O. B. Mokin, Dr. Sc. (Eng.), Prof.; V. B. Mokin, Dr. Sc. (Eng.), Prof.;
B. I. Mokin, Acad. NAPSU, Dr. Sc. (Eng.), Prof.; I. O. Chernova

METHOD OF PROCESSES IDENTIFICATION IN MULTIDIMENTIONAL DYNAMIC OBJECTS, PERMITING LINEARIZATION, BY NOT HIGHER THAN THIRD ORDER MATMEMATICAL MODELS, EQUIVALENT BY THE CUT-OFF FREQUENCY

Method of processes identification in multidimensional dynamic objects, permitting linearization and operating in direct signal transmission mode, by mathematical models of not higher than third order, equivalent by the cut-off frequency is suggested. Algorithm of the method is based on the system of equations, one part of which is synthesized, taking into account limiting conditions, set by cut-off frequency, and the second part is synthesized by the standard procedure of the method of least squares relatively logarithmic frequency characteristics.

Key words: *dynamic system, mathematic model, cut frequency, differential, making equivalent, method of least squares.*

Problem set-up and initial preconditions

In [1] conditions are determined, under which processes in dynamic objects, permitting linearization and described by differential equations of high orders, i. e., differential equations, having the form

$$a_n \frac{d^n y}{dt^n} + a_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + ... + a_1 \frac{dy}{dt} + y = Kx, \quad n > 3, \tag{1}$$

in the range of motion coordinates change can equivalently (without introducing substantial errors) be described by means of differential equations of the order, not greater than third, i. e. differential equations that have the form

$$a_3 \frac{d^3 y}{dt^3} + a_2 \frac{d^2 y}{dt^2} + a_1 \frac{dy}{dt} + y = K_3 x \tag{2}$$

or form

$$a_2 \frac{d^2 y}{dt^2} + a_1 \frac{dy}{dt} + y = K_2 x, \tag{3}$$

or even the form

$$a_1 \frac{dy}{dt} + y = K_1 x. \tag{4}$$

These conditions are the following: only for these dynamic objects, when single input signal is sent to their input

$$x(t) = 1(t) = \begin{cases} 1, t \ge 0, \\ 0, t < 0, \end{cases} \tag{5}$$

for their reaction *y(t)* expressions are valid:

$$y'(0) \ne 0, \tag{6}$$

$$y''(t) \neq 0, \quad \forall t \in [0,\infty) \tag{7}$$

as equivalent model differential equation of the form (4) can be used.

If for *y(t)* reaction of the dynamic objects the expressions are valid:

$$y'(0) = 0 \tag{8}$$

$$y''(t_n) = 0, \quad t_n \in [0,\infty), \tag{9}$$

then as the equivalent model differential equations of the form (3) can be used. And if for *y(t)* reaction of the dynamic object expressions (6) and (9) are valid, then as the equivalent model differential equation of the form (2) can be used.

In the same research [1] it is shown, that for the synthesis of the equivalent models of multidimensional dynamic objects in the classes of equations (2 – 4) conditions (6 – 9) are necessary but not sufficient and sufficient conditions of equivalization of dynamic objects, using their logarithmic frequency characteristics — amplitude (LAFC) $L(\omega)$ and phase (LPFC) $\varphi(\omega)$ [2] which for dynamic object, motion of which in general form is described by differential equation of the n^{th} order (1) approximately may be presented as it is shown in Fig. 1.

