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DISCRETE METHOD AND SYSTEM-STRUCTURAL ANALYSIS OF DYNAMIC PROBLEMS IN PIPE LINES WITH PRESSURE CHANGE AT THE BEGINNING AND FLOW RATE CHANGE AT THE END

The paper considers the application of system-structural analysis in combination with discrete method for the development of technological fundamentals of complex pipeline systems control. Combination of discrete and system-structural methods enables to unify problems in complex systems with distributed parameters in order to elaborate common patterns of analysis and computation of flow parameters.

Key words: system-structural analysis, discrete method, pipe line, distributed parameter, structural-architectural model.

Introduction. Pipeline systems, due to their specific technological features, are one of the examples of the systems with distributed parameters, operating in dynamic modes. Dynamic processes, occurring in such systems, are described by the systems of equations in partial derivatives.

The research performed, showed that long – distance pipe lines (LDPL) are referred to complex systems with distributed parameters. They are of sufficient length with intermediate sources of disturbance are branched, non-uniform systems. As the sources of primary information in these systems are located at a considerable distance, then they represent complex system with distributed parameters and distributed data bases [3].

Nowadays methods of structural-system analysis are successfully applied for obtaining detailed and generalized information, regarding investigated processes, development of the ways of aim-oriented synthesis of complex systems structure and methods of non-stationary measurements [4, 5, 6]. Nowadays while the solution of the problems for complex systems with distributed parameters combination of discrete method [1] and system-structural analysis [2] is used in order to obtain detailed and generalized information regarding the state of investigated processes, determination of the ways of aim-oriented synthesis of system structure and elaboration of the methods of non-stationary measurements. The problem solution is reduced to adaptation of mathematical models to real conditions by means of identification of system parameters for choosing the correct solutions.

The pattern of the analysis and computation used for oil pipe line systems is based on the solution of differential flow equations under corresponding boundary conditions, which allow to calculate stationary, non-stationary pressure and oil flow rate. However, it is not sufficient, as it is not always possible to perform the detailed analysis of physical processes, taking place. This approach does not allow to solve such problems as correction and synthesis of the systems with preset processes of pressure fields formation, and some problems, connected with the control of the process, where it is necessary to define constant coefficients, making part of oil flow equation [4].

Proceeding from the above-mentioned, the development of generalized ideology of calculation and analysis of the behaviour of systems with distributed parameters, applying discrete and system-structural method, elaboration of the technology of adaptation of calculated models to real conditions of systems operation irrespectively of geometric and dynamic topology is actual problem for investigation and analysis of dynamic processes in pipe line systems.

Problem set-up. The given paper considers the application of system-structural analysis [3] in combination with discrete method while development of technological fundamentals of complex pipe line systems control. Combination of discrete and system-structural method allows to unify the problems in complex systems with distributed parameters in order to elaborate common schemes of analysis and computation of flow parameters. The solution of the dynamics problem is considered as a system, presented by structural diagram. The elements of structural diagram are mathematical

operators, establishing the rules of transformation of certain impacts on the object into the reaction, generated by these impacts.

The given problem is reduced to adaptation of mathematical models to real conditions by means of identification of system parameters in order to choose the correct solutions and to solve inverse and pseudoinverse problems on the basis of calculated architectural control models, developed by the authors.

Methods of solution. Twofold and discrete Laplace transform is used in the given paper as mathematical tool. While transition from the image to the original of functions recurrent relations are applied[1].

It is known, that the study of dynamic processes in oil pipe line is reduced to the solution of the equation of motion and continuity, at corresponding initial and boundary conditions [3].

Operation process in oil pipe line with pressure change at the beginning and flow rate at the end of the pipe of ℓ length, located in Cartesian coordinate system along abscissa axis, is described by equations of motion and continuity [3]:

$$\begin{cases} -\frac{\partial P(x,t)}{\partial x} = \kappa_1 \frac{\partial G(x,t)}{\partial t} + \kappa_3 G(x,t), \\ -\frac{\partial P(x,t)}{\partial t} = \kappa_2 \frac{\partial G(x,t)}{\partial x}, \end{cases} \quad (1)$$

At initial and boundary conditions:

$$\begin{aligned} P(x, t)|_{x=0} &= P(0, t) = \varphi_1(t), \\ \frac{\partial P(x, t)}{\partial x}|_{x=\ell} &= \omega(x, t)_{x=\ell} = \varphi_2(t), \end{aligned}$$

where $P(x, t)$ – is pressure; $G(x, t)$ – is flow rate; $\kappa_1, \kappa_2, \kappa_3$ – are constant coefficients; $\varphi_1(t)$ and $\varphi_2(t)$ – are functions of time.

