D. O. Topchyi

THE THEORY OF PLAFALES: CONSTRUCTION OF A STANDARD BASIS OF SFE•12

On the basis of the theory of plafales, the paper shows the main steps of the algorithm for constructing a standard basis of the serendipity finite element – SFE • 12 (bicubic approximation).

Key words: *bicubic approximation, basis functions, plafal (-es).*

Introduction

The history of the finite element method (FEM) started with the idea of R. Courant, an outstanding mathematician, which he published in 1943 [1], [2-4]. Initially, the researchers took no interest in Courant's idea since its realization required huge computational efforts. After the emergence of computers the method started to be actively developed by research engineers. And they, not mathematicians, occupied computers immediately in order to obtain answers to the practical questions. Courant's procedure had become a new step in computational mathematics, though the influence of FDM (the method of finite differences) remained for a certain period of time (before the appearance of Turner's arbitrary triangulation). Rapid growth and popularization of FEM is explained by professional background of the users. On the other hand, some believe (and not without reason) that the lack of mathematical knowledge of engineering-oriented professionals was the main cause for the emergence and spread of false hypotheses and inadequate models in FEM. The majority of errors are associated with the construction of form functions (basis functions) of finite elements, in particular, the elements of serendipity family. These elements were a real breakthrough in FEM.

A square with bilinear interpolation was first used as a computational template in 1964 [5]. This element is wonderfully combined with a triangular simplex, creating a simple and efficient FEM grid. Squares, as a rule, are efficient in the middle of the computational domain and triangles – in the boundary strip. In real two- and three-dimensional problems boundaries of the computational domain, boundaries between the elements as well as interfaces (in inhomogeneous environments) are often curvilinear [5, 6, 7]. Such element was investigated by Ergatoudis, Irons and Zienkiewicz in 1968. It was an example of a successful application of the isoparametric technique that consists in selecting piecewise polynomial functions in order to determine transformation of the coordinates [9]. The term isoparametric means that for coordinate transformation the same polynomials are selected as those which interpolate a physical field, i.e. basis functions play a double role. In 1968 the authors did not take into account that basis functions play a triple role [8]. They are used in the problems of localization of the loads on the finite element. If there are internal nodes, transformation could be sensitive to the displacement of these nodes. Probably, the authors observed the feature and this was the reason for their abandoning the internal node of Lagrangian model [8]. In the early 80-ies of the 20th century, when it became clear that the role of matrix algebra in FEM is exaggerated, geometrical approaches appeared [10] as well as stochastic procedures for constructing the bases [11, 12]. It is no exaggeration to emphasize that bicubic approximation (interpolation) occupies a special place in the theory of polynomial approximation of two-argument functions.

We believe that there is a possibility to propose an algorithm of constructing basis functions, based on the theory of plafales, which is different from the above-mentioned procedures [13, 14, 15]. On our opinion, the role of basis functions is a tetradic one. IT platforms in FEM, that could be created on the basis of the theory of plafales [16], contain the notion of algorithmic complexity: introduction of the basis (and global basis) functions into the software complex as well as search for the problem solution by software-hardware complex are functions of time. Therefore, basis functions are functions of time. Definitely, an integral component of the above-mentioned IT platform is the process of its functional visualization [17].

Analysis of the research

This study is based on the works [13, 14, 15] and on the conference materials [16, 18].

Aim of the work

The paper aims mainly at showing the algorithm of forming surface $L_i(x,y,t)$ (bicubic approximation) of the basis function of time.

Current importance of the work

IT platforms in FEM (based on the algorithms of the theory of plafales) can contain an artificial intelligence component – construction of basis functions in the automatic mode.

Main part

We shall consider a standard square 2×2 with 12 nodes – a serendipity finite element (Fig. 1). Basis functions of bicubic interpolation $L_i(x,y)$ ($i=\overline{1;12}$) must satisfy the interpolation hypothesis:

$$L_i(x_k, y_k) = \delta_{ik}, \quad \sum_{i=1}^{12} L_i(x,y) = 1, \tag{1}$$

where δ_{ik} – Kronecker symbol, i – number of the function, k – number of the node.