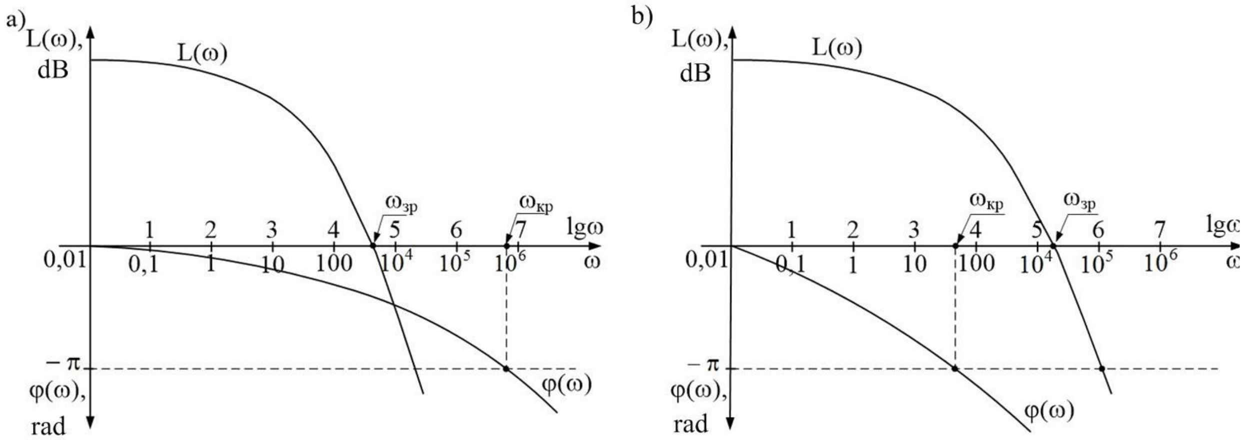

Fig. 1. Approximate LAFC and LPFC graphs of dynamic object of n^{th} order

It should be noted, that transforming differential equations (1 – 4) by Laplace transform [2], we may pass to their equivalents on the complex plane – transfer functions – in the form:

$$W(p) = \frac{y(p)}{x(p)} = \frac{K}{a_n p^n + a_{n-1}p^{n-1} + \ldots + a_1 p + 1}, \tag{10}$$

$$W_1(p) = \frac{K_1}{a_1 p + 1}, \tag{11}$$

$$W_2(p) = \frac{K_2}{a_2 p^2 + a_1 p + 1}, \tag{12}$$

$$W_3(p) = \frac{K_3}{a_3 p^3 + a_2 p^2 + a_1 p + 1}, \tag{13}$$

for which it is valid:

$$W_i(j\omega) = W_i(p)\Big|_{p=j\omega} = R_i(\omega) + jQ_i(\omega) = A_i(\omega)e^{j\varphi_i(\omega)},\ i = \overline{1,3}\ , \tag{14}$$

$$A_i(\omega) = \sqrt{R_i^2(\omega) + Q_i^2(\omega)}, \qquad \varphi_i(\omega) = arctg\frac{Q_i(\omega)}{R_i(\omega)}, \tag{15}$$

$$L_i(\omega) = 20\lg A_i(\omega), \tag{16}$$

It should be remembered that in the expressions (16) for LAFC and LPFC the increments of angular velocity ω are laid off on frequency axis in decades.

As it is known from [2], for LAFC and LPFC two frequencies are characteristic — frequency of the cut-off ω_c and critical frequency ω_{cr}, found from the equations:

$$L(\omega_c) = 0, \tag{17}$$

$$\varphi(\omega_{cr}) = -\pi \tag{18}$$

and have geographical interpretation, shown in Fig. 1.

According to Nyquists criterion [2], if, as it is shown in Fig. 1a, for stable dynamic object the condition is satisfied

$$\omega_c < \omega_{cr}\ , \tag{19}$$

then it remains stable after closing it by a single negative feedback. That is, in such an object the character of processes to its closing by a single negative feedback and after closing does not change.

If for dynamic object, stable in open state, the condition is satisfied (see Fig. 1b),

$$\omega_c > \omega_{cr}\,, \tag{20}$$

then it becomes instable after closing it by a single negative feedback. That is, in such an object character of processes to its closing by a single negative feedback and after closing changes.

That is why if the condition (19) is satisfied for multidimensional dynamic object equivalent models in the form (3) and (4) can be used, as for the first of them, as it is known [2] inequality is valid

$$\varphi_1(\omega) > -\frac{\pi}{2}, \quad \forall\omega \in [0,\infty), \tag{21}$$

and for the second inequality is valid

$$\varphi_2(\omega) > -\pi, \quad \forall\omega \in [0,\infty)\ . \tag{22}$$

If for multidimensional dynamic object the condition (20) is satisfied, then for the description of the processes taking place in it equivalent models (3), (4), cannot be used, but it is necessary to use the equivalent model (2), as the system of inequalities is valid for it

$$\begin{aligned} &\varphi_3(\omega) \ge -\pi, && \forall\omega \in [0,\omega_{cr}], \\ &\varphi_3(\omega) < -\pi, && \forall\omega \in (\omega_{cr},\infty), \end{aligned} \tag{23}$$

with which an inequality (20) coincides near critical frequency ω_{cr}.

