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SYNTHESIS OF THE CONTROL LAW OF OPTIMAL MOTION OF THE BATTERY-DRIVEN VEHICLE ON THE SECTION OF HORIZONTAL HIGHWAY

The paper synthesizes the law of the control of battery- driven vehicle on the horizontal section of the highway, optimal as for the criterion of minimum electric energy consumption of storage battery.

Key words: *synthesis, optimization, mathematical model, battery- driven vehicle, traction motor, storage battery, energy consumption.*

Introduction

Paper, proceeding the research of the issues of optimization and identification of mathematical models of the motion of battery- driven vehicle with the direct current traction motor of series excitation on the horizontal section, highlights the synthesis of the law of control the battery- driven vehicle motion on the horizontal section, optimal as for the criterion of minimum electric energy consumption of the storage battery.

Initial preconditions and problem statement

The work [1] proves that the loaded battery- driven vehicle with direct current series excitation traction motor connected to storage battery, as it is shown in Fig. 1,

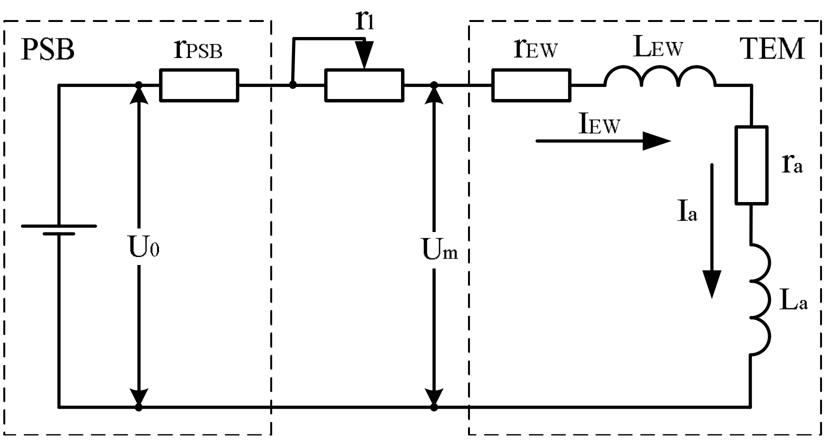

Fig. 1. Electric diagram of the main power circuit of battery- driven vehicle electric motor drive with regulated resistance of the line between the storage battery and the traction motor input terminals

will move along horizontal section of the highway and minimize the storage battery energy consumption only if the current in its power circuit is formed according to the law, mathematical model of which is the following

$$i(\tau)=\frac{1-a_1\left(C_2e^{(f_1+2f_2\nu)\tau}-\dfrac{C_1}{f_1+2f_2\nu}\right)}{2\alpha+2b_1\left(C_2e^{(f_1+2f_2\nu)\tau}-\dfrac{C_1}{f_1+2f_2\nu}\right)},\tag{1}$$

where $i=\frac{I_a}{I_n}$ – relative current of power circuit, $\tau=\frac{t}{T_M}$ – relative time, $\nu=\frac{V}{V_n}$ – relative linear speed of battery-driven vehicle, $\alpha=\frac{I_n}{I_k}$ – slope coefficient of static characteristic $u=f(i)$ of power circuit, that has the form

$$u=1-\alpha i\,, \tag{2}$$

where $u=\frac{U}{U_n}$ – relative voltage, and a_1, b_1 – coefficients of the traction motor magnetization curve, mathematical model of which has the form

$$\phi(i)=\begin{cases}-a_2 i^2+b_2 i, i\in[0,i_c),\\ a_1+b_1 i, i\in[i_c,\infty),\end{cases} \tag{3}$$

where $\phi(i)=\frac{\Phi(I_{ew})}{\Phi(I_n)}$ – relative magnetic flux, which is the function of the relative current i, C_1, C_2 – unknown coefficients, and $f_1=\frac{\mu_1 R V_n}{wk_M I_n \Phi(I_n)}$, $f_2=\frac{\mu_2 R V_n^2}{wk_M I_n \Phi_n(I_n)}$ – coefficients of the mathematical model of the electric drive dynamics, that in relative values has the form

$$\frac{d\nu}{d\tau}=a_1 i+b_1 i^2-f_0-f_1\nu-f_2\nu^2\,, \tag{4}$$

and coefficient $f_0=\frac{\mu_0 R}{wk_M I_n \Phi_n}$.

It should also be noted that electromagnetic time constant of traction electric drive of the vehicle necessary for determination of relative time T_M is determined from the expression

$$T_M=\frac{mV_n R}{wk_M I_n \Phi(I_n)}, \tag{5}$$

where m – vehicle weight, w – gear ratio from the wheel to the traction motor, R – wheel radius, k_M – winding connection factor between the armature torque of the traction motor and the current in the armature winding and the magnetic flux of its excitation winding – this coefficient is determined using the passport data of the traction motor.