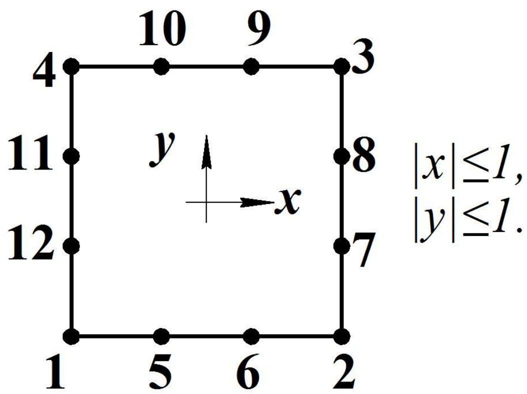

Fig. 1. Serendipity FE of bicubic interpolation (12 nodes)

Standard basis of SFE • 12 is given by [19]:

$$
\begin{aligned}
&L_1(x,y) = \frac{1}{32}(1-x)(1-y)(-10+9(x^2+y^2)), \quad L_2(x,y) = \frac{1}{32}(1+x)(1-y)(-10+9(x^2+y^2)),\\
&L_3(x,y) = \frac{1}{32}(1+x)(1+y)(-10+9(x^2+y^2)), \quad L_4(x,y) = \frac{1}{32}(1-x)(1+y)(-10+9(x^2+y^2)),\\
&L_5(x,y) = \frac{9}{32}(1-x^2)(1-y)(1-3x), \quad L_6(x,y) = \frac{9}{32}(1-x^2)(1-y)(1+3x),\\
&L_7(x,y) = \frac{9}{32}(1-y^2)(1+x)(1-3y), \quad L_8(x,y) = \frac{9}{32}(1-y^2)(1+x)(1+3y),\\
&L_9(x,y) = \frac{9}{32}(1-x^2)(1+y)(1+3x), \quad L_{10}(x,y) = \frac{9}{32}(1-x^2)(1+y)(1-3x),\\
&L_{11}(x,y) = \frac{9}{32}(1-y^2)(1-x)(1+3y), \quad L_{12}(x,y) = \frac{9}{32}(1-y^2)(1-x)(1-3y).
\end{aligned} \tag{2}
$$

Let us consider properties of function $L_i(x,y,t)$:

$$L_i(x,y,t) = L_i(x,y)^\circ G(t) = \begin{cases} L_i(x,y), t \equiv T, i = \overline{1;12}, \\ N_i(x,y), t \in (\gamma;T]; \gamma \geq 0; i = \overline{1;12}, \end{cases} \tag{3}$$

$$z_{x,y} = G(t),$$

where parameter t – time; T – moment of time when surface (2) is formed; respectively $N_i(x,y)$ – surfaces that are formed at the time moments $t \in (\gamma;T]$; *G(t)* – global function of modification time of the applicates of surfaces $L_i(x,y,t)$. In what follows (in an implicit form), *G(t)* will be considered as composition of the objects of the theory of plafales.

Let us formulate an interpolation hypothesis for functions $N_i(x_k, y_k)$ as follows:

$$N_i(x_k, y_k) = \delta_{ik}{}^\circ G(t) \tag{4}$$

For functions $N_i(x,y)$ the following estimation is performed:

$$|N_i(x,y)| \leq 1 \tag{5}$$

Introduction of the systems and preparation of a computational template

Let us introduce the following systems of the theory of plafales: the static canvas of plafal $PF_k^{U^{SP}}$ [15, P.16], the «ensemble» of the points $PF_{r\,(i,j)}^{ens\,(i,e)}$ [15, P. 569 - 575], the imaginary point of plafal $PF^{(i,e)pi}$ [15, P. 29 - 86], the degenerate isolated point of plafal [15, P. 23 - 25], the flickering point of plafal $PF^{(i,e)pid}$ [15, P. 87 - 152].

