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DECOMPOSITION OF MOTION OPTIMIZATION PROBLEM OF TRANSPORT VEHICLE WITH COMBINED DRIVE

Decomposition of motion optimization problem of transport vehicle with combined drive from internal combustion engine and d.c. electric motor on condition that transport vehicle moves along the road, that besides horizontal sections also has descents and grades.

Key words: *transport vehicle with combined drive, internal combustions engine, d.c. electric motor, motion optimization.*

Initial preconditions and problem set-up

Nowadays along with transport vehicles, motion of which is provided by internal combustion engines and transport vehicles, motion of which is provided by electric drives, powered from storage batteries, transport vehicles with combined drive, containing in its structure both internal combustion engine and electric drive, powered from storage battery.

As the distance that can be covered by the transport vehicle with internal combustion engine depends on the volume of the fuel tank and fuel consumption per kilometer of the road, the distance, that can be covered by the transport vehicle with electric drive depends mainly on the capacity of the storage battery and consumption of electric energy per kilometer of the road, then taking into account limited values of the tank volume and capacity of the storage battery, set by manufacturers, the problem of determination of such relation of fuel consumption and electric energy consumption, that allows transport vehicle with combined drive to cover the longest distance without refueling and recharging is very actual.

Taking into account the fact that transport vehicle in real conditions moves not only on horizontal sections of the road but on descents and grades, we realize that the solution of the given optimization problem should be started with its decomposition – the given article considers this given stage of the solution of the problem, formulated above.

The solution of the set problem

We will start the solution of the problem from its connection to the equation [1], known from the course of theoretical mechanics.

$$m\frac{dV}{dt} = F_T - F_S, \tag{1}$$

the given equation describes the motion of the motor vehicle of mass m in time t with the speed V under the action of the thrust force F_S, created by the engine that has to overcome motion resistance force. As it is known from theoretical mechanics [1] and aerodynamics [2], motion resistance force F_S has tree components, first of them is F_O, stipulated by the friction of the wheels with road pavement, is proportional to the weight F_G of the vehicle, the second component F_1, stipulated by the friction of side face of the vehicle with the air, is proportional to the speed of automobile motion, and the third component F_2, stipulated by the pressure of ram air on lateral section of the automobile, is proportional to the squared speed of the automobile, i.e.

$$F_S = F_O + F_1 + F_2 = k_0 F_G + k_1 V + k_2 V^2, \tag{2}$$

where the value of k_0 coefficient is tabulated value and depends on the material of road surface, value of k_2 coefficients depends on the surface of the side face vehicle, and the value of coefficient depends on the area of cross-section of the vehicle.

Substituting the expression for F_S from (2) into (1), we obtain the equation of vehicle dynamics in the form

$$m\frac{dV}{dt} = F_T - k_0 F_G - k_1 V - k_2 V^2, \tag{3}$$

which in general form is valid for the vehicle, thrust force F_T of which is created by the internal combustion engine and for the vehicle, where this force is created by electric motor. But if we recollect that

$$F_T = \frac{M_T}{R}, \tag{4}$$

where M_T - thrust moment of the engine, and R – radius of the wheel and if we take into account that for internal combustion engine

$$M_T = \frac{P}{\omega} = \frac{P}{\frac{V}{2\pi R}} = \frac{2\pi RP}{V}, \tag{5}$$

where P – power of the engine and ω -- angular velocity of wheel rotation and if we take into account that for thrust electric d.c. motor with series excitation.

$$M_T = k_D I\Phi(I), \tag{6}$$

where k_D – coefficient that is determined from the data of electric motor certificate, I - armature current of this electric motor, $\Phi(I)$ – magnetic flux of the inductor, that according to magnetizing curve is the function of armature current then, taking into account the relations (4) - (6), equation of vehicle dynamics (3) will have the form

$$m\frac{dV}{dt} = \frac{2\pi P}{V} - k_0 F_G - k_1 V - k_2 V^2 = 2\pi k_P \frac{Q}{V} - k_0 F_G - k_1 V - k_2 V^2, \tag{7}$$

if thrust force is created by the internal combustion engine as a result of combustion of Q units of fuel per unit of time, where kp – coefficient of proportionality between the flow of fuel Q and power P, created by this flow, it will have the form

$$m\frac{dV}{dt} = \frac{k_D}{R} I\Phi(I) - k_0 F_G - k_1 V - k_2 V^2, \tag{8}$$

if trust force is created by d.c. electric motor with series excitation. Comparing the right parts of equations (7), (8), we see that they differ greatly.