The information, regarding the parameters, which are included in the formulas, determining the above-mentioned relative values, is presented in the paper, referred to above [1].

The work [2] proves that the unloaded battery- driven vehicle with direct current series excitation traction motor, connected to storage battery, as it is shown in Fig. 1, will move along horizontal section of the highway and minimize the storage battery energy consumption only if the current in its power circuit is formed according to the law, mathematical model of which is the following:

$$i(\tau)=\frac{2\alpha+2b_2\lambda_1(\tau)-\sqrt{(2\alpha+2b_2\lambda_1(\tau))^2-12a_2\lambda_1(\tau)}}{6a_2\lambda_1(\tau)}, \tag{6}$$

where

$$\lambda_1(\tau)=C_2^* e^{(f_1+2f_2\nu)\tau}-\frac{C_1^*}{f_1+2f_2\nu}, \tag{7}$$

and mathematical model of the electric drive system dynamics has the form

$$\frac{d\nu}{d\tau} = -a_2 i^3 + b_2 i^2 - f_0 - f_1\nu - f_2\nu^2 . \tag{8}$$

In mathematical models of optimal current (1) and (6), which provides the motion of the battery-driven vehicle on the horizontal section of the highway with minimum losses of energy, the unknown parameters are the pair of constants C_1, C_2 or C_1^*, C_2^*, the problem of these models identification is reduced to determination of these constants which has been solved in [3].

This aim of the research is to find the method of realization of the synthesized and identified models of optimal motion, which set the change of relative current in the relative time, that is, to find the law for control the optimal motion of battery-driven vehicle on the horizontal section of the highway, on the base of synthesized models.

Development of the methods of the given problem solution.

To begin with, we, as it was in [4], compose the equation applying electrical engineering parameters of the traction electric drive , which provide the control over its motion.

Since the schema of the power circuit of the electric drive system does not depend on the fact , which branch of magnetization curve (3) is used for the synthesis of optimal current models, the equation of the second Kirchhoff's law for this circuit will not depend on optimal current models and will remain the same, namely

$$U_0 = I_a r_{PSB} + I_a (r_a + r_{ew}) + I_a r_l + E_\omega + (L_a + L_{ew})\frac{dI_a}{dt} , \tag{9}$$

or

$$U_0 - I_a r_{PSB} - I_a r_l = U_m = I_a (r_a + r_{ew}) + E_\omega + (L_a + L_{ew})\frac{dI_a}{dt} . \tag{10}$$

In these equations U_0 – electromotive force of the power storage battery (PSB), r_{PSB} – internal resistance of this battery, r_l – resistance of the line, connecting the PSB with the input terminals of the traction motor, U_m – voltage on the terminals, r_a, L_a – resistance and inductance of traction motor armature winding, r_{ew}, L_{ew} – resistance and inductance of excitation winding of traction motor, and

$$E_\omega = k_\omega \omega \Phi(I_{ew}) = k_\omega \omega \Phi(I_a) , \tag{11}$$

rotating electromotive force, induced in armature winding and depends on the angular velocity of rotation ω of the armature, winding factor k_ω connection between electromotive force of rotation, angular velocity and magnetic flux Φ of excitation winding, which, in its turn, depends on the current I_{ew} in excitation winding, equal for the series excitation electric motor current I_a of the armature.

It follows from the equations (9) and (10) that it is possible to set the necessary value of traction electric motor (TEM) optimal current either by changing the resistance r_l of the line between the storage battery(PSB) and its input terminals, as it is shown in Fig.1, or changing voltage U_m, supplied to these terminals, for example, by means of the controlled electronic amplifier (EA), as it is shown in Fig. 2

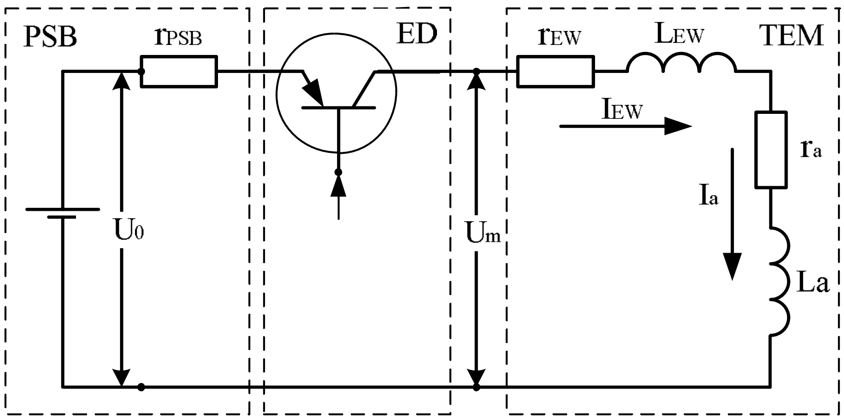

Fig. 2. Diagram of the main power circuit of battery- driven vehicle with the controlled electronic amplifier (EA) between power storage battery(PSB) and input terminals of Traction electric motor (TEM)