On the static canvas (a zero-level surface) $PF_k^{U^{SP}}$ ensemble of the points $PF_{r\,(i,j)}^{ens\,(i,e)}$ [15, P. 569] creates a standard square 2×2: $PF_{r\,(i,j)F^d(x,y)}^{ens\,(i,e)}$ with 12 nodes along the contour (Fig.2):

$$PF_k^{U^{SP}} : PF_{r\,(i,j)}^{ens\,(i,e)} \to PF_{r\,(i,j)F^d(x,y)}^{ens\,(i,e)} \tag{6}$$

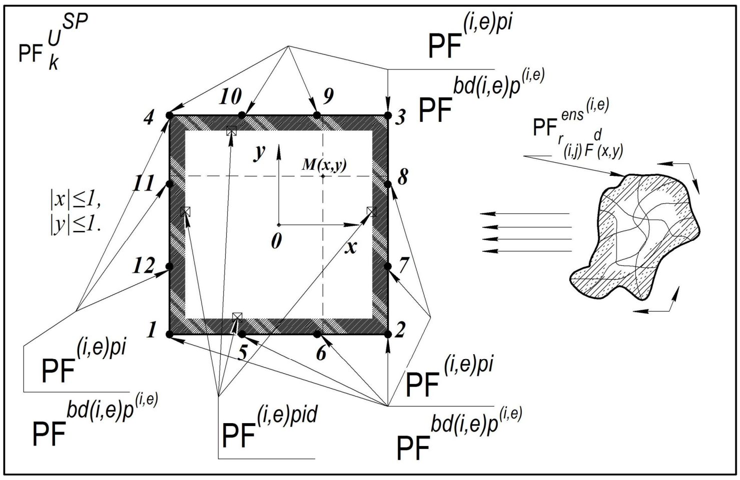

Fig. 2. Creation of the finite element

As a computational template, in a general form, Fig. 3 will be considered. Nodes 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 are imaginary $PF^{(i,e)pi}$ or degenerate isolated points $PF^{bd(i,e)p^{(i,e)}}$ (for each of the 12 basis functions there is its own order of location), between which time-based transition is performed [15, P. 31, P. 41]. Along the sides 1 – 2, 2 – 3, 3 – 4, 4 – 1 (along the contour, respectively) and inside the square (region D) flickering points $PF^{(i,e)pid}$ are located.

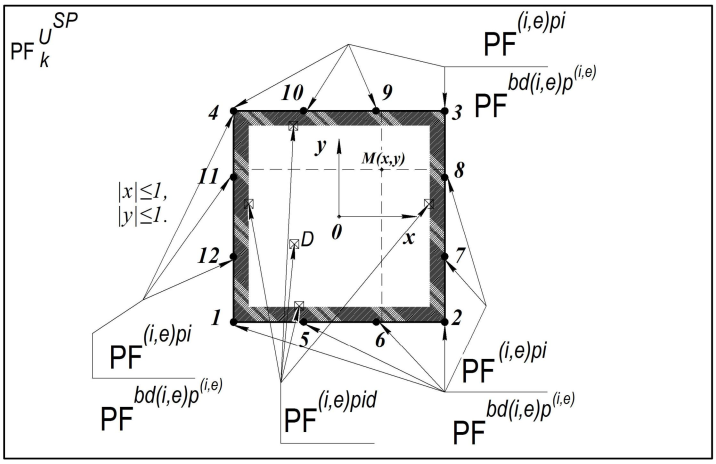

Fig. 3. Computational template

Computational template configuration

Using the interpolation hypothesis (1), we assume [15, P. 30]:

$$N_i(x_i, y_i) = PF^{(i,e)pi} = a(m(t)) \pm 1 = 1, \tag{7}$$

where $a(m(t)) \pm 1$ – characteristic function of an imaginary point; $m(t)$ – time function of the characteristic function. Using the interpolation hypothesis (1), we assume [15, P. 30]:

$$N_i(x_k, y_k) = PF^{bd(i,e)p^{(i,e)}} = a(n(t)) \pm 1 = 0, \tag{8}$$

where $a(n(t)) \pm 1$ – characteristic function of the degenerate isolated point; $n(t)$ – time function of the characteristic function. Using the property $|N_i(x, y)| \le 1$, we assume that values of the applicates along the contour 1 – 2, 2 – 3, 3 – 4, 4 – 1 and in region D [15, P. 88]:

$$N_i(x, y) = PF^{(i,e)pid} = a(h(t)) \pm 1; \quad |a(h(t)) \pm 1| < 1, \tag{9}$$

where $a(h(t)) \pm 1$ – characteristic function of a flickering point; $h(t)$ – time function of the characteristic function.

