O. B. Mokin, Dr. Sc. (Eng.), Prof.; V. B. Mokin, Dr. Sc. (Eng.), Prof.;
B. I. Mokin, Acad. NAPSU, Dr. Sc. (Eng.), Prof.

METHOD OF IDENTIFICATION OF AUTOREGRESSION-SLIDING MEAN (ARSM) MODEL (P,Q) WITH RANDOM VALUES OF P, Q, ORDERS THAT GENERALIZES YULE–WORKER TECHNIQUE

Method of identification of autoregression sliding mean (ARSM) (p,q) model with random values of p,q,orders, that generalizes Yule-Worker technique, developed for identification of the model of autoregression AP(p), in case, when autoregressive component of AP(p) model is complemented with the component of sliding mean SM(q, is suggested). Algorithm of the suggested method contains both the system of linear equations of Yule-Worker type for determination of p parameters of autoregression and additional non-linear equations for identification of q parameters of sliding mean. Operation of the algorithm was demonstrated on the example p = 3, q = 3.

Key words: *time series, model of autoregression-sliding mean, Yule-Worker technique, non-linear complement of Yule-Worker technique.*

Problem set-up and initial preconditions

As it is known [1, 2], it is possible to identify model AP(p), that has the form

$$z_t = \varphi_1 z_{t-1} + \varphi_2 z_{t-2} + \ldots + \varphi_p z_{t-p} + a_t, \tag{1}$$

where z_t – centered value of time series at the moment of time t, a_t – white noise pulse with dispersion σ_a^2, generated at the same moment of time t, applying Yule-Worker technique, according to which, numerical values of regression coefficients φ_i, $i = 1, 2, \ldots, p$, are determined from matrix equation

$$\varphi = M^{-1}\rho, \tag{2}$$

where matrix M^{-1} is inverse to matrix M, and matrices φ, M, ρ have the form:

$$\varphi = \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \ldots \\ \varphi_p \end{bmatrix}, \quad M = \begin{bmatrix} 1\ \rho_1\ \rho_2\ \rho_3 \cdots \rho_{p-1} \\ \rho_1\ 1\ \rho_1\ \rho_2 \cdots \rho_{p-2} \\ \rho_2\ \rho_1\ 1\ \rho_1 \ldots \rho_{p-3} \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ \rho_{p-1}\rho_{p-2}\rho_{p-3}\ldots 1 \end{bmatrix}, \quad \rho = \begin{bmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \\ \ldots \\ \rho_p \end{bmatrix}. \tag{3}$$

It should be noted, that in matrices (3) ρ_i, i = 1, 2, …, p – these are autocorrelations of the centered time series z_t, determined from the expression

$$\rho_i = \frac{\gamma_i}{\gamma_0}, i = 1,2,\ldots,p, \tag{4}$$

where γ_i, $i = 1, 2, \ldots, p$ – autocovariations, determined for centered time series z_t by the expression

$$\gamma_i = \frac{1}{N-i}\sum_{t=1}^{N-i} z_t z_{t+i}, \ i = 1, 2, \ldots, p \tag{5}$$

and are used, besides the solution of matrix equation (2), also for determination of σ_a^2 dispersion of white noise a_t by the expression

$$\sigma_a^2 = \gamma_0 - \varphi_1\gamma_1 - \varphi_2\gamma_2 - \ldots - \varphi_p\gamma_p, \tag{6}$$

where the results of the solution of matrix equation (2) are substituted.

Expressions (1) – (6) assign initial preconditions for the problem of creation of identification method of the model of autoregression-sliding mean ARSM(*p,q*) with random values of *p, q*, orders structure of which for centered time series z_t has the form

$$z_t = \varphi_1 z_{t-1} + \varphi_2 z_{t-2} + \ldots + \varphi_p z_{t-p} + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \ldots - \theta_q a_{t-q}, \tag{7}$$

by means of further development of Yule-Worker technique.