Generalizing the above-mentioned, we can state that, if the vehicle creates trust force only by means of internal combustion engine, its dynamics will be described by the equation (7), if thrust force is created only by means of d.c. electric motor with series excitation, then the dynamics of the vehicle will be described by the equation (8), and if trust force is created by parallely working on one shaft both by internal combustion engine and d.c. electric motor with series excitation, then the dynamics of the vehicle will be described by the equation

$$m\frac{dV}{dt} = 2\pi k_P \frac{Q}{V} + \frac{k_D}{R} I\Phi(I) - k_0 F_G - k_1 V - k_2 V^2, \tag{9}$$

that we obtain by means of superposition of equations (7) and (8).

But it should be noted, that in the given form the equations (7) - (9) describe the dynamics of the vehicle only in the process of motion along the road, pavement of which is placed on horizontal plane.

If the vehicle moves on the descent, as it is shown in Fig. 1a or on grade, as it is shown in Fig. b, then in the equation (7) - (9) instead of the term ($-k_0 F_G$) binomial ($-k_0 F_G \cos\beta + F_G \sin\beta$) should be introduced for the descent or binomial ($-k_0 F_G \cos\beta - F_G \sin\beta$) for the grade, i.e., these equations will have the following form:

- for the descent-

$$m\frac{dV}{dt} = = 2\pi k_P \frac{Q}{V} - k_0 F_G \cos\beta + F_G \sin\beta - k_1 V - k_2 V^2, \quad (10)$$

$$m\frac{dV}{dt} = \frac{k_D}{R} I\Phi(I) - k_0 F_G \cos\alpha + F_G \sin\alpha - k_1 V - k_2 V^2, \quad (11)$$

$$m\frac{dV}{dt} = 2\pi k_P \frac{Q}{V} + \frac{k_D}{R} I\Phi(I) - k_0 F_G \cos\beta + F_G \sin\beta - k_1 V - k_2 V^2, \quad (12)$$

- for the grade-

$$m\frac{dV}{dt} = = 2\pi k_P \frac{Q}{V} - k_0 F_G \cos\beta - F_G \sin\beta - k_1 V - k_2 V^2, \quad (13)$$

$$m\frac{dV}{dt} = \frac{k_D}{R} I\Phi(I) - k_0 F_G \cos\beta - F_G \sin\beta - k_1 V - k_2 V^2, \quad (14)$$

$$m\frac{dV}{dt} = 2\pi k_P \frac{Q}{V} + \frac{k_D}{R} I\Phi(I) - k_0 F_G \cos\beta - F_G \sin\beta - k_1 V - k_2 V^2. \quad (15)$$

Now we will define the criteria, minimizing their values we solve the optimization problem.

It is obvious that for the problem of vehicle motion optimization by means of internal combustion engine this criteria has the form of the functional

$$E_Q = \int_0^{T_Q} k_P Q dt, \quad (16)$$

that characterizes the amount of mechanical energy E_Q, spent during time T_Q of the vehicle motion, using the internal combustion engine.

