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CALCULATION OF THE DISTRIBUTION NETWORK MODE UNDER ASYMMETRICAL LOAD

The paper substantiates a new method for calculation of the distribution network mode under asymmetrical load, which is based on the application of orthogonal coordinate system.

Keywords: *asymmetrical load, orthogonal coordinate system, method of symmetric components .*

Introduction

With the development of industry the range of electric consumers negatively affecting the voltage unbalance is expanding. This is especially characteristic of the networks supplying powerful asymmetrical consumers such as railway traction networks with a two-phase load, electric arc furnaces and others which are the cause for voltage unbalance. Considerable voltage unbalance is observed in the networks with the power of 0,38 kW. The unbalance is also caused by open-phase modes, arising due to broken wires and open contacts of the switch. Voltage unbalance leads to the reduction of reliability and operation efficiency of electrical equipment of electric receivers [1].

To analyze asymmetrical modes the method of symmetric components or the phase coordinate method is used [2, 3]. At each point of the electric network voltages and currents are characterized by phase values in accordance with:

$$\mathbf{U} = \begin{bmatrix} \dot{U}_A \\ \dot{U}_B \\ \dot{U}_C \end{bmatrix}, \ \mathbf{I} = \begin{bmatrix} \dot{I}_A \\ \dot{I}_B \\ \dot{I}_C \end{bmatrix}.$$

In the system of symmetric coordinates the same voltages and currents will be given by

$$\mathbf{U}_s = \begin{bmatrix} \dot{U}_1 \\ \dot{U}_2 \\ \dot{U}_0 \end{bmatrix}, \ \mathbf{I}_s = \begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \\ \dot{I}_0 \end{bmatrix}.$$

Transition from symmetrical components to phase coordinates is carried out using Fortescue matrix according to formulas

$$\mathbf{U} = \mathbf{s}\mathbf{U}_s, \quad \mathbf{I} = \mathbf{s}\mathbf{I}_s.$$

The reverse transition from the phase coordinates to the symmetrical components is performed using the inverse matrix of Fortescue

$$\mathbf{U}_s = \mathbf{s}^{-1}\mathbf{U}, \quad \mathbf{I}_s = \mathbf{s}^{-1}\mathbf{I}.$$

Transition matrices are given by

$$\mathbf{s} = \begin{bmatrix} \mathbf{s}_1 & \mathbf{s}_2 & \mathbf{s}_0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{bmatrix}; \ \mathbf{s}^{-1} = \frac{1}{3}\begin{bmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{bmatrix}.$$

Voltage at each point of the radial network under asymmetrical load can be calculated by the formula

$$\mathbf{U}_s = \mathbf{E}_s - \underline{\mathbf{Z}}_s\mathbf{J}_s, \tag{1}$$

or

$$\mathbf{U} = \mathbf{E} - \mathbf{s} \cdot \underline{\mathbf{Z}}_s \cdot \mathbf{J}_s, \tag{2}$$

where $\underline{\mathbf{E}}_s = \begin{bmatrix} \underline{E}_1 \\ \underline{E}_2 \\ \underline{E}_0 \end{bmatrix}$, $\underline{\mathbf{E}} = \begin{bmatrix} \underline{E}_A \\ \underline{E}_B \\ \underline{E}_C \end{bmatrix}$, $\underline{\mathbf{Z}}_s = \begin{bmatrix} \underline{Z}_1 & & \\ & \underline{Z}_2 & \\ & & \underline{Z}_0 \end{bmatrix}$ – vectors of the symmetrical components, phase EMF of the source and the matrix of symmetrical components of the transmission line resistances.

For the analysis of asymmetric modes the phase coordinate method could be also used:

$$\mathbf{U} = \mathbf{E} - \underline{\mathbf{Z}} \cdot \mathbf{J}. \tag{3}$$

In this case the matrix of transmission line phase resistances is determined by the formula [4]

$$\mathbf{Z} = \mathbf{s}\mathbf{Z}_s\mathbf{s}^{-1}. \tag{4}$$

E. g., to matrix

$$\underline{\mathbf{Z}}_s = \begin{bmatrix} 0{,}1 + j0{,}2 & & \\ & 0{,}1 + j0{,}2 & \\ & & 3(0{,}1 + j0{,}2) \end{bmatrix}$$

the following matrix corresponds:

$$\underline{\mathbf{Z}} = \begin{bmatrix} 0{,}167 + j0{,}333 & 0{,}067 + j0{,}133 & 0{,}067 + j0{,}133 \\ 0{,}067 + j0{,}133 & 0{,}167 + j0{,}333 & 0{,}067 + j0{,}133 \\ 0{,}067 + j0{,}133 & 0{,}067 + j0{,}133 & 0{,}167 + j0{,}333 \end{bmatrix}.$$

Due to the interdependence between quantities represented in phase coordinates, calculation of asymmetric modes is complicated considerably in this case.