Respectively, for characteristic functions $a(m(t)) \pm 1$, $a(n(t)) \pm 1$, $a(h(t)) \pm 1$ the following possibilities arise:

– For (7), (8), (9) there is a common time value $t = T$, for which they exist. Therefore, $m(t)$, $n(t)$ and $h(t)$ are interrelated as follows:

$$\begin{cases} a(m(t)) = 0, \\ a(n(t)) = 1. \end{cases} \Rightarrow \quad n(t) = f(m(t)); \tag{10}$$

$$h(t) = h_1(t) \circ h_2(t) = m(t, \beta) \circ n(t, \beta); \quad |\beta| < 1 \tag{11}$$

This construction is a “soft” modeling [20] of the computational pattern configuration.

– $m(t)$ and $n(t)$ are arbitrary functions. Therefore, for each of them its own time moment may exist, when equalities (7), (8), (9) are realized.

The sequence (time-based transition) of constructing the basis functions consists from the following chain: $L_1(x, y) \to L_2(x, y) \to L_3(x, y) \to L_4(x, y) \to \ldots \to L_{12}(x, y)$. Between the basis functions smooth time-based transitions are performed. In what follows, construction of the chains of basis functions will be performed in accordance with the two above-mentioned possibilities. Timing will be started from $t = 0$.

Construction of the basis function in the first node

In accordance with (4), (7), (8), (9) we obtain configuration of the basis function in the first node (Fig. 4):

$$\begin{cases} N_1(x_1, y_1) = PF^{(i,e)pi} = a(m(t)) \pm 1 = 1, \\ N_1(x_k, y_k) = PF^{bd(i,e)p^{(i,e)}} = a(n(t)) \pm 1 = 0; \quad k = \overline{2;12}, \\ N_1(x, y)_{1-2,4-1,D} = PF^{(i,e)pid} = a(h(t)) \pm 1 = \alpha; \quad |\alpha| < 1, \\ N_1(x, y)_{2-3,3-4} = PF^{(i,e)pid} = a(h(t)) \pm 1 = PF^{bd(i,e)p^{(i,e)}} \equiv 0. \end{cases} \tag{12}$$

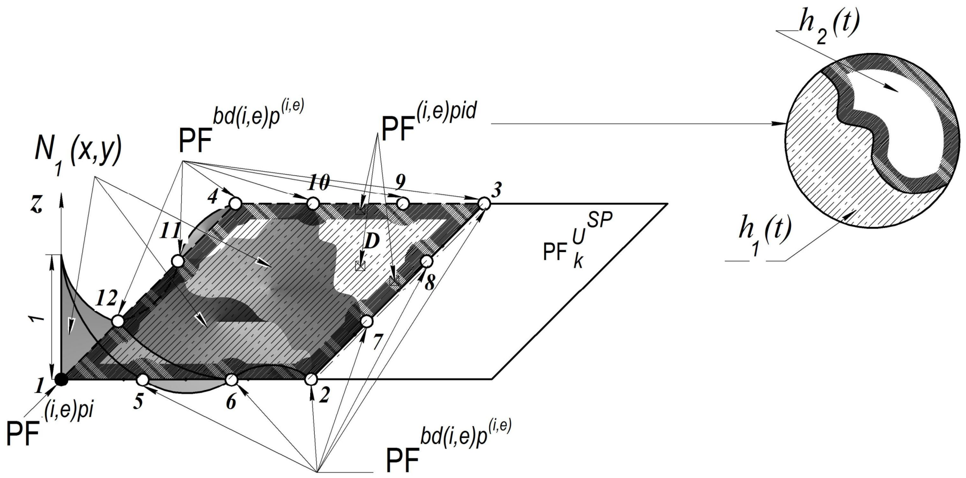

Fig. 4. Configuration of the basis function in the first node

In the correspondence of two cases 5.2 (of the computational template configuration), we obtain

– According to (10):

$$\exists\, a = \ln: \begin{cases} \ln(m(t)) = 0, \\ \ln(n(t)) = 1. \end{cases} \Rightarrow n(t) = e \times m(t); \tag{13}$$