It is obvious that from the position of the reliability of electric drive system operation, it is necessary to choose the variant with the controlled electronic amplifier.We will consider it later

Having divided the equation (10) by U_0 and performed a number of auxiliary actions, we obtain its the analogue in relative units

$$u = \theta i + \beta \frac{di}{dt} + \gamma v \phi(i), \tag{12}$$

where

$$\beta = \frac{(L_a + L_{ew})I_n}{U_0 T_M}, \gamma = \frac{k_\omega V_n \Phi(I_n)}{U_0 R}, \theta = \frac{(r_a + r_{ew})}{U_0},$$

$$u = \frac{U_m}{U_0}, T_M = \frac{m V_n R}{w k_M I_n \Phi(I_n)}. \tag{13}$$

Proceeding from the obtained models it is possible to state, that two cases are possible for optimal control battery- driven vehicle, namely:

1) vehicle is loaded and control is performed using the controlled electronic amplifier;

2) vehicle is not loaded and control is performed using the controlled electronic amplifier.

In the first case – while synthesis of the law of optimal control over the loaded vehicle – it is necessary to substitute the expression (1) and the low branch of the expression (3) into the expression (12), as a result we obtain

$$u^*(\tau) = \theta i + \beta \frac{di}{dt} + \gamma v \phi(i) = \theta i + \beta \frac{di}{dt} + \gamma v (a_1 + b_1 i) =$$

$$= \beta \frac{d}{d\tau} \left(\frac{f_1 + 2f_2 v - (f_1 + 2f_2 v) a_1 C_2 e^{(f_1 + 2f_2 v)\tau} + a_1 C_1}{2\alpha(f_1 + 2f_2 v) + 2b_1 C_2 (f_1 + 2f_2 v) e^{(f_1 + 2f_2 v)\tau} - 2b_1 C_1} \right) +$$

$$+ (\theta + \gamma v b_1) \left(\frac{f_1 + 2f_2 v - (f_1 + 2f_2 v) a_1 C_2 e^{(f_1 + 2f_2 v)\tau} + a_1 C_1}{2\alpha(f_1 + 2f_2 v) + 2b_1 C_2 (f_1 + 2f_2 v) e^{(f_1 + 2f_2 v)\tau} - 2b_1 C_1} \right) + \gamma v a_1, \tag{14}$$

or

$$F_1(u^*, v, \tau) = 0, \tag{15}$$

In the second case – while synthesis of the law of optimal control over the not loaded vehicle – is necessary to substitute the expression (6) and the upper branch of the expression (3) into the expression (12), as a result we obtain

$$u^*(\tau) = \theta i + \beta \frac{di}{dt} + \gamma \nu \phi(i) = \theta i + \beta \frac{di}{dt} + \gamma \nu (-a_2 i^2 + b_2 i) =$$

$$= \beta \frac{d}{d\tau}\left(\frac{2\alpha(f_1+2f_2\nu) + 2(f_1+2f_2\nu) b_2 C_2^* e^{(f_1+2f_2\nu)\tau} - 2b_2 C_1^*}{6\alpha_2 (C_2^*(f_1+2f_2\nu)e^{(f_1+2f_2\nu)\tau} - C_1^*)} - \right.$$

$$- \frac{\sqrt{(2\alpha(f_1+2f_2\nu) + 2b_2 C_2^*(f_1+2f_2\nu)e^{(f_1+2f_2\nu)\tau} - 2b_2 C_1^*)^2 -}}{6\alpha_2 (C_2^*(f_1+2f_2\nu)e^{(f_1+2f_2\nu)\tau} - C_1^*)} \times$$

$$\left. \times \frac{\sqrt{-12a_2(f_1+2f_2\nu)^2 C_2^* e^{(f_1+2f_2\nu)\tau} + 12\alpha_2 C_1^*(f_1+2f_2\nu)}}{6\alpha_2 (C_2^*(f_1+2f_2\nu)e^{(f_1+2f_2\nu)\tau} - C_1^*)} \right) +$$

$$+ (\theta + \gamma \nu a_2)\left(\frac{2\alpha(f_1+2f_2\nu) + 2(f_1+2f_2\nu) b_2 C_2^* e^{(f_1+2f_2\nu)\tau} - 2b_2 C_1^*}{6\alpha_2 (C_2^*(f_1+2f_2\nu)e^{(f_1+2f_2\nu)\tau} - C_1^*)} - \right.$$