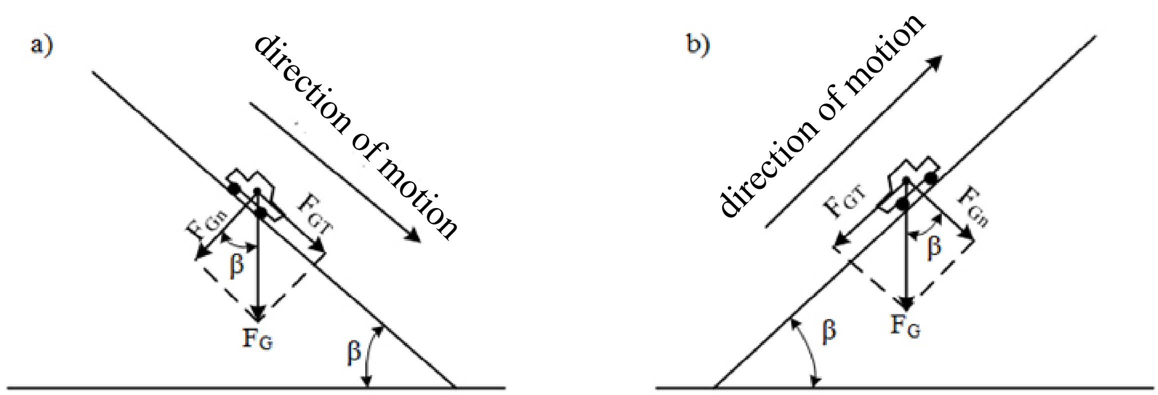

Fig. 1. Vector diagram of forces, created by the force of weight F_G of the vehicle, during its motion on the descent (a) and grade (b)

For the problem of optimization of vehicle motion by means of d.c. electric motor with serial excitation optimization criterion will have the form of the functional

$$E_I = \int_0^{T_I} UIdt, \qquad (17)$$

that characterizes the amount of electric energy E_I, spent during time T_I of the vehicle motion, using d.c. electric motor with serial excitation, in the armature of which current I, created by the voltage U at its terminals, flows. It is seen from the circuit, shown in Fig. 2, where U_V - voltage of storage battery B, that has internal resistance r_B, that the criterion (17) is easily reduced to

$$E_I = \int_0^{T_I} UIdt = \int_0^{T_I} (U_B - \Delta U)Idt = \int_0^{T_I} U_B(1 - \frac{r_B}{U_B} I)Idt, \qquad (18)$$

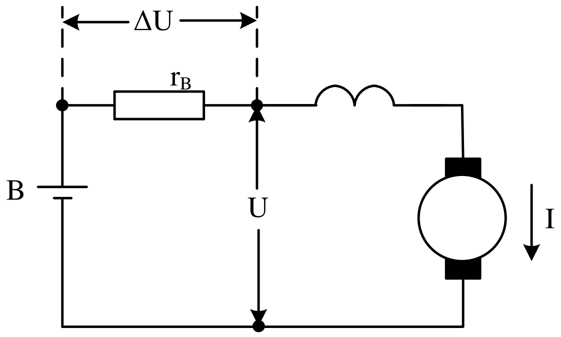

Fig. 2. Connecting scheme of d.c. electric motor with serial excitation to storage battery B, having internal resistance r_B

For the problem of vehicle motion optimization by means of parallely working on one shaft of internal combustion engine and d.c. electric motor with serial excitation optimization criterion will have the form of the functional

$$E_{QI} = \gamma \int_0^{T_{QI}} U_B(1 - \frac{r_B}{U_B} I)Idt + \int_0^{T_{QII}} k_P Qdt, \qquad (19)$$

where T_{QI} - time of the vehicle motion with simultaneous usage of both motors, and γ - coefficient of reduction of electric energy to equivalent amount of mechanical energy.

As a rule, when we move in the automobile, for us it is interesting not only the amount of fuel or electric energy, consumed during time T_Q, T_I or T_{QI}, but we are interested in what distance L_Q, L_I or L_{QI} we will cover during this time. It is evident, that knowing speed of the automobile V, we can determine these distances by means of the functionals:

$$L_Q = \int_0^{T_Q} Vdt, \qquad (20)$$

$$L_I = \int_0^{T_I} Vdt, \qquad (21)$$

$$L_{QI} = \int_0^{T_{QI}} Vdt \qquad (22)$$

And taking into account, functionals (20) - (22) will transfer this optimization problem into the class of isoperimetric problems [3, 4], as we will have to look for such law of automobile speed change that minimizes one of the functionals (16), (18) or (19) – on condition that the dynamics of the automobile is described by corresponding equation from the set (7) - (9) or (10) - (15) and the distance, covered by the automobile, is set by corresponding functional from the set (20) - (22).