To these six preconditions we should add one more – the seventh precondition, that is stipulated by the lack of correlation between neibouring pulses of white noise and can be set by the known expressions [1, 2]:

$$\operatorname{cov}(z_t a_{t-i}) = \begin{bmatrix} \sigma_a^2,\ for\ (i=0), \\ 0,\ for\ (i \neq 0). \end{bmatrix} \tag{8}$$

$$\operatorname{cov}(z_{t-k} a_{t-i}) = \begin{bmatrix} \sigma_a^2,\ for\ (i=k), \\ 0,\ for\ (i \neq k). \end{bmatrix} \tag{9}$$

Completing the introduction, to underline non-triviality of the method of ARSM (*p,q*) model identification at random values of *p, q* orders, suggested by us, we will quote the following passage from page 97 of Box and Jankins monograph [3]: «As in case of fixed σ_a^2, and in case of fixed σ_γ^2 (equal by definition γ_0 – authors remark) optimal choice leads to certain random processes, parameters of which are the functions of unknown dynamic parameters.That is why, we find ourselves in well known paradoxical situation when the best data can be collected only under condition that something is known about the answer, we are looking for. Consecutive approach, when we improve the mode, obtaining new information regarding the parameters, opens the possibilities, deserving further study» .

Solution of the set problem

At the first stage of creation of the method of ARSM (*p,q*) model identification and its algorithm in the model (7) we specify the values of parameters at the level when it will be possible to make generalizations but which do not lead to rather cumbersome mathematical expressions. It is obvious that if we set $p = 1$, $q = 1$ or $p = 2$, $q = 2$, that is characteristic for [1], then the obtained information will not be sufficient for generalization, that is why let $p = 3$ and $q = 3$. Then from the expression (7) for ARSM (3,3) we will have:

$$z_t = \varphi_1 z_{t-1} + \varphi_2 z_{t-2} + \varphi_3 z_{t-3} + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \theta_3 a_{t-3}. \tag{10}$$

Multiplying both parts of the equation (10) by turn into $z_t, z_{t-1}, z_{t-2}, z_{t-3}, z_{t-4}, z_{t-5}, z_{t-6}$, averaging the obtained products using the expression for autocovariation (5) and taking into account conditions (8), (9), we obtain the following system of equations:

$$\begin{cases} \gamma_0 = \varphi_1\gamma_1 + \varphi_2\gamma_2 + \varphi_3\gamma_3 + \sigma_a^2 - \theta_1(\varphi_1 - \theta_1)\sigma_a^2 - \theta_2(\varphi_2 - \theta_2)\sigma_a^2 - \theta_3(\varphi_3 - \theta_3)\sigma_a^2, \\ \gamma_1 = \varphi_1\gamma_0 + \varphi_2\gamma_1 + \varphi_3\gamma_2 - \theta_1\sigma_a^2 - \theta_2(\varphi_1 - \theta_1)\sigma_a^2 - \theta_3(\varphi_2 - \theta_2)\sigma_a^2, \\ \gamma_2 = \varphi_1\gamma_1 + \varphi_2\gamma_0 + \varphi_3\gamma_1 - \theta_2\sigma_a^2 - \theta_3(\varphi_1 - \theta_1)\sigma_a^2, \\ \gamma_3 = \varphi_1\gamma_2 + \varphi_2\gamma_1 + \varphi_3\gamma_0 - \theta_3\sigma_a^2, \\ \gamma_4 = \varphi_1\gamma_3 + \varphi_2\gamma_2 + \varphi_3\gamma_1, \\ \gamma_5 = \varphi_1\gamma_4 + \varphi_2\gamma_3 + \varphi_3\gamma_2, \\ \gamma_6 = \varphi_1\gamma_5 + \varphi_2\gamma_4 + \varphi_3\gamma_3. \end{cases} \tag{11}$$

Analyzing the obtained system of equations (11), we see that from the first four equations, applying Yule-Worker technique it is easy to obtain all three parameters of autoregression. It is obvious, that in this case, matrices, used in Yule-Worker technique, will have the form:

$$\varphi = \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{bmatrix}, \quad M_\gamma = \begin{bmatrix} \gamma_3 & \gamma_2 & \gamma_1 \\ \gamma_4 & \gamma_3 & \gamma_2 \\ \gamma_5 & \gamma_4 & \gamma_3 \end{bmatrix}, \quad \gamma = \begin{bmatrix} \gamma_4 \\ \gamma_5 \\ \gamma_6 \end{bmatrix}, \tag{12}$$

and analog of matrix solution (2) will be matrix expression

$$\varphi = M_\gamma^{-1}\gamma, \tag{13}$$

where M_γ^{-1} – matrix, inverse to matrix M_γ, set by the expression (12).