Let us assume that $A: m(t) = t \Rightarrow n(t) = e \times t$. Then (11) takes on the form:

$$\begin{cases} h(t)_{1-2,4-1} = t \times (-\frac{9}{16}\beta^3 + \frac{9}{16}\beta^2 + \frac{1}{16}\beta - \frac{1}{16}) + (e \times t) \times (1 - (-\frac{9}{16}\beta^3 + \frac{9}{16}\beta^2 + \frac{1}{16}\beta - \frac{1}{16})); \quad |\beta| < 1, \\ h(t)_{1-2}: \beta = x; \quad h(t)_{4-1}: \beta = y. \end{cases}$$

$$\tag{14}$$

$$\begin{cases} h(t)_D = t \times \zeta + (e \times t) \times \tau; \quad |\zeta| < 1; \quad |\tau| < 1, \\ h(t)_D: (\zeta; \tau) = (x; y). \end{cases}$$

From (13), (14) and A we determine that system (12) comes into force (surface $L_i(x, y)$ is formed) for $t = T = 1$:

$$\begin{cases} N_1(x_1, y_1) = PF^{(i,e)pi} = \ln(1) + 1 = 1, \\ N_1(x_k, y_k) = PF^{bd(i,e)p^{(i,e)}} = \ln(e) - 1 = 0; \quad k = \overline{2;12}, \\ N_1(x, y)_{1-2,4-1,D} = PF^{(i,e)pid} \equiv L_1(x, y); \\ N_1(x, y)_{2-3,3-4} = PF^{(i,e)pid} = a(h(t)) \pm 1 = PF^{bd(i,e)p^{(i,e)}} = \ln(e) - 1 \equiv 0. \end{cases} \tag{15}$$

For $0<t<1$ system (12) takes on the following form:

$$\begin{cases} N_1(x_1, y_1) = PF^{(i,e)pi} = \ln(t) \pm 1, \\ N_1(x_k, y_k) = PF^{bd(i,e)p^{(i,e)}} = \ln(e \times t) \pm 1; \quad k = \overline{2;12}, \\ N_1(x, y)_{1-2,4-1,D} = PF^{(i,e)pid} = \ln(h(t)_{1-2,4-1,D}) \pm 1, \\ N_1(x, y)_{2-3,3-4} = PF^{(i,e)pid} = \ln(h(t)) \pm 1 = PF^{bd(i,e)p^{(i,e)}} \equiv \ln(e \times t) \pm 1. \end{cases} \tag{16}$$

Function $N_1(x, y)$ is given by:

$$N_1(x, y) = (\ln(t) \pm 1)^{\circ} (\ln(e \times t) \pm 1)^{\circ} (\ln(h(t)_{1-2,4-1,D}) \pm 1); \quad 0 < t \le 1. \tag{17}$$

where compositions of the functions in (17) are equivalent to (15) and (16).

Smooth transition from $L_1(x, y) \to N_2(x, y)$ is performed according to the rules of «absolute transition» and «absolute reversionary returning back transition after a certain time» [15, P. 31; 15, P. 41]:

$$\begin{cases} PF^{(i,e)pi} \to^{\Omega(g(t))} PF^{bd(i,e)p^{(i,e)}}, \\ g(t) = (\ln(m(1+c))^{\circ} \ln(n(1+c;\infty)) + (-1)^{[m(1+c)^{\circ} n(1+c;\infty)]}). \end{cases} \tag{18}$$

$$\begin{cases} PF^{bd(i,e)p^{(i,e)}} \to^{\Omega(g^r(t))} PF^{(i,e)pi}, \quad (k=2) \\ g_1(t) = (\ln(m(1+c))^{\circ} \ln(n(T_2)) + (-1)^{[m(1+c)^{\circ} n(T_2)]}); t \in [1+c; T_2], \\ g_2(t) = (\ln(m(T_2+d))^{\circ} \ln(n(T_2+d;\infty)) + (-1)^{[m(T_2+d)^{\circ} n(T_2+d;\infty)]}), \\ g^r(t) = g_1(t) \cup g_2(t). \end{cases} \tag{19}$$

where $(1+c)$ – time moment of smooth transition $L_1(x, y) \to N_2(x, y)$; (T_2+d) – time moment of smooth transition $L_2(x, y) \to N_3(x, y)$; T_2 – time moment at which system (27) comes into force for function $N_2(x, y)$