To represent the results obtained in most generalized form, it is expedient to pass to relative values, using as basic values: for thrust force F_T its value F_N in nominal mode; for the moment M on the shaft its nominal value M_N; for the speed V of automobile motion its nominal value V_H; for current I of the armature of traction electric motor its nominal value I_H; for magnetic flux $\Phi(I)$ of traction electric motor its value at nominal current $\Phi_N = \Phi_N(I_N)$; for voltage U, applied to the terminals of traction electric motor, voltage U_B of storage battery it in case of its full charge; for fuel consumption Q their values in nominal mode Q_N, i.e., in the mode, when the internal combustion engine creates power P, that equals its value P_N in nominal operation mode. As derivative basic values we will use: for time t – electric mechanical constant T_M, that is connected with main basic values by the relation –

$$T_M = \frac{mV_N}{F_N}, \tag{23}$$

for distance L – value L_N, that is connected with main basic values by the relation –

$$L_N = V_N T_N, \tag{24}$$

for energy E of direct current of storage battery--- value E_N^I, that is connected with main basic values by the relation –

$$E_N^I = U_B I_N T_M, \tag{25}$$

for mechanical energy E, that is produced by internal combustion engine, – value E_N^Q, that is connected with main basic values by the relation–

$$E_N^Q = k_P Q_N T_M. \tag{26}$$

Taking into account main basic and derivative basic values:

- as the equivalent to the equation (7) in relative values we obtain the equation–

$$\frac{d\nu}{d\tau} = \frac{q}{\nu} - f_0 - f_1\nu - f_2\nu^2, \tag{27}$$

where:

$$\nu = \frac{V}{V_H}, \ \ q = \frac{Q}{Q_H}, \ \ \tau = \frac{t}{T_M}; \tag{28}$$

$$\frac{2\pi k_P Q_H T_M}{mV_H^2} = 1,; \tag{29}$$

$$f_0 = \frac{k_0 F_G T_M}{mV_H}, \ \ f_1 = \frac{k_1 T_M}{m}, \ \ f_2 = \frac{k_2 V_H T_M}{m}; \tag{30}$$

- as the equivalent to the equation (8) in relative values we obtain the equation –

$$\frac{d\nu}{d\tau} = i\phi(i) - f_0 - f_1\nu - f_2\nu^2, \tag{31}$$

where:

$$i = \frac{I}{I_H}, \ \ \phi(i) = \frac{\Phi(I)}{\Phi_H(I_H)}; \tag{32}$$

$$\frac{k_D I_H \Phi_H(I_H) T_M}{RmV_H} = 1, \tag{33}$$

and relative variables ν, τ and coefficients f_0, f_1, f_2 are determined by the relations (28) and (30), correspondingly;

- as the equivalent to the equation (9) in relative values we obtain the equation –

$$\frac{d\nu}{d\tau} = \frac{q}{\nu} + i\phi(i) - f_0 - f_1\nu - f_2\nu^2; \tag{34}$$

- as the equivalents to the equations (10), (11), (12) in relative values, we obtain, correspondingly, the equations –

$$\frac{d\nu}{d\tau}=\frac{q}{\nu}+f_0^*\sin\beta-f_0\cos\beta-f_1\nu-f_2\nu^2, \tag{35}$$

$$\frac{d\nu}{d\tau}=i\phi(i)+f_0^*\sin\beta-f_0\cos\beta-f_1\nu-f_2\nu^2, \tag{36}$$

$$\frac{d\nu}{d\tau}=\frac{q}{\nu}+i\phi(i)+f_0^*\sin\beta-f_0\cos\beta-f_1\nu-f_2\nu^2, \tag{37}$$

where in addition to the already determined variables and coefficients we have one more coefficient –

$$f_0^*=\frac{F_G T_M}{mV_H}: \tag{38}$$

- as the equivalents to the equations (13),(14),(15) in relative values we obtain, correspondingly, the equations –

$$\frac{d\nu}{d\tau}=\frac{q}{\nu}-f_0^*\sin\beta-f_0\cos\beta-f_1\nu-f_2\nu^2, \tag{39}$$

$$\frac{d\nu}{d\tau}=i\phi(i)-f_0^*\sin\beta-f_0\cos\beta-f_1\nu-f_2\nu^2, \tag{40}$$

$$\frac{d\nu}{d\tau}=\frac{q}{\nu}+i\phi(i)-f_0^*\sin\beta-f_0\cos\beta-f_1\nu-f_2\nu^2, \tag{41}$$

