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SYNTHESIS OF MATHEMATICAL MODEL FOR DIAGNOSTICS OF PNEUMATIC BRAKE SYSTEM OF ELECTRIC TRAIN

Synthesis of mathematic model for diagnostics of pneumatic brake line of electric train has been performed, applying this model the location of line failure can be detected, without leaving the driver's compartment.

Key words: *electric train, diagnostics, pneumatic actuator, brake system.*

Initial preconditions and problem set-up. Transmission of the signal for braking in brake system of electric train is realized across pneumatic brake pipe, that starts at electric locomotive and ends in the last van, having intermediate connections between vans in the form of brake coupling pipes (Fig 1).

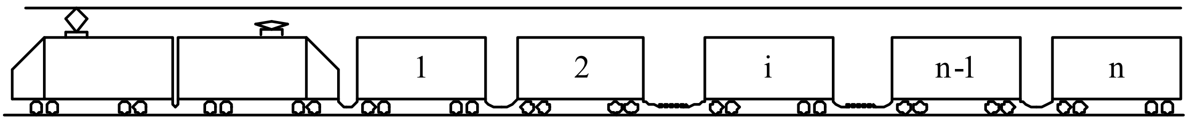

Fig. 1. Simplified functional diagram of electric train, presenting brake coupling pipes of pneumatic brake system

In the process of electric train motion in these intermediate connections under the influence of vibrations, additional rail deflections at curves and air masses vortices, that can be saturated with solid particles, taken by vortices from gauge line surface breaks may emerge, across which the leakage of compressed air from pneumatic pipe and pressure drop can occur. In this case emergency brake system starts its operation, electric train stops, train crew starts to look for the location of the break. There are cases when brake coupling pipe is not broken but crack emerges in it, that makes visual inspection rather difficult, since it is not always the member of train crew manages to inspect the brake coupling pipe, before the whistle of compressed air, escaping from the damaged line stops. In such cases the search of failure is delayed and train schedule is violated as a result of emergency stop of the train. Such emergency stops violate operation of dispatcher's service and yield economic losses as a result of the decrease of trains motion intensity.

Proceeding from the above – mentioned, the problem of automatic search of compressed air leakage from pneumatic pipe-line of electric train brake system by means of diagnostic system, installed in the compartment of traction electric locomotive, corresponding information being brought on the terminal, becomes very actual. To develop such diagnostic system, mathematical model must be synthesized, this model would be able to describe adequately the process of pressure change in pneumatic pipe-line of electric train brake system, caused by the break of brake pipe or the crack in it.

The given paper considers the synthesis of this mathematical model, connected with brake system of electric train with traction electric locomotive of ВЛ series.

Main results. In [1], using the results, obtained in [2], it is shown that the process of pressure change in pneumatic pipe-line with the open end if intermediate leakages across its walls are missing, is described by the equation in partial derivatives, that has the form:

$$\frac{\partial^2 p(x,t)}{\partial x^2} - \frac{1}{v_*^2}\frac{\partial^2 p(x,t)}{\partial t^2} - \frac{8\pi\mu_d}{S\rho v_*^2}\frac{\partial p(x,t)}{\partial t} = 0\,, \tag{1}$$

where $p(x,t)$ – is the pressure in pneumatic pipe-line, measured in H/m^2 and during transient process, specified by the break, is the function both of axial coordinate x of pneumatic pipe-line (Fig. 2), measured in

m, and time t, measured in sec.; v_* – is velocity of sound in compressed air $m/\sec$; S – is cross-section area of pneumatic pipeline (Fig. 2), measured in m^2; ρ – is the density of compressed air, measured in kg/m^3; μ_d – is dynamic coefficient of compressed air viscosity, measured in $\frac{H \cdot \sec}{m^2}$.