– Let $\forall\, m(t): \exists\, t = T_1$ so that (7) is satisfied :

$$N_1(x_1, y_1) = PF^{(i,e)pi} = a(m(T_1)) \pm 1 = 1 \tag{20}$$

Let $\forall\, n(t): \exists\, t = T_2 \ne T_1$ so that (8) is satisfied:

$$N_1(x_k, y_k) = PF^{bd(i,e)p^{(i,e)}} = a(n(T_2)) \pm 1 = 0 \tag{21}$$

Taking into account (14), we reduce (20) i (21) to uniform conditions (22), (23):

$$\begin{cases} T_2 > T_1: \\ N_1(x_1,y_1) = PF^{(i,e)pi} = a(m(t))\pm 1; \quad \sup_t(a(m(t))\pm 1)=1; \quad t\in(0;T_2], \\ N_1(x_k,y_k) = PF^{bd(i,e)p^{(i,e)}} = a(n(t))\pm 1; \inf_t(a(n(t))\pm 1)=0; \quad t\in(0;T_2]; \quad k=\overline{2;12}, \\ PF^{(i,e)pid} = a((-\frac{9}{16}\beta^3+\frac{9}{16}\beta^2+\frac{1}{16}\beta-\frac{1}{16})\times m(t)+(1-(-\frac{9}{16}\beta^3+\frac{9}{16}\beta^2+\frac{1}{16}\beta-\frac{1}{16}))\times n(t))\pm 1 = \\ = N_1(x,y)_{1-2,4-1} = \lambda_1\beta^3+\lambda_2\beta^2+\lambda_3\beta+\lambda_4; \; \lambda_j\in R; \quad j=\overline{1;4}; \quad T_1<t<T_2, \\ PF^{(i,e)pid} = a(\zeta\times m(t)+\tau\times n(t))\pm 1 = N_1(x,y)_D = \sum_{k,l=0}^{4}\mu_{kl}\times\zeta^k\tau^l; \; \mu_{kl}\in R, \quad T_1<t<T_2, \\ PF^{(i,e)pid} = a(h(t)_{1-2,4-1,D})\pm 1 = N_1(x,y)_{1-2,4-1,D} = L_1(x,y), \quad t=T_2, \\ N_1(x,y)_{2-3,3-4} = PF^{(i,e)pid} = a(h(t))\pm 1 = \inf_t(a(n(t))\pm 1) = 0; \quad t\in(0;T_2]. \end{cases} \tag{22}$$

$$\begin{cases} T_2 < T_1: \\ N_1(x_1,y_1) = PF^{(i,e)pi} = a(m(t))\pm 1; \quad \sup_t(a(m(t))\pm 1)=1; \quad t\in(0;T_1], \\ N_1(x_k,y_k) = PF^{bd(i,e)p^{(i,e)}} = a(n(t))\pm 1; \inf_t(a(n(t))\pm 1)=1; \quad t\in(0;T_1]; \quad k=\overline{2;12}, \\ PF^{(i,e)pid} = a((-\frac{9}{16}\beta^3+\frac{9}{16}\beta^2+\frac{1}{16}\beta-\frac{1}{16})\times m(t)+(1-(-\frac{9}{16}\beta^3+\frac{9}{16}\beta^2+\frac{1}{16}\beta-\frac{1}{16}))\times n(t))\pm 1 = \\ = N_1(x,y)_{1-2,4-1} = \lambda_1\beta^3+\lambda_2\beta^2+\lambda_3\beta+\lambda_4; \; \lambda_j\in R; \quad j=\overline{1;4}; \quad T_2<t<T_1, \\ PF^{(i,e)pid} = a(\zeta\times m(t)+\tau\times n(t))\pm 1 = N_1(x,y)_D = \sum_{k,l=0}^{4}\mu_{kl}\times\zeta^k\tau^l; \; \mu_{kl}\in R, \quad T_2<t<T_1, \\ PF^{(i,e)pid} = a(h(t)_{1-2,4-1,D})\pm 1 = N_1(x,y)_{1-2,4-1,D} = L_1(x,y), \quad t=T_1, \\ N_1(x,y)_{2-3,3-4} = PF^{(i,e)pid} = a(h(t))\pm 1 = \inf_t(a(n(t))\pm 1) = 1; \quad t\in(0;T_1]. \end{cases} \tag{23}$$

Conditions (22) and (23) are generalized to configuration (12).