$$- \frac{\sqrt{(2\alpha(f_1+2f_2\nu) + 2b_2 C_2^*(f_1+2f_2\nu)e^{(f_1+2f_2\nu)\tau} - 2b_2 C_1^*)^2 -}}{6\alpha_2 (C_2^*(f_1+2f_2\nu)e^{(f_1+2f_2\nu)\tau} - C_1^*)} \times$$

$$\left. \times \frac{\sqrt{-12a_2(f_1+2f_2\nu)^2 C_2^* e^{(f_1+2f_2\nu)\tau} + 12\alpha_2 C_1^*(f_1+2f_2\nu)}}{6\alpha_2 (C_2^*(f_1+2f_2\nu)e^{(f_1+2f_2\nu)\tau} - C_1^*)} \right) -$$

$$- \gamma \nu a_2 \left(\frac{2\alpha(f_1+2f_2\nu) + 2(f_1+2f_2\nu) b_2 C_2^* e^{(f_1+2f_2\nu)\tau} - 2b_2 C_1^*}{6\alpha_2 (C_2^*(f_1+2f_2\nu)e^{(f_1+2f_2\nu)\tau} - C_1^*)} - \right.$$

$$- \frac{\sqrt{(2\alpha(f_1+2f_2\nu) + 2b_2 C_2^*(f_1+2f_2\nu)e^{(f_1+2f_2\nu)\tau} - 2b_2 C_1^*)^2 -}}{6\alpha_2 (C_2^*(f_1+2f_2\nu)e^{(f_1+2f_2\nu)\tau} - C_1^*)} \times$$

$$\left. \times \frac{\sqrt{-12a_2(f_1+2f_2\nu)^2 C_2^* e^{(f_1+2f_2\nu)\tau} + 12\alpha_2 C_1^*(f_1+2f_2\nu)}}{6\alpha_2 (C_2^*(f_1+2f_2\nu)e^{(f_1+2f_2\nu)\tau} - C_1^*)} \right)^2, \tag{16}$$

or

$$F_2(u^*, \nu, \tau) = 0. \tag{17}$$

Equations (15), (17) may be rewritten

$$\begin{cases} \nu = F_1^*(u^*, \tau); \\ \nu = F_2^*(u^*, \tau). \end{cases} \tag{18}$$

Computational algorithm for dependences (18) built using the expressions (14) and (16), and applicable for realization in board computer of the vehicle, will be presented in the separate paper, and in this paper we will proceed from the assumption that we already have these dependences.

From the expressions (14), (16), we see that they are nonlinear differential equations of the first order relatively the coordinate ν, therefore, transferring to the discrete time k, dependences (18) may be presented

$$\begin{cases} v_{k+1} = F_1^{**}(u_k^*, v_k, k); \\ v_{k+1} = F_2^{**}(u_k^*, v_k, k). \end{cases} \tag{19}$$

Let v_{k+1} be the real value of the relative speed of the vehicle at the discrete time moment $k+1$, which we see on the speed indicator , installed on the panel of the vehicle, and

$$\Delta v_{k+1} = \left| v_{k+1}^{p} - v_{k+1} \right|. \tag{20}$$

– the error, which arises at supply of relative voltage u_k^* to the input terminals of the traction motor at the discrete time moment k. Then, setting the values u_k^* so that to perform equality, (14), (16) and to minimize the error (20), we will make the vehicle move at the speed which in accordance with the expressions (14), (16) and (1), (6), will minimize the battery losses.

In real conditions of driving the vehicle it means that if the driver drives at the speed of v_k at a discrete time moment k, using the accelerator, connected to the controlled unit of the electronic amplifier, the driver will set such a value u_k^*, which will ensure at the discrete time moment $k+1$ according to the expression (19) the value of the relative speed v_{k+1}, input on the display of the board computer, the control over which, according to the criterion (20), is performed by the value, indicated on electronic speed indicator of the vehicle than traction motor of the vehicle will consume the current which will provide the realization of the preset program of motion with minimal losses of the energy of the storage battery.

Conclusions

1. There had been built the mathematical model for the determination of the speed of battery-driven vehicle the e on the horizontal section of the highway form one point to the other, adequate current of the traction motor, optimal as for the criterion of minimum energy losses of storage battery.
2. There had been determined the structure of the law and the algorithm of the optimal motion of the vehicle on the horizontal section of the highway by transferring from the models of optimal current of traction motor to the parameters of motion speed, corresponding to this current.
3. There had been presented the ways for the realization of the constructed law of optimal motion by the driver of battery -driven vehicle.

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