While solving the system of equations (1) by operator method, using two-fold Laplace transform, at the first stage we obtain image for unknown functions $P(x, t)$ and $G(x, t)$. Then by means of inverse transformation we restore one-fold image of functions $P(x, t)$, $G(x, t)$ and apply discrete Laplace transform.

Solution of equations in partial derivatives, found by means of two-fold Laplace transform does not depend on the sequence of application of direct and inverse transforms. It is clear, that successfully chosen order in two-fold transform can considerably facilitate the solution of the problem [1].

Solving the problem relatively $P(x, t)$ in the area of images, we obtain:

$$\bar{P}(x, s) = \frac{ch\gamma(l-x)}{ch\gamma l} \cdot \bar{\varphi}_1(s) + \frac{sh\gamma x}{\gamma ch\gamma l} \cdot \bar{\varphi}_2(s).$$

We represent the obtained expression in the form of the sum of two pressures:

$$\bar{P}(x, s) = \bar{P}_1(x, s) + \bar{P}_2(x, s), \quad (2)$$

$$\text{where } \bar{P}_1(x, s) = \frac{ch\gamma(l-x)}{ch\gamma l} \cdot \bar{\varphi}_1(s), \quad \bar{P}_2(x, s) = \frac{sh\gamma x}{\gamma ch\gamma l} \cdot \bar{\varphi}_2(s),$$

Here $\bar{\gamma}(s) = \sqrt{sk_2(k_3 + sk_1)}$ – is the coefficient of wave propagation.

Then from the expression (2) for $\bar{P}_1(x, s)$ in different sections of pipe line we have:

$$\begin{aligned}
\bar{p}_1(0, s) &= \bar{\varphi}_1(s), \\
\bar{p}_1(l, s) &= \bar{p}_1(0, s) \cdot \frac{1}{ch \gamma l}, \\
\bar{p}_1(x, s) &= \bar{p}_1(l, s) \cdot ch \gamma (l - x),
\end{aligned} \tag{3}$$

pressure gradient

$$\frac{\partial \bar{p}_1(x, s)}{\partial x} = -\gamma \cdot \bar{p}_1(x, s) \cdot th \gamma (l - x). \tag{4}$$

mass velocity

$$\bar{G}_1(x, s) = -\frac{1}{\bar{b}_1(s)} \cdot \frac{\partial \bar{p}_1(x, s)}{\partial x}, \tag{5}$$

where $\bar{b}_1(s) = \sqrt{(sk_1 + k_3)/k_2}$ - is wave resistance of the pipe line.

Analogously for $\bar{P}_2(x, s)$ and $\bar{G}_2(x, s)$ we obtain:

$$\begin{aligned}
\bar{p}_2(0, s) &= 0, \\
\bar{p}_2(x, s) &= \bar{p}_2(l, s) \frac{sh \gamma x}{sh \gamma l},
\end{aligned} \tag{6}$$

$$\bar{p}_2(l, s) = \frac{1}{\gamma} \cdot \bar{\varphi}_2(s) \cdot th \gamma l,$$

$$\frac{\partial \bar{p}_2(x, s)}{\partial x} = \gamma \cdot \bar{p}_2(x, s) \cdot th \gamma x, \tag{7}$$

$$\bar{G}_2(x, s) = \frac{1}{\bar{b}_1(s)} \cdot \frac{\partial \bar{p}_2(x, s)}{\partial x}. \tag{8}$$

Expressions (2) – (8) allow to compose structural model, characterizing dynamic processes in the investigated system, in the form of the sum of two pressures $\bar{P}_1(x, s)$ and $\bar{P}_2(x, s)$ (Fig. 1).