Function $N_1(x,y)$ has the form of

$$N_1(x,y) = (a(m(t))\pm 1)^\circ(a(n(t))\pm 1)^\circ(a(h(t)_{1-2,4-1,D})\pm 1); \; t\in(0;T_1] \quad or \quad t\in(0;T_2]. \tag{24}$$

where compositions of functions in (24) are equivalent to (22) and (23).

Smooth transition from $L_1(x,y)\to N_2(x,y)$ is performed according to the rules [15, P. 31; 15, P. 41]:

$$\begin{cases} PF^{(i,e)pi} \quad \to^{\Omega(g(t))} \quad PF^{bd(i,e)p^{(i,e)}}, \\ g(t) = (a(m(T_{1,2}+c))^\circ a(n(T_{1,2}+c;\infty)) + (-1)^{[m(T_{1,2}+c)^\circ n(T_{1,2}+c;\infty)]}). \end{cases} \tag{25}$$

$$\begin{cases} PF^{bd(i,e)p^{(i,e)}} \quad \to^{\Omega(g^r(t))} \quad PF^{(i,e)pi}, \quad (k=2) \\ g_1(t) = (a(m(T_{1,2}+c))^\circ a(n(T_3)) + (-1)^{[m(T_{1,2}+c)^\circ n(T_3)]}); \; t\in[T_{1,2}+c;T_3], \\ g_2(t) = (a(m(T_3+d))^\circ a(n(T_3+d;\infty)) + (-1)^{[m(T_3+d)^\circ n(T_3+d;\infty)]}), \\ g^r(t) = g_1(t)\cup g_2(t). \end{cases} \tag{26}$$

where $(T_{1,2}+c)$ – time moment of smooth transition $L_1(x,y) \to N_2(x,y)$; (T_3+d) – time m0ment of smooth transition $L_2(x,y) \to N_3(x,y)$; T_3 – time moment at which system (27) comes to force for function $N_2(x,y)$.

Construction of basis functions in the 2 – 12 nodes

Construction of functions $N_j(x,y);\ j=\overline{2;12}$ is performed similar to function $N_1(x,y)$ taking into account configurations of basis functions (27):

$$\begin{cases} N_i(x_i,y_i) = PF^{(i,e)pi} = a(m(t)) \pm 1 = 1; \quad i=\overline{2;12}, \\ N_i(x_k,y_k) = PF^{bd(i,e)p^{(i,e)}} = a(n(t)) \pm 1 = 0; \quad k=\overline{1;12}/\{i\}, \\ N_i(x,y)_{i \in m \cap n, D} = PF^{(i,e)pid} = a(h(t)) \pm 1 = \alpha\ ; \quad |\alpha|<1, \\ N_i(x,y)_{i \notin m \cap n} = PF^{(i,e)pid} = a(h(t)) \pm 1 = PF^{bd(i,e)p^{(i,e)}} \equiv 0. \end{cases} \tag{27}$$

where $i \in m \cap n$ is the node with which the side (-s) of a standard square are associated.

Assembling of surfaces

Proceeding from the everything mentioned above, for (3) we obtain that *G(t)* is a global time function of changing the applicates of surfaces $L_i(x,y,t)$:

$$\begin{aligned} G(t) = N_1(x,y)^\circ N_2(x,y)^\circ N_3(x,y)^\circ N_4(x,y)^\circ N_5(x,y)^\circ N_6(x,y)^\circ N_7(x,y)^\circ N_8(x,y)^\circ N_9(x,y)^\circ \\ {}^\circ N_{10}(x,y)^\circ N_{11}(x,y)^\circ N_{12}(x,y)^\circ g(t)^\circ g^r(t) \end{aligned} \tag{28}$$

Conclusions and prospects for the research

In the case of successful testing of IT platforms, that can be created on the basis of the algorithms of the theory of plafales [16, 18], it will be possible not to use the already known standard basis $L_i(x,y)$. Instead of it, functions $N_i(x,y)$ can be directly used. Respectively, the investigated function $L_i(x,y,t)$ will have the following form: $L_i(x,y,t) \equiv G(t)$. All the known informational platforms in FEM, which are used in engineering calculations, contain a known set of standard basis functions of O. Zienkiewicz. The proposed algorithm does not break monumentality of the standard basis functions of Zenkevitch or alternative basis functions of A. N. Khomchenko. IT platform in FEM (based on the algorithms of the theory of plafales) will perform the following functions: 1.To construct, in an automatic mode, a basis function on the computational template, where people have not found the basis yet. For a platform (in a final form) to be able to construct a monumental surface, it must “”analyze” intermediate surfaces that are formed before the final monolithic (basis) surface. To realize this, a key parameter – time – is introduced. 2. To represent formation (the relief) of a non-stationary field with dynamic thermocouples. 3. To function as an integral software complex that will find physically adequate models on complex computational templates and perform its role in the engineering applications.