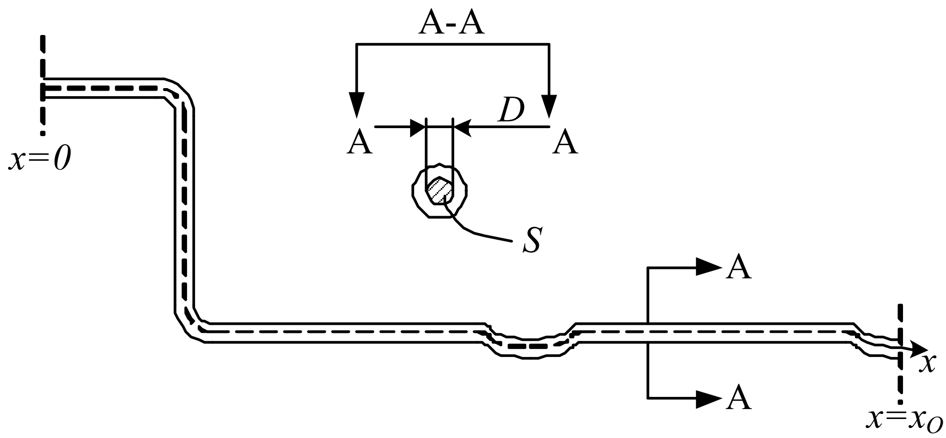

Fig. 2. Simplified functional diagram of the section of pneumatic brake system of electric train from the traction electric locomotive to the second, broken brake coupling pipe

As it is shown in [3], one of the methods, used for the solution of differential equations in partial derivatives of the second order, is the method of separation of variables, main idea of the method is that the function of two variables $p(x,t)$, that is the solution of the equation (1), is searched in the form of the product of two functions $p_*(x), p_{**}(t)$, each of which is the function of one variable, i. e., in the form

$$p(x,t) = p_*(x)p_{**}(t). \tag{2}$$

Substituting the expression (2) in the equation (1), and taking into account that after separation of variables, partial derivatives transform into ordinary ones, we obtain:

$$p_{**}(t)\frac{d^2 p_*(x)}{dx^2} - \frac{1}{v_*^2} p_*(x)\frac{d^2 p_{**}(t)}{dt^2} - \frac{8\pi\mu_d}{S\rho v_*^2} p_*(x)\frac{dp_{**}(t)}{dt} = 0. \tag{3}$$

As the rate of pressure drop in case of brake coupling pipe break is proportional to current pressure, both by spatial coordinate x, and in each point of this pneumatic pipe-line by time coordinate t, i. e., because for fixed values x_* and t_* differential equations will be valid

$$\frac{dp_*(x,t_*)}{dx} = -k_x(t_*)p_*(x,t_*), \tag{4}$$

$$\frac{dp_{**}(x_*,t)}{dt} = -k_t(x_*)p_{**}(x_*,t), \tag{5}$$

then as the functions $p_*(x), p_{**}(t)$ in the expression (2) and equation (3), it is expedient to take the solutions of equations (4), (5), which will have the form(4):

$$p_*(x,t_*) = p_*(x_*,t_*)e^{-k_x(t_*)(x-x_*)}, \tag{6}$$

$$p_{**}(x_*,t) = p_{**}(x_*,t_*)e^{-k_t(x_*)(t-t_*)}. \tag{7}$$

Physically, it is stipulated by the fact, that pneumatic pipe is the accumulator of only one type of energy – potential energy of compressed air, which, in case of pipe break has only one channel of transformation into another type of energy – kinetic energy of air flux in to the environment without even partial intermediate restoration.

Substituting the expressions (6), (7) in the equation (3) and carrying out differentiation, we obtain

$$p_{**}(x_*,t_*)e^{-k_t(x_*)(t-t_*)}\left(-k_x(t_*)\right)^2 p_*(x_*,t_*)e^{-k_x(t_*)(x-x_*)}-$$

$$-\frac{1}{v_*^2}p_*(x_*,t_*)e^{-k_x(t_*)(x-x_*)}\left(-k_t(x_*)\right)^2 p_{**}(x_*,t_*)e^{-k_t(x_*)(t-t_*)}-$$

$$-\frac{8\pi\mu_d}{S\rho v_*^2}p_*(x_*,t_*)e^{-k_x(t_*)(x-x_*)}\left(-k_t(x_*)\right)p_{**}(x_*,t_*)e^{-k_t(x_*)(t-t_*)}=0. \quad (8)$$