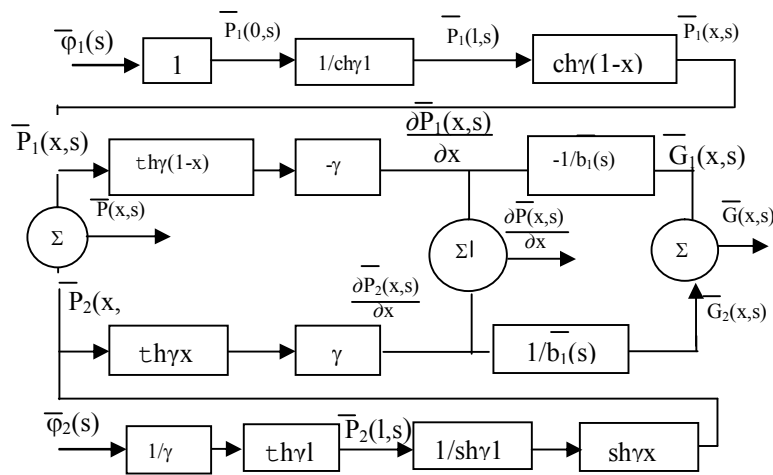


Fig. 1. Structural model of fluid motion in the pipe line

As it is seen from Fig. 1, dynamic processes, taking place in pipe lines, are rather complex problems in complex systems with distributed parameters.

Let us consider the solution of the problem of structural analysis of dynamic processes in pipe

lines with known change of pressure at the beginning and flow rate at the end according to structural architectural model (Fig. 1) and on the basis of discrete method [1].

According to Fig. 1, information element of structural diagram, establishing communication between $\bar{P}_1(0, s)$ and $\bar{\varphi}_1(s)$, can be represented as in Fig. 2.

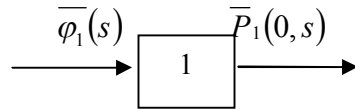


Fig. 2. Element of block diagram, establishing communication between $\bar{P}_1(0, s)$ and $\bar{\varphi}_1(s)$

According to Fig. 2, we can consider that:

$$\bar{P}_1(0, s) = 1 \cdot \bar{\varphi}_1(s). \quad (9)$$

Applying the discrete transformation and convolution theorem, equations (9) in the area of originals we can present:

$$P_1[0, n] = \sum_{m=0}^n \varphi_1[m] \cdot 1[n-m] - \sum_{m=0}^{n-1} P_1[0, m] \cdot 1[n-m]. \quad (10)$$

Link between $\bar{P}_1(1, s)$ and $\bar{P}_1(0, s)$ is determined by the operator function of the element, shown in Fig. 3.

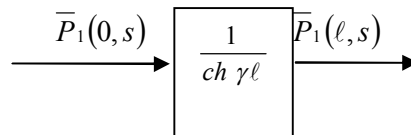


Fig. 3. Element of block diagram, establishing link between pressures in points $x=0$ and $x=\ell$

According to Fig. 3 we can write:

$$\bar{P}_1(1, s) = \frac{1}{ch \gamma \ell} \bar{P}_1(0, s), \quad (11)$$

or taking into account $ch \gamma \ell = \frac{1}{2}(\ell^{\gamma \ell} + \ell^{-\gamma \ell})$, equations (11) can be presented:

$$\bar{P}(\ell, s) = \frac{2e^{-\gamma \ell}}{1 + e^{-2\gamma \ell}} \bar{P}_1(0, s). \quad (12)$$

Hence, according to discrete method and convolution theorem, equations (12) in the area of originals can be represented:

$$P_1[\ell, n] = \frac{1}{1 + K_4[0]} \left(2 \sum_{m=0, 5\lambda}^n K'_5[m] P_1[0, n-m] - \sum_{m=\lambda}^n K'_4[n-m] P_1[\ell, m] - \sum_{m=0}^{n-1} P_1[\ell, m] \cdot 1[n-m] \right) \quad (13)$$

Link between $\bar{P}_1(x, s)$ and $\bar{P}_2(\ell, s)$ is determined by operator function of the element, shown in Fig. 4.

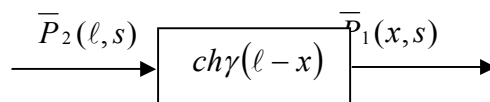


Fig. 4. Element of block diagram, establishing the link between pressures $\bar{P}_1(x, s)$ and $\bar{P}_2(\ell, s)$

Then, according to Fig. 4, in operator form we obtain:

$$2 \bar{P}_1(x, s) \frac{1}{s} e^{-2\gamma \ell(0, 5-\delta)} = \frac{1}{s} (1 + e^{-2\gamma \ell(1-\delta)}) \bar{P}_2(\ell, s). \quad (14)$$

Applying the discrete method and convolution theorem we obtain:

$$P_2[\delta, n] = \frac{1}{2K_{11}[0]} \left(\sum_{m=0}^{n-1} P_2[\ell, m] \cdot 1[n-m] + \sum_{m=\lambda(1-\delta)}^m K'_{11}[m] P_1[\ell, n-m] - 2 \sum_{m=\lambda(0.5-\delta)}^n K_{11}[n-m] P_1[\delta, m] \right), \quad (15)$$

where $K_{11}[n]$ and $K'_{11}[n]$ – are the originals of the functions $\bar{K}_{11}(s)$ and $\bar{K}'_{11}(s)$, determined by the table of images [7].