Further we will proceed in the following manner. For each of the first four equations of the system (11), we will find numerical values of algebraic sum of the components – we will denote them by A, B, C, D, using such expressions:

$$\gamma_0 - \varphi_1\gamma_1 - \varphi_2\gamma_2 - \varphi_3\gamma_3 = A, \tag{14}$$

$$\gamma_1 - \varphi_1\gamma_0 - \varphi_2\gamma_1 - \varphi_3\gamma_2 = B, \tag{15}$$

$$\gamma_2 - \varphi_1\gamma_1 - \varphi_2\gamma_0 - \varphi_3\gamma_1 = C, \tag{16}$$

$$\gamma_3 - \varphi_1\gamma_2 - \varphi_2\gamma_1 - \varphi_3\gamma_0 = D. \tag{17}$$

Substituting the expression (17) in the fourth equation of the system (11), we will find that

$$\sigma_a^2 = -\frac{D}{\theta_3}, \tag{18}$$

and substituting expressions (14)–(16) and expression (18) in the first three equations of the system (11), we obtain new system of equations in the form:

$$\begin{cases} A\theta_3 = D(-1 + \theta_1(\varphi_1 - \theta_1) + \theta_2(\varphi_2 - \theta_2) + \theta_3(\varphi_3 - \theta_3)), \\ B\theta_3 = D(\theta_1 + \theta_2(\varphi_1 - \theta_1) + \theta_3(\varphi_2 - \theta_2)), \\ C\theta_3 = D(\theta_2 + \theta_3(\varphi_1 - \theta_1)), \end{cases} \tag{19}$$

from the third equation of this system, we will find that

$$\theta_3 = \frac{D\theta_2}{C - D(\varphi_1 - \theta_1)}. \tag{20}$$

Substituting the expression (20) in the first two equations of the system (19) and, simplifying, we obtain new system of two equations with two unknowns θ_1, θ_2, having the form:

$$\begin{cases} f_1(\theta_1,\theta_2)=0, \\ f_2(\theta_1,\theta_2)=0, \end{cases} \tag{21}$$

where:

$$\begin{gathered} f_1(\theta_1,\theta_2)=A\theta_2(C-D(\varphi_1-\theta_1))-(C-D(\varphi_1-\theta_1))^2(-1+\theta_1(\varphi_1-\theta_1)+\theta_2(\varphi_2-\theta_2))- \\ -D\theta_2(\varphi_3(C-D(\varphi_1-\theta_1))-D\theta_2), \end{gathered} \tag{22}$$

$$f_2(\theta_1,\theta_2)=B\theta_2-(C-D(\varphi_1-\theta_1))(\theta_1+\theta_2(\varphi_1-\theta_1))-D\theta_2(\varphi_2-\theta_2). \tag{23}$$

Thus, the second stage of the suggested method of identification will be the determination of numerical values of θ_1^*,θ_2^* , θ_1,θ_2 parameters, sliding mean of the model.

Due to the fact that equations, the system (21) consists of, are non-linear, for the solution of this system of non-linear equations we will apply one of successive approximation methods. If, for example, for the solution of this system we apply Neuton method [4], then, knowing *n-approximation* of θ_1,θ_2 parameters, their (*n+1*)-approximation, we will find by means of recurrent relations:

$$\theta_{1(n+1)}=\theta_{1(n)}+\frac{\Delta f_{\theta_2(n)}}{\Delta f_{\theta_1\theta_2(n)}},\quad \theta_{2(n+1)}=\theta_{2(n)}+\frac{\Delta f_{\theta_1(n)}}{\Delta f_{\theta_1\theta_2(n)}},\quad n=0,1,2,..., \tag{24}$$

where:

$$\Delta f_{\theta_1\theta_2(n)}=\begin{vmatrix} \dfrac{\partial f_1(\theta_{1(n)},\theta_{2(n)})}{\partial\theta_1} & \dfrac{\partial f_1(\theta_{1(n)},\theta_{2(n)})}{\partial\theta_2} \\ \dfrac{\partial f_2(\theta_{1(n)},\theta_{2(n)})}{\partial\theta_1} & \dfrac{\partial f_2(\theta_{1(n)}\theta_{2(n)})}{\partial\theta_2} \end{vmatrix}, \tag{25}$$

$$\Delta f_{\theta_2(n)}=\begin{vmatrix} -f_1(\theta_{1(n)},\theta_{2(n)}) & \dfrac{\partial f_1(\theta_{1(n)},\theta_{2(n)})}{\partial\theta_2} \\ -f_2(\theta_{1(n)},\theta_{2(n)}) & \dfrac{\partial f_2(\theta_{1(n)}\theta_{2(n)})}{\partial\theta_2} \end{vmatrix},\ \Delta f_{\theta_1(n)}=\begin{vmatrix} \dfrac{\partial f_1(\theta_{1(n)},\theta_{2(n)})}{\partial\theta_1} & -f_1(\theta_{1(n)},\theta_{2(n)}) \\ \dfrac{\partial f_2(\theta_{1(n)},\theta_{2(n)})}{\partial\theta_1} & -f_2(\theta_{1(n)},\theta_{2(n)}) \end{vmatrix}. \tag{26}$$

Iteration process is stopped when relative error of the next approximation becomes smaller than its a priori assigned value δ , i. e., when

$$\left|\frac{\theta_{1(n+1)}-\theta_{1(n)}}{\theta_{1(n)}}\right|\le\delta,\qquad \left|\frac{\theta_{2(n+1)}-\theta_{2(n)}}{\theta_{2(n)}}\right|\le\delta\ . \tag{27}$$

Those numerical values of θ_1,θ_2 parameters of which satisfy the inequality (27) and which we take as real values of these parameters, are found as a result of identification problem solution. That is, we assume that

$$\theta_1^*=\theta_{1(n)},\qquad \theta_2^*=\theta_{2(n)}\,. \tag{28}$$

At the final, third stage of identification problem solution it is necessary to perform only two actions, namely – first by means of substitution of the expressions (28) into relation (20), it is necessary to determine numerical value θ_3^* of the third unknown parameter of ARSM(3,3), model and then substituting this already found value of the third parameter in the expression (18) determine numerical value of σ_a^2 dispersion of white noise a_t, pulses of which are generated by standard computer program at the set value of the dispersion, it is necessary to add into time series (10) as this requires the structure of this model.

It is easy to see, that for obtaining calculated expressions of the suggested method for values of p and q orders less than three, it is necessary, in the expressions synthesized by us, to equate to zero those parameters, simpler ARSM(p,q) model, chosen by us, does not contain in its structure. For instance, if p = 3, q = 2, then instead of the system of equations (11), we will have the system of equations

$$\begin{cases} \gamma_0 = \varphi_1\gamma_1 + \varphi_2\gamma_2 + \varphi_3\gamma_3 + \sigma_a^2 - \theta_1(\varphi_1 - \theta_1)\sigma_a^2 - \theta_2(\varphi_2 - \theta_2)\sigma_a^2, \\ \gamma_1 = \varphi_1\gamma_0 + \varphi_2\gamma_1 + \varphi_3\gamma_2 - \theta_1\sigma_a^2 - \theta_2(\varphi_1 - \theta_1)\sigma_a^2, \\ \gamma_2 = \varphi_1\gamma_1 + \varphi_2\gamma_0 + \varphi_3\gamma_1 - \theta_2\sigma_a^2, \\ \gamma_3 = \varphi_1\gamma_2 + \varphi_2\gamma_1 + \varphi_3\gamma_0, \\ \gamma_4 = \varphi_1\gamma_3 + \varphi_2\gamma_2 + \varphi_3\gamma_1, \\ \gamma_5 = \varphi_1\gamma_4 + \varphi_2\gamma_3 + \varphi_3\gamma_2, \end{cases} \tag{29}$$

and instead of matrices M_γ, γ in the form (12) we will have them in the form

$$M_\gamma = \begin{bmatrix} \gamma_2 & \gamma_1 & \gamma_0 \\ \gamma_3 & \gamma_2 & \gamma_1 \\ \gamma_4 & \gamma_3 & \gamma_2 \end{bmatrix}, \quad \gamma = \begin{bmatrix} \gamma_3 \\ \gamma_4 \\ \gamma_5 \end{bmatrix}. \tag{30}$$