After corresponding reductions equation (8) is simplified to the equation

$$\left(-k_x(t_*)\right)^2-\frac{1}{v_*^2}\left(-k_t(x_*)\right)^2-\frac{8\pi\mu_d}{S\rho v_*^2}\left(-k_t(x_*)\right)=0. \quad (9)$$

From the equation (9) we obtain

$$k_x(t_*)=\frac{k_t(x_*)}{v_*}\sqrt{1-\frac{8\pi\mu_d}{S\rho k_t(x_*)}}. \quad (10)$$

Having obtained the expression(10), we obtained the relation that connects differential equation in partial derivatives of the 2nd order (1) and solutions (6), (7) of ordinary differential equations of the 1st order (4), (5) and which allows, further for the description of the processes in pneumatic line on the section from its beginning to the point of break, to use each of these differential equations separately on condition that, one of the variables in each of them is fixed at the initial level.

Substituting the expression (10) in equation (6), dividing both parts of this equation by $p_*(x_*,t_*)$ and taking the logarithm of the new equation, transformed in this way, we obtain

$$-(x-x_*)\frac{k_t(x_*)}{v_*}\sqrt{1-\frac{8\pi\mu_d}{S\rho k_t(x_*)}}=\ln\frac{p_*(x,t_*)}{p_*(x_*,t_*)}, \quad (11)$$

whence

$$x=x_*-\frac{v_*}{k_t(x_*)\sqrt{1-\frac{8\pi\mu_d}{S\rho k_t(x_*)}}}\ln\frac{p_*(x,t_*)}{p_*(x_*,t_*)}. \quad (12)$$

If we denote the coordinate of the point of pneumatic line break as x_o (Fig. 2) and take a reading of coordinate x from the beginning of this pneumatic line, that is, take in the formula (12)

$$\begin{cases} x_*=0, \\ x=x_o, \end{cases} \quad (13)$$

then, the expression (12) will have the form

$$x_o=-\frac{v_*}{k_t(0)\sqrt{1-\frac{8\pi\mu_d}{S\rho k_t(0)}}}\ln\frac{p_*(x_o,t_*)}{p_*(0,t_*)}. \quad (14)$$

This expression (14) will be the first component of mathematical model intended for the diagnostics of pneumatic pipe of electric train brake system, as it enables to determine the coordinate x_o of the pipe break point. But, as it is seen, this expression includes the parameter $k_t(0)$, which can be determined only from another differential equation, i. e., equation (7).

That is why, as the second component of the above-mentioned mathematical model, we will construct the expression, intended for finding $k_t(0)$ parameter.

To find this parameter we will return to the equation (7). Dividing its both parts by $p_{**}(x_*, t_*)$ and taking the logarithm of the new equation transformed in this way, we obtain

$$-k_t(x_*)(t - t_*) = \ln \frac{p_{**}(x_*, t\ )}{p_{**}(x_*, t_*)}, \tag{15}$$

either (taking into account the first of relations (13))

$$k_t(0) = \frac{1}{t_* - t} \ln \frac{p_{**}(0, t\ )}{p_{**}(0, t_*)}. \tag{16}$$

The expression (16) is the second component of our mathematical model, by means of which, setting the value of the moment of time $t = t_1$ at the known moment of time t_*, the value of $k_t(0)$ parameter is easily found, this parameter numerically equals k_t^*, i. e., from the expression (16) we will obtain

$$k_t^* = \frac{1}{t_* - t_1} \ln \frac{p_{**}(0, t_1)}{p_{**}(0, t_*)}. \tag{17}$$

However, in real conditions it is difficult to fix exactly the moment of time t_*, when the break occurs in pneumatic pipe, that is why, for its analytical determination it is expedient, having equated right parts of the expression (16) for two moments of time t_1, t_2, synthesize the auxiliary expression

$$\frac{1}{t_* - t_1} \ln \frac{p_{**}(0, t_1)}{p_{**}(0, t_*)} = \frac{1}{t_* - t_2} \ln \frac{p_{**}(0, t_2)}{p_{**}(0, t_*)}, \tag{18}$$

and find the parameter t_* from it. The obtained expressions (17), (18) we will identify as the third component of the above-mentioned mathematical model.