From Fig. 1 it is seen that for dynamic object of n^{th} order the following system of inequalities is valid

$$\begin{aligned} &L(\omega) \geq 0, \qquad &\forall \omega \in [0, \omega_c], \\ &L(\omega) < 0, \qquad &\forall \omega \in (\omega_c, \infty). \end{aligned} \tag{24}$$

Upper inequality from the system (24) testifies that the dynamic object behaves as an amplifier on the frequencies up to the frequency of the cut-off, the following inequality is valid for it

$$A(\omega) \geq 1, \tag{25}$$

and lower inequality from the system (24) testifies that at the values of frequency, greater than frequencies of the cut, the dynamic object behaves like a filter, for which the following inequality is valid

$$A(\omega) < 1 \,. \tag{26}$$

It follows from the above-mentioned, that while synthesis of the equivalent mathematical model for multidimensional dynamic object, which functions in the mode of input signal converter into output signal without the necessity of its closing by a single negative feedback, the conditions (25), (26) must be taken into account. While synthesis of the equivalent mathematical model for multidimensional dynamic object, control of which will be performed by means a single negative feedback, conditions (20), (23) must be taken into account.

In the given paper we present the method of equivalent mathematic model synthesis for multidimensional dynamic object, that functions in the mode of input signal converter into output signal, and the method of equivalent mathematical model synthesis for multidimensional dynamic object, the control over which will be performed using single negative feedback, we will present in our next paper.

Solution of the set-up problem

As for multidimensional dynamic object, functionning in signal transmission mode from the input to output, while its equivalenting it is necessary to provide identical character of the main mathematical model solution (1) and equivalent model from the set (2 – 4), by which we will describe processes in this object, both in frequency region of signal amplification and in frequency region of their filtration, then the graphs of LAFC $L(\omega)$ of the basic model (1) and graphs of asymptotic LAFC $L_1(\omega)$, $L_2(\omega)$, $L_3(\omega)$ of the equivalent models (2) – (4) must start from one and the same point at the least initial value ω_n of the frequency ω, in which

$$L(\omega_n) = L_1(\omega_n) = L_2(\omega_n) = L_3(\omega_n), \tag{27}$$

and pass across one and the same point at the frequency cut ω_c, i. e., have approximate form, shown in Fig. 2.

For further solution of the problem we will assume that we have the possibility to provide at the input of dynamic object equivalent mathematic model of which we synthesis, single stepwise signal (5), response to which allows us to determine, which of the requirements (6)–(9) are realized and there is a possibility to measure experimentally amplitude $A(\omega)$ and phase $\varphi(\omega)$ frequency characteristics of the object in the form of sequences $A(\omega_s)$, $\varphi(\omega_s)$, $s = 1, 2, ..., N$ by means of standard set instruments, that comprises sinusoidal signals generator, double peak voltmeter, frequency meter-phase meter and is equipped with intercoupler devices with input and output of dynamic object. Relations (16) will help up to pass to logarithmic frequency characteristics of the object. That is why, having LAFC $L(\omega)$, by means of relation (17) we easily find cut-off frequency ω_c of the dynamic object.