$$K_{11}[n] = \begin{cases} 0, & \text{if } m < (0.5 - \delta)\lambda \\ e^{-\alpha T} + \alpha T \sum_{m=(0.5-\delta)\lambda}^n e^{-\frac{\alpha m T}{\lambda}} \frac{I_0\left(\frac{\alpha T}{\lambda} \sqrt{m^2 - [(0.5 - \delta)\lambda]^2}\right)}{\sqrt{m^2 - [(0.5 - \delta)\lambda]^2}} & \text{if } m > (0.5 - \delta)\lambda \end{cases}$$

$$K'_{11}[n] = \begin{cases} 0, & \text{if } n < (1 - \delta)\lambda \\ e^{-\alpha T} + \alpha T \sum_{m=(1-\delta)\lambda}^n e^{-\frac{\alpha T m}{\lambda}} \frac{I_1\left(\frac{\alpha_2 T}{\lambda} \sqrt{m^2 - [(1 - \delta)\lambda]^2}\right)}{\sqrt{m^2 - [(1 - \delta)\lambda]^2}} & \text{if } m > (1 - \delta)\lambda \end{cases}$$

Link between gradient $\text{grad } \bar{P}_1(x, s)$ and $P_1(x, s)$ according to Fig. 1, is determined by the operator function of the element, shown in Fig. 5.

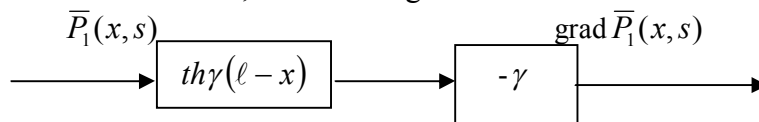


Fig. 5. Element of block diagram, establishing link between pressures $\text{grad } \bar{P}_1(x, s)$ and $\bar{P}_1(x, s)$

According to Fig. 5. $\text{grad } \bar{P}_1(x, s)$ in operator form is determined:

$$\text{grad } \bar{P}_1(x, s) = -\gamma \text{th} \gamma(\ell - x) \bar{P}_1(x, s), \quad (16)$$

whence, according to theory of discrete systems and convolution theorem

$$\begin{aligned} \text{grad } P_1[\delta, n] = \frac{1}{1 - K'_{11}[0]} \sum_{m=0}^n K_{14}[m] P_1[\delta, n] - \sum_{m=\lambda(1-\delta)}^n K'_{11}[n-m] \text{grad } P_1[\delta, m] - \\ - \sum_{m=0}^{n-1} \text{grad } P_1[\delta, m] \cdot 1[n-m]. \end{aligned} \quad (17)$$

where $K_{14}[n]$, $K_{15}[n]$ – are originals of the functions, correspondingly

$$\bar{K}_{14}(s) = \frac{1}{s} \sqrt{\frac{s + \alpha_2}{s}} \gamma, \quad \bar{K}_{15}(s) = \frac{1}{s} \sqrt{\frac{s + \alpha_2}{s}} e^{-2\gamma \ell (1-\delta)}, \quad \alpha_2 = \frac{\bar{K}_2}{\bar{K}_1}.$$

$$K_{14}[n] = e^{-\frac{\alpha_2 n T}{2\lambda}} I_0\left(\frac{\alpha_2 n T}{2\lambda}\right) + \alpha_2 \sum_{m=0}^n e^{-\frac{\alpha_{12} m T}{2\lambda}} I_0\left(\frac{\alpha_2 m T}{2\lambda}\right) \text{if } m > 0.$$

Link between $\bar{G}_1(x, s)$, $\text{grad } \bar{P}_1(x, s)$ is determined by operator function of the element, shown in Fig. 6.