Instead of the expression (18) we will have

$$\sigma_a^2 = -\frac{C}{\theta_2}, \tag{31}$$

And instead of the system of equations (19) we will have the system of equations

$$\begin{cases} A\theta_2 = C(-1 + \theta_1(\varphi_1 - \theta_1) + \theta_2(\varphi_2 - \theta_2)), \\ B\theta_2 = C(\theta_1 + \theta_2(\varphi_1 - \theta_1)), \end{cases} \tag{32}$$

From the second equation, that instead of the expression (20) we will have

$$\theta_2 = \frac{C\theta_1}{B - C(\varphi_1 - \theta_1)}. \tag{33}$$

Instead of the system of non-linear equations (21) we will have only one non-linear equation

$$f_1(\theta_1) = 0, \tag{34}$$

where

$$\begin{gathered} f_1(\theta_1) = A\theta_1(B - C(\varphi_1 - \theta_1)) - (-1 + \theta_1(\varphi_1 - \theta_1))(B - C(\varphi_1 - \theta_1))^2 - \\ \theta_1(\varphi_2(B - C(\varphi_1 - \theta_1)) - C\theta_1). \end{gathered} \tag{35}$$

In this case, instead of the procedure, set by the expressions (24)–(26), we will search the solution of the equation (34) using the expression

$$\theta_{1(n+1)} = \theta_{1(n)} - \frac{f_1(\theta_{1(n)})}{\dfrac{df_1(\theta_{1(n)})}{d\theta_1}}, \quad n = 0, 1, 2, ... \tag{36}$$

Generalization of the suggested method on the model of ARSM(p,q) at values of orders p, q greater than three and algorithm of model structure optimization we will present in the next paper. And in the conclusion of this paper we suggest to draw your attention to the fact, that unlike algorithm of model ARSM(p,q) optimization, using the technique, described in [1], where both the regression parameters and sliding mean parameters are suggested to define, using the same values of autocovariations and applying the procedure of deviations squares sum minimization for the search of sliding mean parameters values, at determined earlier at the same parameters of autocovariations, using Yule-Worker technique, autoregression parameters, in our method, first, parameters of autoregression are calculated, using one set of autocovariations, and parameters of sliding mean are calculated, using another set of autocovariations, that corresponds to cybernetic principle of using «fresh points» while expansion of parameters set, estimations of which are searched, secondly, for determination of sliding mean parameters, direct procedure is used, that does not require restoration of the procedure of deviations square sum minimization while transfer to other values of autoregression and sliding mean orders.

Conclusions

1.Method of identification of autoregression sliding mean ARSM(p,q) model with random value of p, q, orders is suggested, the given method generalizes Yyle-Worker technique, developed for identification of autoregression AP(p) model, in case when autoregression component of AR(p) model, is complemented with sliding mean SM(q) component.

2. For realization of the algorithm of the suggested method, it is necessary at the first stage to solve the system of linear equations of Yule-Worker type for determination of p parameters of autoregression using one set of autocavariations, at the second stage the system of additional non-linear equations for identification of q–1 parameters of sliding mean, using the second set of autocovatiations must be solved, at the third stage formula definition of the last parameter of sliding mean should be performed and value of white noise dispersion that must be added to ARSM(p,q) model to provide its adequacy to real time series should be determined.

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Mokin Oleksandr – Doctor of Science (Eng), Professor, Head of the Chair of Renewable Power and Transport Electric Systems and Complexes (RETESC), e-mail: abmokin@gmail.com.

Mokin Vitaliy – Doctor of Science (Eng), Professor, Head of the Chair of Computer Ecological and Economic Monitoring and Engineering Graphics (CEEMEG).

Mokin Borys – Academician of NAPSU, Doctor of Science (Eng), Professor, Department of Renewable Power and Transport Electric Systems and Complexes.

Vinnytsia National Technical University.