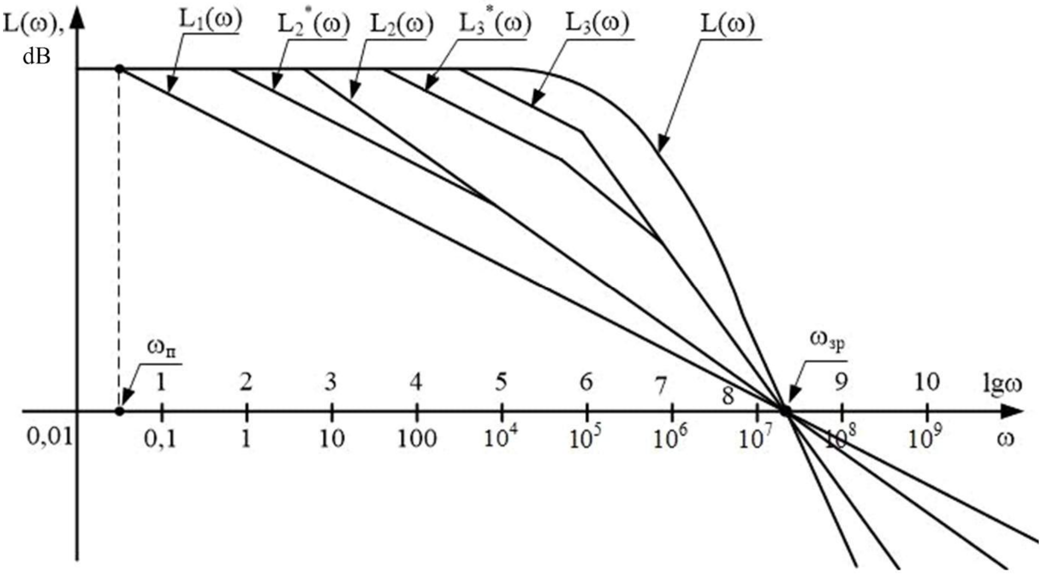

Fig. 2. Approximate view of LAFC graph of the basic model of multidimensional dynamic object and graphs of asymptotic LAFC of its equivalent models

As it is known from the theory of automatic control [2] and is shown in Fig. 2, the graph of asymptotic LAFC $L_1(\omega)$ of mathematical model of the first order has only two straight line segments — initial with zero slope and final with 20 dB slope per decade. Graph of asymptotic LAFC of the second order mathematical model has either two straight line segments (if the poles of transfer function (12) is the pair of complex conjugate numbers), the first of which has zero slope, and the second one has the slope of 40 dB per decade — this is graph $L_2(\omega)$ in Fig. 2 or three straight line segments (if the poles of transfer function (12) is the pair of real negative numbers), the first of which has zero slope, the second has 20 dB slope per decade and the third one has 40 dB slope per decade — this is graph $L_2^*(\omega)$ in Fig. 2. The graph of asymptotic LAFC of the mathematical model of the third order has two or three straight line segments (if the poles of the transfer function (13) is one real negative number and two complex conjugate numbers) the first of which has zero slope, the second had 20 dB slope per decade (or 40 dB per decade) and the third has 60 dB slope per decade — this is graph $L_3(\omega)$ in Fig. 2, or four straight line segments (if poles of the transfer function (13) are three real negative numbers), the first of which has zero slope, the second has 20 dB slope per decade, the third has 40 dB slope per decade, and the fourth has 60 dB slope per decade — this is graph $L_3^*(\omega)$ in Fig. 2. We will only remember that the slope of the last straight line segment of asymptotic LAFC graph, crossing frequency axis in point ω_c for the object with the first order model (4) equals 20 dB per decade, for the object with second order model (3) equals 40 dB per decade and for the third order object (2) equals 60 dB per decade.

Further, using the expressions (11) – (16), we will find that

$$L_1(\omega) = 20\lg A_1(\omega) = 20\lg K_1 - 10\lg(1 + a_1^2\omega^2),$$
$$\varphi_1(\omega) = -arctg(a_1\omega); \qquad (28)$$

$$L_2(\omega) = 20\lg A_2(\omega) = 20\lg K_2 - 10\lg\left[(1 - a_2\omega^2)^2 + a_1^2\omega^2\right],$$
$$\varphi_2(\omega) = -arctg\frac{a_1\omega}{1 - a_2\omega^2}; \qquad (29)$$

$$L_3(\omega) = 20\lg A_3(\omega) = 20\lg K_3 - 10\lg\left[(1-a_2\omega^2)^2 + (a_1\omega - a_3\omega^3)^2\right],$$

$$\varphi_3(\omega) = -arctg\frac{a_1\omega - a_3\omega^3}{1-a_2\omega^2}. \tag{30}$$