- as the equivalents to optimization criteria (16), (18), (19) in relative values we obtain, correspondingly, criterial functionals -

$$e_q=\int_0^{\tau_q} q\,d\tau, \tag{42}$$

$$e_i=\int_0^{\tau_i}(1-\alpha i)i\,d\tau, \tag{43}$$

$$e_{qi}=\int_0^{\tau_{qi}} q\,d\tau+\gamma\int_0^{\tau_{qi}}(1-\alpha i)i\,d\tau, \tag{44}$$

where:

$$e_q=\frac{E_Q}{E_N^Q}=\frac{E_Q}{k_q Q_N T_M},\ \ e_i=\frac{E_I}{E_N^I}=\frac{E_I}{U_B I_N T_M},\ \ e_{qi}=\frac{E_{QI}}{E_N^Q}=\frac{E_{QI}}{k_q Q_N T_M}; \tag{45}$$

$$\tau_q=\frac{T_Q}{T_M},\ \ \tau_i=\frac{T_I}{T_M},\ \ \tau_{qi}=\frac{T_{QI}}{T_M}; \tag{46}$$

$$\alpha=\frac{r_B I_N}{U_B},\ \ \gamma=\frac{U_B I_N}{k_P Q_N}; \tag{47}$$

- as the equivalents of limitations (20), (21), (22) in relative values, we obtain, correspondingly, functionals -

$$l_q=\int_0^{\tau_q}\nu\,d\tau, \tag{48}$$

$$l_i=\int_0^{\tau_i}\nu\,d\tau, \tag{49}$$

$$l_{qi} = \int_{0}^{\tau_{qi}} v d\tau, \tag{50}$$

where:

$$l_q = \frac{L_Q}{V_N T_M}, \ l_i = \frac{L_I}{V_N T_M}, \ l_{qi} = \frac{L_{QI}}{V_N T_M} \tag{51}$$

Now we have all the relations, necessary for the solution of optimization problem of automobile motion both in case of creation of trust force only by internal combustion engine or only by d.c. electric motor with serial excitation, and in case of thrust force creation by two engines simultaneously – so we can pass to the decomposition of this task, in our opinion, logically the following scheme will be the most suitable.

1) If electric drive is out of order or storage battery is completely discharged, the rest of the route will have to be covered by means of internal combustion engine, in this case, it is necessary to synthesize the laws of optimal motion both on horizontal sections of the road and on descents and grades.
2) If internal combustion engine is out of order or there is no fuel in the tank, the rest of the route will have to be covered only by means of electric drive, in this case, it is necessary to synthesize the laws of optimal motion both on horizontal sections of the road and an descents and grades.
3) If both engines are operational, volume of fuel is sufficient and expensive storage batteries are used, their price and operation expenses are included in corresponding proportion, determined by operation term, to the cost of the consumed electric energy, it is expedient to cover the whole route by means of two engines, simultaneously operating on the common shaft, that leads to the necessity of synthesis of optimal motion laws for this case both on horizontal sections and on descents and grades.
4) If both engines are operational and cheap storage batteries are used, that makes the route covering mode more economic by means of electric drive, it is expedient to overcome horizontal sections and descents only by means of electric drive and on the grades connect to the shaft, besides electric drive also the internal combustion engine, that leads to the necessity of the synthesis that leads to the necessity of the synthesis of optimal motion laws, taking into account this peculiarity. This mode of economic fuel consumption, that should be kept to, in places, where there is no possibility to charge the storage battery, that is why, the route will be completeв only by means of using the internal combustion engine. This mode is also expedient in case of the deficit of time, needed for storage battery charging, in cases, where there is a possibility to charge the storage battery.

Procedures of optimal motion laws synthesis of hybrid automobile for all the cases, described above, will be given in our further publications.

Conclusions

1). Problem of optimal motion laws synthesis of hybrid automobile both on horizontal section of the road and on descents and grades is formulated, equations of automobile dynamics, criterial functionals and isoperimetric limitations are reduced to relative values.

2). Decomposition of the set problem in order to include all possible cases of hybrid automobile motion organization by optimal laws is realized.

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