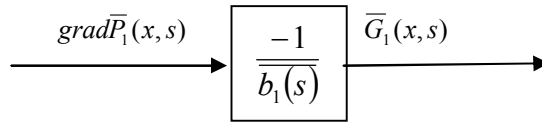


Fig. 6. Element of block diagram, establishing the link between $\text{grad } \bar{P}_1(x, s)$ and $\bar{G}_1(x, s)$

We obtain from Fig. 6:

$$\bar{G}_1(x, s) = -\frac{1}{\bar{b}_1(s)} \text{grad} \bar{P}_1(x, s). \quad (18)$$

Equation (18) in the area of the originals can be represented by the following recurrent relation

$$G_1[\delta, n] = -\alpha_3 \sum_{m=0}^n K_3[m] \cdot \text{grad} P_1[\delta, n-m] - \sum_{m=0}^{n-1} G_1[\delta, m] \cdot l[n-m], \quad (19)$$

where $K_3[n] = e^{-\alpha_3 \frac{nT}{\lambda}} I_0\left(\frac{\alpha_2}{2} n \frac{T}{\lambda}\right)$ - is the original of the function $\bar{K}_3(s) = \frac{1}{s} \sqrt{\frac{s}{s+\alpha_2}}$. determined by the table of images [7].

In the same way recurrent relations determining link between $\bar{P}_2(l, s)$ and $\bar{\varphi}_2(s)$ can be obtained for the second branch of block diagram (Fig. 1):

$$\begin{aligned} \varphi_2[n] = \frac{1}{\alpha_1(K_{14}[0] + K_{26}[0])} \{ \sum_{m=0}^n \varphi_2[m] l[n-m] - \sum_{m=\lambda}^n K'_4[m] \varphi_2[n-m] - \alpha_1 \sum_{m=0}^{n-1} P_2[\ell, m] K_{14}[n-m] - \\ - \alpha_1 \sum_{m=\lambda}^{n-1} K_{26}[n-m] P_2[\ell, m] \}, \end{aligned} \quad (20)$$

where $K_{26}[n]$ is the original of the function $\bar{K}_{26}(s) = \sqrt{\frac{s+\alpha_2}{s}} e^{-2\alpha_1 \sqrt{(s+\alpha_2)s} \ell}$ determined by the table of images [7].

$$K_{26}[n] = \begin{cases} 0, & \text{if } 0 \leq m \leq \lambda \\ e^{-\alpha_n \frac{t}{\lambda}} I_0\left(\frac{\alpha T}{\lambda} \sqrt{n^2 - \lambda^2}\right) + \alpha \sum_{m=\lambda}^n e^{-\alpha \frac{nT}{\lambda}} \cdot I_0\left(\frac{\alpha T}{\lambda} \sqrt{m^2 + \lambda^2}\right), & \text{if } m > \lambda \end{cases}$$

Between $\bar{P}_2(x, s)$ and $\bar{P}_2(\ell, s)$:

$$\begin{aligned} P_2[\ell, n] = \sum_{m=\lambda(\delta-0,5)}^n K_{27}[m] P_2[\delta, n-m] + \sum_{m=\lambda(\delta-0,5)}^n K_{28}[m] P_2[\delta, n-m] + \\ + \sum_{m=0,5\lambda\delta}^n K_{16}[m] P_2[\delta, -n-m] - \sum_{m=0}^{n-1} P_2[\ell, m] l[n-m], \end{aligned} \quad (21)$$

where $K_{16}[n]$, $K_{27}[n]$, $K_{28}[m]$ - are originals of functions, correspondingly

$$\bar{K}_{27}[s] = \frac{1}{s} e^{-\gamma(x-\ell)}, \quad \bar{K}_{28}[s] = \frac{1}{s} e^{-\gamma(\ell+x)}.$$

$$K_{16}[n] = \begin{cases} 0, & \text{if } n < \delta\lambda \\ e^{-\frac{\alpha_2 n T}{2\lambda}} + \delta\lambda \sum_{m=\delta\lambda}^n e^{-\frac{\alpha_2 m T}{2\lambda}} \frac{I_1\left(\frac{\alpha_2 T}{2\lambda} \sqrt{m^2 - (\delta\lambda)^2}\right)}{\sqrt{m^2 - (\delta\lambda)^2}}, & \text{if } m > \delta\lambda \end{cases}$$

$$K_{27}[n] = \begin{cases} 0, & \text{if } m < (\delta - 0,5)\lambda \\ e^{-\alpha_2 T} + \alpha_2 T \sum_{m=(\delta-0,5)\lambda}^n e^{-\frac{\alpha_2 m T}{\lambda}} \frac{I_0\left(\frac{\alpha_2 T}{\lambda} \sqrt{m^2 - [(\delta - 0,5)\lambda]^2}\right)}{\sqrt{m^2 - [(\delta - 0,5)\lambda]^2}}, & \text{if } m > (\delta - 0,5)\lambda \end{cases}$$

$$K_{28}[n] = \begin{cases} 0, & \text{if } m < (0,5 + \delta)\lambda \\ e^{-\alpha_2 T} + \alpha_2 T \sum_{m=(0,5+\delta)\lambda}^n e^{-\frac{\alpha_2 m T}{\lambda}} \frac{I_0\left(\frac{\alpha_2 T}{\lambda} \sqrt{m^2 - [(0,5 + \delta)\lambda]^2}\right)}{\sqrt{m^2 - [(0,5 + \delta)\lambda]^2}}, & \text{if } m > (0,5 + \delta)\lambda \end{cases}$$