In accordance with generally adopted approach in the theory of identification [3], as equivalenting criteria in our problem we will make use of standard criteria of the least squares, in our case, relatively experimentally measured LAFC of multidimensional object $L(\omega_s),\ s = 1, 2, ..., N$ and LAFC of equivalent models $L_1(\omega),\ L_2(\omega),\ L_3(\omega)$, that is, we will make use of the functionals:

$$\Sigma_1 = \sum_{s=1}^{N}\left(L(\omega_s) - L_1(\omega_s)\right)^2, \tag{31}$$

$$\Sigma_2 = \sum_{s=1}^{N}\left(L(\omega_s) - L_2(\omega_s)\right)^2, \tag{32}$$

$$\Sigma_3 = \sum_{s=1}^{N}\left(L(\omega_s) - L_3(\omega_s)\right)^2, \tag{33}$$

substituting in which the expressions for $L_1(\omega), L_2(\omega), L_3(\omega)$, taken from the relations (28) – (30), we will obtain:

$$\Sigma_1 = \sum_{s=1}^{N}\left(L(\omega_s) - 20\lg K_1 + 10\lg(1 + a_1^2\omega_s^2)\right)^2, \tag{34}$$

$$\Sigma_2 = \sum_{s=1}^{N}\left(L(\omega_s) - 20\lg K_2 + 10\lg((1 - a_2\omega_s^2)^2 + a_1^2\omega_s^2)\right)^2, \tag{35}$$

$$\Sigma_3 = \sum_{s=1}^{N}\left(L(\omega_s) - 20\lg K_3 + 10\lg((1 - a_2\omega_s^2)^2 + (a_1\omega_s - a_3\omega_s^3)^2)\right)^2. \tag{36}$$

Further, according to standard procedure of the least square method, in case if we select as equivalent mathematic model differential equation (4), then partial derivatives from Σ_1 with respect to unknown coefficients $K_1,\ a_1,$ would have to be taken, equated with zero, and having solved the obtained system of two equations with respect to two unknown $K_1,\ a_1$, obtain numerical values of these two unknown coefficients.

Similarly, in case, when we select as the equivalent mathematical model differential equation (3), then, we would have to take partial derivatives from Σ_2 with respect to unknown coefficients $K_2,\ a_1,\ a_2,$ equate these derivatives with zero and having solved the obtained system of three equations with respect to three unknown $K_2,\ a_1,\ a_2$, and obtain numerical values of these unknown coefficients.

If we select as the equivalent mathematical model differential equation (2), we would have to take partial derivatives from Σ_3 with respect to unknown coefficients $K_3, a_1,\ a_2,\ a_3$, equate these derivatives with zero and having solved the obtained system of four equations with respect to four unknown $K_2,\ a_1,\ a_2,\ a_3$, and obtain numerical values of these unknown coefficients.

But if we act in this way, then we will obtain models of the processes, taking place in dynamic objects, solutions of which will approach to real processes in root-mean-square, not coinciding with the values of real processes at the end of ranges, which in frequency area will be set by the frequencies ω_i and ω_c, for which the requirements (17) and (27) must be satisfied, that at equivalenting , must be satisfied obligatory.

That is why, it follows that while equivalenting mathematical model of multidimentional dynamic object by mathematical model (4), containing only two unknown parameters K_1, a_1, for their definition we should have the system, consisting only of two equations, that can be built without taking partial derivative with respect to the equation (34), but making use of limiting conditions (17), (27), because from (27) due to the small value of ω_i, it follows, that

$$L(\omega_i) = 20\lg K_1, \tag{37}$$

from (17) it follows that

$$20\lg K_1 - 20\lg a_1 - 20\lg \omega_c = 0. \tag{38}$$

Solving the system of equations (37), (38), we will obtain:

$$K_1 = 10^{\frac{L(\omega_i)}{20}}, \qquad a_1 = \frac{1}{\omega_c} 10^{\frac{L(\omega_i)}{20}}. \tag{39}$$