To understand, how the calculations are to be performed by means of synthesized mathematical model, Fig. 3 shows geometric interpretation of pressure $p(x,t)$ change process along space coordinate x on coordinate plane xt_*p and in time t on coordinate plane tt_*p in case of break of pneumatic pipe of brake system in point x_o at the moment of time $t_* = 0$.

It is seen from Fig. 3 that to calculate space coordinate x_o of the point of pneumatic pipe break at the moment of time $t = t_* = 0$, on condition that $x_* = 0$, it is not obligatory to know the total model surface $p(x,t)$, but it is sufficient to know only the lines of its intersection $p(x, t_*)$ and $p(0, t - t_*)$ with corresponding coordinate planes and initial $p(0, t_*)$ and limiting $p(x_o, t_*)$, $p(0, t_k - t_*)$ conditions, which are the components of synthesized mathematical model, determined by the expressions (14) – (18). And since these lines of intersection on these planes are solutions (6), (7) of differential equations (4), (5), then, working with models (14) – (18) we should remember that

$$p_*(0, t_*) = p_{**}(0, t_*) = p(0, t_*), \tag{19}$$

$$p_*(x_o, t_*) = p(x_o, t_*) = p_{**}(0, t_k - t_*) = p(0, t_k - t_*), \tag{20}$$

$$p_{**}(0, t) = p(0, t), \ \ p_{**}(0, t_1) = p(0, t_1), \ \ p_{**}(0, t_2) = p(0, t_2). \tag{21}$$

In traction electric locomotive the pressure in non-damaged pneumatic pipe of brake system in operation mode is maintained at the level of 5.2 at (note, that technical atmosphere(at) equals 735.66 mm Hg or $0.981 \cdot 10^5\ H/m^2$) [5]. Proceeding from these data we have