REFERENCES

1. Courant R. Variational methods for the solution of problems of equilibrium and vibrations / R. Courant / – Bull. Amer. Math. Soc. – 1943. – V. 49. – P. 1 – 23.

2. Сильвестер П. Метод конечных элементов для радиоинженеров и инженеров-электриков / П. Сильвестер, Р. Феррари. – М.: Мир, 1986. – 229 с.

3. Сегерлинд Л. Применение метода конечных элементов / Сегерлинд Л. – М.: Мир, 1979. – 392 с.

4. Марчук Г. И. Введение в проекционно-сеточные методы / Г. И. Марчук, В. И. Агошков. – М.: Мир, 1981. – 416 с.

5. Оден Дж. Конечные элементы в нелинейной механике сплошных сред / Оден Дж. – М.: Мир, 1976. – 464 с.

6. Зенкевич О. Метод конечных элементов в технике / Зенкевич О. – М.: Мир, 1975. – 541 с.

7. Митчелл Э. Метод конечных элементов для уравнений с частными производными / Э. Митчелл, Р. Уэйт. – М.: Мир, 1981. – 216 с.

8. Ergatoudis I. Curved isoparametric quadrilateral elements for finite element analysis / I. Ergatoudis, B. M. Irons, O. C. Zienkiewicz // Int. J. Solids Struct. – 1968. – V. 4. – P. 31 – 42

9. Стренг Г. Теория метода конечных элементов / Г. Стринг, Дж. Фикс. – М.: Мир, 1981. – 416 с.

10. Wachspress E. I. A rational finite element basis / E. I. Wachspress. – Academic Press: New York, 1975. – 216 p.

11. Хомченко А. Н. Некоторые вероятностные аспекты МКЭ / А. Н. Хомченко. – Ив. - Франк. ин-т нефти и газа: Ивано-Франковск, 1982. – 6 с. Деп. в ВИНИТИ, № 1213.

12. Хомченко А. Н. Метод конечных элементов: стохастический подход / А. Н. Хомченко. – Ив. - Франк. ин-т нефти и газа: Ивано-Франковск, 1982. – 7 с. Деп. в ВИНИТИ, № 5167.

13. Topchyi D. The theory of plafales: the proof of P versus NP problem / Topchyi D. – Best Global Publishing, 2011. – 634 p.

14. Topchyi D. The theory of plafales: the proof algorithms for millennium problems / Topchyi D. – Best Global Publishing, 2013. – 695 p.

15. Topchyi D. The theory of plafales: the proof algorithms for millennium problems [Електронний ресурс] / Topchyi D. – Best Global Publishing, 2013. – 695 p. – Режим доступу до ресурсу: http://eleanor-cms.ru/uploads/book.pdf.

16. The theory of plafales: Applications of new cryptographic algorithms and platforms in Military complex, IT, Banking system, Financial market [Електронний ресурс] / Topchyi D. // XLII KONFERENCJA ZASTOSOWAŃ MATEMATYKI. – 2013. – Режим доступу до ресурсу: http://www.impan.pl/~zakopane/42/Topchyi.pdf.

17. Зенкин А. А. Когнитивная компьютерная графика / Зенкин А. А. – М.: Наука, 1991. – 192 с.

18. XLII KONFERENCJA ZASTOSOWAŃ MATEMATYKI, 2013 / [Електронний ресурс] // Режим доступу до ресурсу: http://www.impan.pl/KZM/42/.

19. Коннор Дж. Метод конечных элементов в механике жидкости / Дж. Коннор, К. Бреббиа. – Л.: Судостроение, 1979. – 264 с.

20. Арнольд В. И. Жесткие и мягкие математические модели / Арнольд В. И. – М.: МЦНМО, 2008. – 32 с.

Topchyi Dmytro **–** Researcher of the Department of Applied and Higher Mathematics.
Petro Mohyla Black Sea State University.