However, while equivalenting mathematical model of multidimensional dynamic object by mathematical model (3), containing three unknown parameters K_1, a_1, a_2, for their determination we must have the system, consisting of three equations, but we can not build this system using only limiting conditions (17), (27), as these conditions give us only two equations, namely:

$$L(\omega_n) = 20\lg K_2, \tag{40}$$

$$20\lg K_2 - 20\lg a_2 - 40\lg \omega_c = 0, \tag{41}$$

from which we will get:

$$K_2 = 10^{\frac{L(\omega_n)}{20}}, \qquad a_2 = \frac{1}{\omega_c^2} 10^{\frac{L(\omega_n)}{20}}. \tag{42}$$

Regarding the third unknown parameter a_1, in this case for its determination partial derivatives must be taken from the expression (35) and obtain, having put this derivative to zero, in addition to equations (42), still the third equation —

$$\sum_{i=1}^{N} \left[L(\omega_i) - \lg \frac{10K_2^2}{1 + (a_1^2 - 2a_2)\omega_i^2 + a_2^2\omega_i^4} \right] \frac{\omega_i^2}{1 + (a_1^2 - 2a_2)\omega_i^2 + a_2^2\omega_i^4} = 0 \tag{43}$$

non-linear equation, but with one unknown, for solution of which, for instance, standard procedure is put in the package of applied programs Mathcad

Regarding equivalenting mathematical model of multidimensional dynamic object by mathematical model (2), containing four unknown parameters K_1, a_1, a_2, a_3, for their determination we must have the system consisting of four equations, which can not be built using only limiting conditions (17), (27), as these conditions give us only two equations, namely:

$$L(\omega_n) = 20\lg K_3, \tag{44}$$

$$20\lg K_3 - 20\lg a_3 - 60\lg \omega_c = 0, \tag{45}$$

we will get from them:

$$K_3 = 10^{\frac{L(\omega_n)}{20}}, \qquad a_3 = \frac{1}{\omega_c^3} 10^{\frac{L(\omega_n)}{20}}. \tag{46}$$

Regarding two unknown parameters a_1, a_2, in this case for their determination two partial derivatives must be taken with respect to expression (36) and obtain, having put these derivatives equal to zero, in addition to equations (46) two equations more:

$$\sum_{s=1}^{N} \left\{ \left[L(\omega_s) - \lg \frac{10K_3^2}{1+(a_1^2-2a_2)\omega_s^2+(a_2^2-2a_1a_3)\omega_s^4+a_3^2\omega_s^6} \right] \cdot \right.$$
$$\left. \cdot \frac{a_1\omega_s^2 - a_3\omega_s^4}{1+(a_1^2-2a_2)\omega_s^2+(a_2^2-2a_1a_3)\omega_s^4+a_3^2\omega_s^6} \right\} = 0, \tag{47}$$

$$\sum_{s=1}^{N} \left[L(\omega_s) - \lg \frac{10K_3^2}{1+(a_1^2-2a_2)\omega_s^2+(a_2^2-2a_1a_3)\omega_s^4+a_3^2\omega_s^6} \right] \frac{\omega_s^2 - a_2\omega_s^4}{1+(a_1^2-2a_2)\omega_s^2+(a_2^2-2a_1a_3)\omega_s^4+a_3^2\omega_s^6} = 0, \tag{48}$$

having solved them as the system of two non-linear equations with two unknown, using standard procedure in the package of applied programs Mathcad, we obtain in addition to already found by the expressions (46) numerical values of parameters K_3, a_3 numerical values of parameters a_1, a_2.

But the procedure of the synthesis of the equivalent mathematical model of multidimensional dynamic object is not over when the full set of equivalent models (2), (3) (4) parameters have been determined, the error of equivalenting must be evaluated, we will pass to the procedure of determination technique construction.