$$p(0, t_*) = 5{,}101 \cdot 10^5 H / м^2. \tag{22}$$

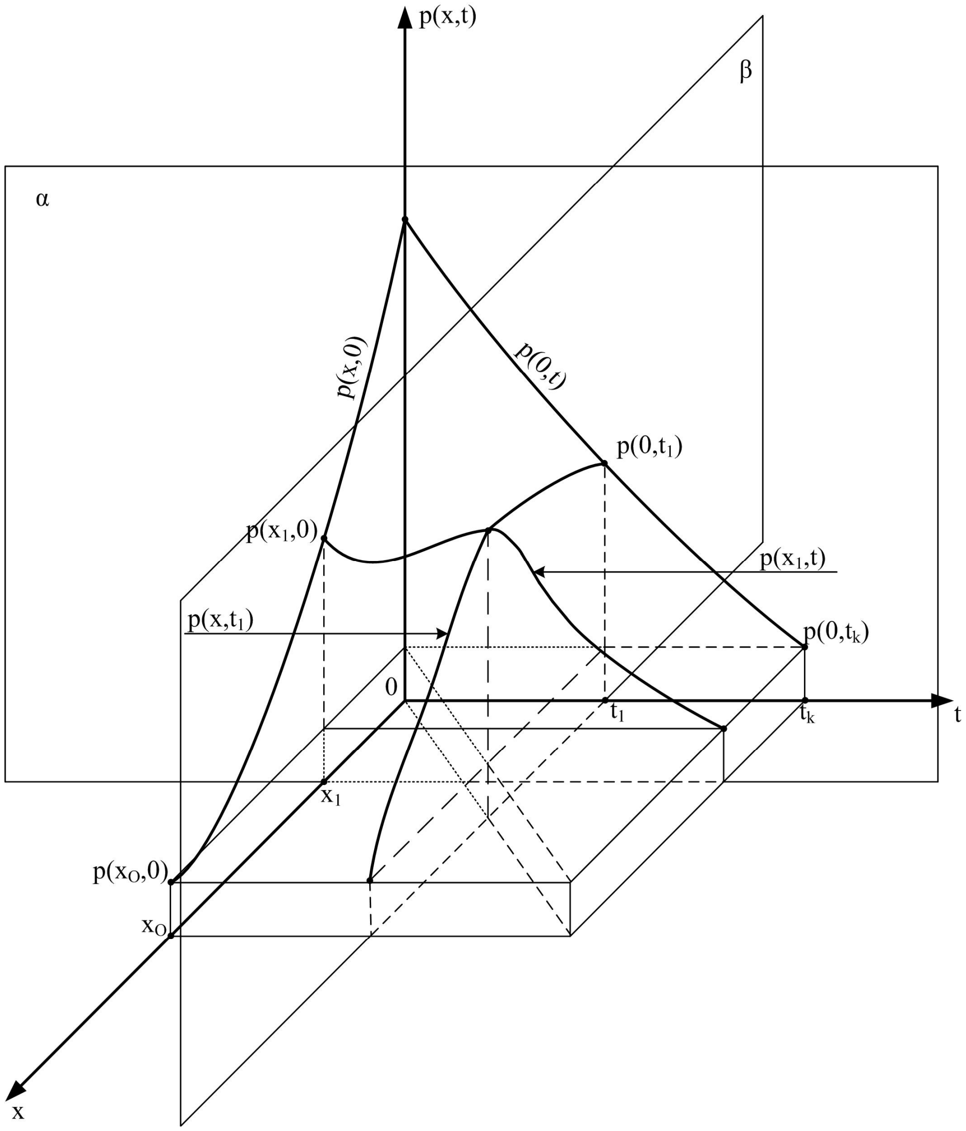

Fig. 3. Geometric interpretation of pressure $p(x,t)$ change process along space coordinate x on coordinate plane xt_*p and in time t on coordinate plane tt_*p in case of break of pneumatic pipe of brake system in point x_o at the moment of time $t_* = 0$

Pressure in the place of break of pneumatic line of brake system equals atmospheric pressure, which, for instance, in Kyiv, at normal weather conditions equals 760 mm Hg or $1.013 \cdot 10^5 H/m^2$, and in Vinnytsia, located higher over the sea level than Kyiv, atmospheric pressure at normal weather conditions equals 735.66 mm Hg or $0.981 \cdot 10^5\ H/m^2$.

That is why, for Kyiv we have

$$p(x_o, t_*) = p(0, t_k - t_*) = 1{,}013 \cdot 10^5 H/m^2, \tag{23}$$

and for Vinnytsia –

$$p(x_o, t_*) = p(0, t_k - t_*) = 0{,}981 \cdot 10^5 H/m^2. \tag{24}$$

Values $p(0,t_1)$, $p(0,t_2)$, needed for calculation of break moment t_* and $k_t(0)$ parameter by the formulas (17) and (18) are determined by means of measurement of falling pressure at the beginning of pneumatic pipe of brake system, i. e., at point $x_* = 0$ at the moments of time t_1, t_2.

For determination of space coordinate x_o of break point of pneumatic pipe of brake system by the formula (14), besides already mentioned values, we will require the number of parameters, namely, S, ρ, μ_d, v_*.

If we know the diameter D of the internal circle of the cross-section of brake coupling pipe of the brake system, the area S of the cross-section can be found by the formula

$$S = \frac{\pi D^2}{4}, \tag{25}$$

two other parameters ρ and μ_d we will take from [2], and v_* parameter – from the reference book [5]. These parameters are

$$\rho = 1.2\ \kappa g/m^3, \quad \mu_d = 1.81 \cdot 10^{-5} \frac{\kappa g}{m \cdot \sec}, \quad v_* = 348 \frac{m}{\sec}. \tag{26}$$

The structure of diagnostics system of pneumatic pipe of electric train brake system, using the developed mathematical model and algorithm of its functioning in automatic mode will be discussed in the next paper, that will continue the research, carried out in this paper.

Conclusions:

1. Mathematical model, intended for diagnostics of pneumatic pipe of electric train brake system has been synthesized, by means of this model space coordinate of break location of the pipe can be determined.

2. It is shown, that for determination of space coordinate of break location of brake system pneumatic pipe only lines of the surface intersection , by which the solution of differential equation in partial derivatives of the second order, with the coordinates planes «space coordinate-pressure» and «time coordinate-pressure», that allows to construct for the solution of this problem simpler mathematical model ; not connected with decomposition of the solution of the given differential equation in Fourier series by space coordinate, modulated by exponents and search of necessary and sufficient quantity of the members of this series to provide the set accuracy of the solution.

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