{A}_{2}=L C R_{2} ; \underline{A}_{1}=L+C R_{1} R_{2}+j 2 \omega L C R_{2} ; \underline{A}_{0}=R_{1}+R_{2}-\omega^{2} L C R_{2}+j \omega\left(L+C R_{1} R_{2}\right) ; \underline{B}_{0}=1+j \omega C R_{2} .
$$

Initial conditions of Cauchy problem in the given set up should be calculated by means of complex scheme of precommunication circuit on the base of independent initial conditions and communication laws, presented by complex formula [3]:

$$
\underline{I}_{m_{L}}\left(0_{+}\right)=\underline{I}_{m_{L}}(0)=\underline{I}_{m_{L}}\left(0_{-}\right) ; \underline{U}_{m_{C}}\left(0_{+}\right)=\underline{U}_{m_{C}}(0)=\underline{U}_{m_{C}}\left(0_{-}\right) .
$$

In our case, two initial conditions of the desired Cauchy problem will be relations:

$$
\begin{equation*}
\underset{-m_{1}}{I}\left(0_{+}\right)=\frac{U}{R_{1}+j\left(\omega L-\frac{1}{\omega C}\right)} ; \quad \frac{d I_{-m_{1}}\left(0_{+}\right)}{d t}=0 \tag{18}
\end{equation*}
$$

the second of which is easy to obtain, making use of the third equation of the system (16).
In order to check the result obtained we will compose differential equation of transient process relatively instantaneous current $i_{1}(t)$. Making use of the scheme of output electric circuit (see Fig. 1, a), first we will compose the system of equations on the base of Kirchhoff's laws and component relations (10), (12), (14), written in classical form

$$
\left\{\begin{array}{l}
i_{1}(t)-i_{2}(t)-i_{3}(t)=0  \tag{19}\\
i_{2}(t)-C \frac{d}{d t} u_{C}(t)=0 \\
L \frac{d}{d t} i_{1}(t)+R_{1} i_{1}(t)+u_{C}(t)=u(t) \\
R_{2} i_{3}(t)-u_{C}(t)=0
\end{array}\right.
$$

where $u(t)=U_{m} \sin \left(\omega t+\psi_{u}\right)$, and then we will rewrite (19) relatively current $i_{1}(t)$. As a result we obtain differential equation of the 2 -nd order of the form (4)

$$
\begin{equation*}
a_{2} \frac{d^{2}}{d t^{2}} i_{1}(t)+a_{1} \frac{d}{d t} i_{1}(t)+a_{0} i_{1}(t)=b_{1} \frac{d}{d t} u(t)+u(t) \tag{20}
\end{equation*}
$$

with coefficients $a_{2}=L C R_{2} ; a_{1}=L+C R_{1} R_{2} ; a_{0}=R_{1}+R_{2} ; b_{1}=C R_{2} ; b_{0}=1$.
At such setting of Cauchy problem, equation (20) must be completed by corresponding initial conditions, certainly, different from (18):

$$
\begin{align*}
& i_{1}\left(0_{+}\right)=\frac{U_{m}}{\sqrt{R_{1}^{2}+\left(\omega L-\frac{1}{\omega C}\right)^{2}}} \sin \left(\psi_{u}-\operatorname{arctg} \frac{\omega L-\frac{1}{\omega C}}{R_{1}}\right) ;  \tag{21}\\
& \frac{d i_{1}\left(0_{+}\right)}{d t}=\frac{u\left(0_{+}\right)-u_{C}\left(0_{+}\right)-R_{1} i_{1}\left(0_{+}\right)}{L},
\end{align*}
$$