Evaluation of equivalenting error

As it is seen from Fig. 3 equivalenting error by models (2) – (4) of multidimensional dynamic object in frequency region, can be written as:

$$\Delta Y_i(j\omega) = Y(j\omega) - Y_i(j\omega) = W(j\omega)X(j\omega) - W_i(j\omega)X(j\omega) =$$
$$= (W(j\omega) - W_i(j\omega))X(j\omega) = W_{\Delta i}(j\omega)X(j\omega), \quad i = 1,\ 2,\ 3, \tag{49}$$

where APFC of dynamic object by equivalenting error $W_{\Delta i}(j\omega)$ may be presented as:

$$W_{\Delta i}(j\omega) = W(j\omega) - W_i(j\omega) = A(\omega)e^{j\varphi(\omega)} - A_i(\omega)e^{j\varphi_i(\omega)} =$$
$$= [A(\omega)\cos\varphi(\omega) - A_i(\omega)\cos\varphi_i(\omega)] + j[A(\omega)\sin\varphi(\omega) - A_i(\omega)\sin\varphi_i(\omega)] = A_{\Delta i}(\omega)e^{j\varphi_{\Delta i}(\omega)},$$
$$i = 1,\ 2,\ 3, \tag{50}$$

where amplitude frequency characteristic (AFC) of dynamic object by equivalenting error – it is:

$$A_{\Delta i}(\omega) = \sqrt{A^2(\omega) + A_i^2(\omega) - 2A(\omega)A_i(\omega)\cos(\varphi(\omega) - \varphi_i(\omega))}, \tag{51}$$

and phase frequency characteristic (PFC) —it is:

$$\varphi_{\Delta i} = arctg \frac{A(\omega)\sin\varphi(\omega) - A_i(\omega)\sin\varphi_i(\omega)}{A(\omega)\cos\varphi(\omega) - A_i(\omega)\cos\varphi_i(\omega)}. \tag{52}$$

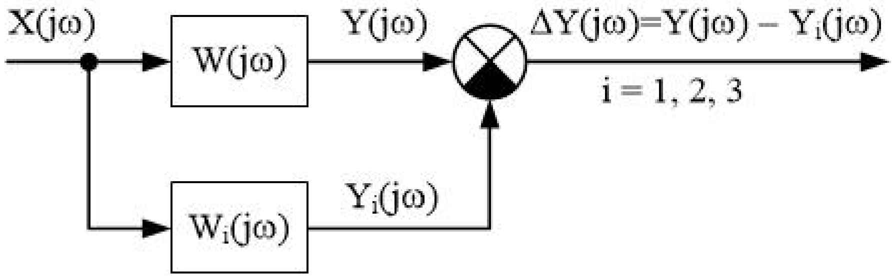

Fig. 3. Structural diagram of equivalenting error formation of multidimensional dynamic object

If all complex values in demonstrative form are set in the expression (49), then we obtain the expression –

$$\Delta Y_i(\omega)e^{j\varphi_{\Delta yi}(\omega)} = A_{\Delta i}(\omega)X(\omega)e^{j(\varphi_{\Delta i}(\omega)+\varphi_X(\omega))}, \tag{53}$$

from which it follows that AFC of the dynamic object by equivalenting error $\Delta Y_i(\omega)$ will be equal to –

$$\Delta Y_i(\omega) = A_{\Delta i}(\omega)X(\omega), \tag{54}$$

where $X(\omega)$ – AFC of dynamic object input signal.

As it is known [4], applying Parserval equality, root-mean-square error $\overline{\Delta y_i(t)}$, characterizing its average power, can be determined from the expression:

$$\overline{\Delta y_i(t)} = \frac{1}{\pi}\int_0^\infty [\Delta Y_i(\omega)]^2 d\omega, \tag{55}$$

substituting in it the expression (54) and replacing the integral by the sum of values in the points of spectrum quantization, we obtain:

$$\overline{\Delta y_i(t)} \approx \frac{\Delta\omega}{\pi}\sum_{s=1}^{m}[A_{\Delta i}(\omega_s)X(\omega_s)]^2 =$$

$$= \frac{\Delta\omega}{\pi}\sum_{s=1}^{m}\left[A^2(\omega_s) + A_i^2(\omega_s) - 2A(\omega_s)A_i(\omega_s)\cos(\varphi(\omega_s) - \varphi_i(\omega_s))\right]X^2(\omega_s), \tag{56}$$

where $\Delta\omega$ — quantization interval by frequency in logarithmic scale, $\omega_{\max}$ — upper frequency of the bandwidth of the dynamic object, the number of discretes m is found from the expression:

$$m = \frac{\omega_{\max} - \omega_n}{\Delta\omega}. \tag{57}$$

Mean error $\overline{\Delta y_i(t)}$ will be the greatest while processing by the dynamic object jump-like input signal (5), for which

$$X(j\omega) = \frac{1}{j\omega} = \frac{1}{\omega}e^{-j\frac{\pi}{2}}. \tag{58}$$