Between $\text{grad}\bar{P}_2(x, s)$ and $\bar{P}_2(x, s)$:

$$\begin{aligned} \text{grad}P_2(\delta, n) = \frac{1}{1 + K_{16}[0]} \{ & -\alpha_1 \left(\sum_{m=0}^n K_5[m]P_2[\delta, n-m] - \sum_{m=\lambda\delta}^n K_{18}[m]P_2[n-m] \right) - \\ & - \sum_{m=0, 5\lambda\delta}^n K_{16}[n-m]\text{grad}P_2[\delta, m] - \sum_{m=0}^{n-1} \text{grad}P_2[\delta, m] \cdot l[n-m] \} \quad , \end{aligned} \quad (22)$$

where $K_{16}[n]$, $K_{18}[n]$ - are originals of functions $\bar{K}_{16}[s]$, $\bar{K}_{18}[s]$.

Link between $\bar{G}_2(x, s)$ and $\text{grad}\bar{P}_2(x, s)$ is expressed as:

$$G_2[\delta, n] = -\alpha_3 \sum_{m=0}^n K_3[m]\text{grad}P_2[\delta, n-m] - \sum_{m=0}^{n-1} G_2[\delta, m] \cdot l[n-m]. \quad (23)$$

The results obtained. For different problems of dynamics, according to the developed structural-architectural model of pipe line, shown in Fig. 1 in the area of originals, corresponding recurrent relations were found, they have rather simple form, consisting of the sums of parameters, describing processes, occurring in pipe lines.

Similarly, other technological situations, observed in practice, can be solved.

The obtained recurrent relations can be easily realized by means of modern computation facilities, that is very important for investigation of complex dynamic systems with distributed parameters.

Conclusions. The constructed structural architectural models, as the model of physical process, are one of the most convenient methods of description and analysis of interconnected processes. On the basis of structural architectural models the equations of the element of group of elements can be written. That enables to establish the link between the coefficients and variables of the equations. Structural architectural models, composed in this way, enable to visualize the interactions and transformations of pressure and rate flow, are information mathematical model of dynamic processes, occurring in oil pipe line systems. Structural architectural models represent continuous information, regarding the object state. If necessary, passing to the area of the originals, we can obtain approximate or accurate information at a given moment of time.

Thus, combination of discrete and system structural method allows to unify problems in complex systems with distributed parameters in order to elaborate common patterns of analysis and flow parameters computation.

REFERENCES

1. Кадымов Я. Б. Переходные процессы в системах с распределенными параметрами / Кадымов Я. Б. – М.: Наука, 1968. – 192 с.
2. Шашков А. Г. Системно-структурный анализ процесса теплообмена и его применение / Шашков А. Г. – М.: Энергоатомиздат, 1983. – 280 с.
3. Мусаев В. Г. Сложные системы трубопроводного транспорта нефти и нефтепродуктов / Мусаев В. Г. – Баку : Элм, 2004. – 301 с.
4. Мусаев В. Г. Дискретный метод и системно-структурного анализ при решении динамических задач в магистральных трубопроводных системах / В. Г. Мусаев // Вестник машиностроения. – 2007. – № 10. – С. 29 – 33.
5. Структурный анализ динамических процессов в магистральных нефтепроводах [Электронный ресурс] / Мусаев В. Г. // Электронный научный журнал «Нефтегазовое дело». – №1. – 2007. Режим доступа до журн.: http://www.ogbus.ru/authors/Musaev/Musaev_1.pdf.
6. Мамедов Г. А. Структурный анализ динамических процессов в системах с иерархической структурой управления / Г. А. Мамедов, К. Э. Рустамов, В. Г. Мусаев // Изв. НАН Азерб., серия физико-технических и математических наук, том XXVI. – 2006. – № 2. – С. 108 – 113.
7. Диткин В. А. Справочник по операционному исчислению / В. А. Диткин, А. П. Прудников. – М.: Высшая школа, 1965. – 465 с.

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