where $u\left(0_{+}\right)=U_{m} \sin \psi_{u} ; \quad u_{C}\left(0_{+}\right)=\frac{-U_{m}}{\omega C \sqrt{R_{1}^{2}+\left(\omega L-\frac{1}{\omega C}\right)^{2}}} \cos \left(\psi_{u}-\operatorname{arctg} \frac{\omega L-\frac{1}{\omega C}}{R_{1}}\right)$.
We should pay attention to the fact, that in each separate case, the derivative of instantaneous current at the moment of communication $\frac{d i_{1}\left(0_{+}\right)}{d t}$ as a rule, does not obtain the same values but depends on initial phase of input voltage and relations between parameters of the elements of the $d I \quad\left(0_{+}\right)$
given circuit. Simultaneously, the value of the derivative $\frac{-m_{1}}{d t}$ is invariant to the given parameters and always equals zero.

Comparing the coefficients of both differential equations (17) and (20) it is easy to note, that

$$
\underset{-2}{A}=a_{2} ; \underset{-1}{A}=a_{1}+j 2 \omega a_{2} ; \quad \underset{-0}{A}=a_{0}-\omega^{2} a_{2}+j \omega a_{1} ; \underset{-0}{B}=b_{0}-\omega^{2} b_{2}+j \omega b_{1},
$$

where in the last relation $b_{2}=0$.
Then, for the set, by the conditions of the example of the case, when $n=2$ and $q=1$ (one source of supply) relations between the groups of corresponding coefficients of differential equations correspond to formulas (6), that is why, the solutions of these equations, taking into consideration the above-mentioned initial conditions (18) and (21) will not contredict to the formula (3)

$$
i_{1}(t)=\operatorname{Im}\left\{{\left.\underset{-m_{1}}{ }(t) e^{j \omega t}\right\} .}\right.
$$

This means, that if the solution $i_{1}(t)$ of Cauchy problem, built for the given electric circuit on the base of classic Kirchhoff's laws and component relations is adequate to objectively existing reality, then solution $\underline{I}_{m_{1}}(t)$ of Cauchy problem, that can be formulated in another way - on the base of the suggested transient complex scheme and by means of Kirchhoff's laws and component relations in complex-temporal from, by its quality will not concede the first solution and will be adequate to realities of transient process in the given electric circuit on another equal conditions.

## Closing part, conclusions

For the example in Fig. 2 two graphs, which separately and qualitative form represent the transient process. The first graph (Fig, 2, a) - it is transient wave diagram of instantaneous current $i_{1}(t)$ the second graph (Fig. 2, b) - time locus [3], built on the base of instantaneous complex current $\underline{I}_{m_{1}}(t)$ on complex plane.


Fig. 2. Transient wave diagram of instantaneous current $i_{1}(t)$ and time locus of instantaneous complex current

$$
\underline{I}_{m_{1}}(t)
$$

As it is seen from the figures, both mathematical models are adequately able to function and selfcontained. In particular, they obviously and definitely represent two characteristic features of the current transient process: firstly, almost three times (!),growth of transient current in the given circuit and secondly radical change of the character of the input impedance of the circuit from capacitive into inductive. Simultaneously, as each model shows its possibilities in the manner, inherent to it, it is worth to apprehend these models and, correspondingly, both forms of Cauchy problem, not in opposition by the number of possibility inherent drawbacks but in natural addition of possible advantages, immanent only to each of them, irrespective of the number and depth of the latter.

Only applying such an approach the theory of transient process in the circuits of sinusoidal current gradually will obtain the necessary completeness and absoluteness. Such an approach will promote to reveal the essence of scientific results, obtained in this research.

Then, notion of transient complex scheme of electric circuit is introduced and developed, basic characteristics an main principles of its composition are described, both Kirchhoff's laws and basic
component relations are formulated in complex-temporal form and substantiated. All this, taken together creates theoretical basis of symbolic-classic method, that enables to set directly and solve fundamental Cauchy problem not in conventional form in terms of instantaneous complex currents and voltages, the method broadens and increases analytical possibilities in the process of calculation and study of transient process in linear electric circuits of sinusoidal and periodic non-sinusoidal.

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