Substituting $X(\omega)$ from (58) into (56) we will have:

$$\overline{\Delta y_i(t)} \approx \frac{\Delta\omega}{\pi}\sum_{s=1}^{m}[A_{\Delta i}(\omega_s)X(\omega_s)]^2 =$$

$$= \frac{\Delta\omega}{\pi}\sum_{s=1}^{m}\left[A^2(\omega_s) + A_i^2(\omega_s) - 2A(\omega_s)A_i(\omega_s)\cos(\varphi(\omega_s) - \varphi_i(\omega_s))\right]\frac{1}{\omega_s^2}. \qquad (59)$$

By analogy with the expressions (55), (56), (58) and (59) average power $\overline{y(t)}$ of initial signal $y(t)$ of the dynamic object as its reaction on the input single signal (5) can be presented in the form —

$$\overline{y(t)} = \frac{1}{\pi}\int_0^\infty [Y(\omega)]^2 d\omega \approx \frac{\Delta\omega}{\pi}\sum_{s=1}^{m}[A(\omega_s)X(\omega_s)]^2 = \frac{\Delta\omega}{\pi}\sum_{s=1}^{m}A^2(\omega_s)\frac{1}{\omega_s^2}. \qquad (60)$$

Taking into account the expressions (59) and (60), relative root-mean-square error of equivalenting $\overline{\delta\, y_i(t)}$, expressed in per cent, may be presented in the form —

$$\overline{\delta\, y_i(t)} = \frac{\overline{\Delta y_i(t)}}{\overline{y(t)}}100\%, \quad i = \overline{1,\ 3}. \qquad (61)$$

Calculating by the expression (61), taking into account the expressions (59) and (60) relative root-mean-square of equivalenting error, we will take a decision if the selected equivalent model of multidimensional dynamic object satisfies us or not.

Conclusions

1. Method of processes identification in multidimensional objects, allowing linearization and operating in direct signal transmission mode, by means of mathematical models of not higher than the third order, equivalent by the cut-off frequency is suggested.
2. Algorithm of the method is based on the system of equations , one part of which is synthesized, taking into account limiting conditions, set by the frequency of the cut-off and the second part is synthesized by the standard procedure of the least square method relatively logarithmic frequency characteristics.
3. As the criterion of equivalenting possibility of multidimensional dynamic object by the selected equivalent mathematical model it is suggested to use relative root-mean-square of equivalenting error, for which computational form is obtained.

REFERENCES

1. Мокін О. Б. Визначення умов, за яких рух динамічних об’єктів з порядком математичних моделей, вищим трьох, можна описувати еквівалентними моделями з порядком, не вищим трьох / О. Б. Мокін, В. Б. Мокін, Б. І. Мокін // Вісник Вінницького політехнічного інституту. — 2014. — №4. — С. 7 – 15.
2. Макаров И. М. Линейные автоматические системы / И. М. Макаров, Б. М. Менский. — М.: Машиностроение, 1977. — 464 с.
3. Цыпкин Я. З. Основы информационной теории идентификации / Цыпкин Я. З. — М.: Наука, 1984. — 320 с.
4. Бабак В. П. Обробка сигналів / В. П. Бабак, В. С. Хандецький, Е. Шрюфер. — К.: Либідь, 1999. — 496 с.

Mokin Oleksandr – Doctor of Science (Eng), Professor, Head of the Chair of Renewable Power and Transport Electric Systems and Complexes (RETESC), e-mail: abmokin@gmail.com.

Mokin Vitaliy – Doctor of Science (Eng), Professor, Head of the Chair of Computer Ecological and Economic Monitoring and Engineering Graphics (CEEMEG), vbmokin@gmail.com.

Mokin Borys – Academician of NAPSU, Doctor of Science (Eng), Professor, Department of Renewable Power and Transport Electric Systems and Complexes.

Chernova Iryna – Engineer of the Research Department.
Vinnytsia National Technical University.