Goal of the work

The paper aims at the substantiation of the method for calculating the distribution network mode under asymmetric load, which is based on the application of the orthogonal coordinate system.

Substantiation of the results

A new approach to the analysis of asymmetric modes is based on application of the variables represented in αβ0 coordinates. At any point of the electric network voltages and currents are characterized by orthogonal components in accordance with

$$\mathbf{U}_p = \begin{bmatrix} \dot{U}_\alpha \\ \dot{U}_\beta \\ \dot{U}'_0 \end{bmatrix}, \ \mathbf{I}_p = \begin{bmatrix} \dot{I}_\alpha \\ \dot{I}_\beta \\ \dot{I}'_0 \end{bmatrix}.$$

Transition from orthogonal components to the phase coordinates is performed using matrix $\mathbf{p}$:

$$\mathbf{U} = \mathbf{p}\mathbf{U}_p, \quad \mathbf{I} = \mathbf{p}\mathbf{I}_p. \tag{5}$$

Transition from phase coordinates to symmetrical components is performed using the inversr matrix $\mathbf{p}^{-1}$ by the formulas

$$\mathbf{U}_p = \mathbf{p}^{-1}\mathbf{U}, \quad \mathbf{I}_p = \mathbf{p}^{-1}\mathbf{I}, \tag{6}$$

where $$\mathbf{p}=\begin{bmatrix}\sqrt{\frac{2}{3}} & 0 & \sqrt{\frac{1}{3}}\\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{3}}\\ -\sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{2}} & \sqrt{\frac{1}{3}}\end{bmatrix};\quad \mathbf{p}^{-1}=\begin{bmatrix}\sqrt{\frac{2}{3}} & -\sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{6}}\\ 0 & \sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}}\\ \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{3}}\end{bmatrix}=\mathbf{p}^{\text{т}}. \tag{7}$$

At each point of the radial network orthogonal components of the votage can be calculated by the formula

$$\mathbf{U}_p=\mathbf{E}_p-\underline{\mathbf{Z}}_p\cdot\mathbf{J}_p, \tag{8}$$

and phase values

$$\mathbf{U}=\mathbf{E}-\mathbf{p}\cdot\underline{\mathbf{Z}}_p\cdot\mathbf{J}_p. \tag{9}$$

The matrix of orthogonal resistances is determined by the formula

$$\mathbf{Z}_p=\mathbf{p}^{\text{т}}\mathbf{Z}\mathbf{p}. \tag{10}$$

E. g., to matrix

$$\underline{\mathbf{Z}}_s=\begin{bmatrix}0{,}1+j0{,}2 & & \\ & 0{,}1+j0{,}3 & \\ & & 0{,}2+j0{,}6\end{bmatrix}$$

the following matrix corresponds

$$\underline{\mathbf{Z}}_p=\begin{bmatrix}0{,}1+j0{,}25 & 0{,}05 & 0\\ -0{,}05 & 0{,}1+j0{,}25 & 0\\ 0 & 0 & 0{,}2+j0{,}6\end{bmatrix}.$$

If $\underline{Z}_1=\underline{Z}_2$, then

$$\mathbf{Z}_p=\mathbf{p}^{\text{т}}\mathbf{Z}\mathbf{p}=\mathbf{Z}_s. \tag{11}$$

In this case the complexity of the usage of orthogonal components corresponds to the method of symmetrical components, all the advantage of the phase coordinate method being preserved.

Asymmetrical load is given by the vectors of complex phase powers when they are connected into a “star” or a “triangle”, respectively

$$\underline{\mathbf{S}}_{\text{Y}}=\begin{bmatrix}\underline{S}_A\\ \underline{S}_B\\ \underline{S}_C\end{bmatrix},\quad \underline{\mathbf{S}}_{\Delta}=\begin{bmatrix}\underline{S}_{AB}\\ \underline{S}_{BC}\\ \underline{S}_{CA}\end{bmatrix}$$

or by diagonal matrices

$$\underline{\mathbf{S}}_{\text{Y д}}=\begin{bmatrix}\underline{S}_A & & \\ & \underline{S}_B & \\ & & \underline{S}_C\end{bmatrix},\quad \underline{\mathbf{S}}_{\Delta\text{ д}}=\begin{bmatrix}\underline{S}_{AB} & & \\ & \underline{S}_{BC} & \\ & & \underline{S}_{CA}\end{bmatrix}.$$

In the case when phases are connected into a “star” the vector of load current [4]

$$\mathbf{J}=\frac{\sqrt{3}}{U_{\text{н}}}\hat{\underline{\mathbf{S}}}_{\text{Y д}}\mathbf{s}_1. \tag{12}$$

In this case the direction of driving current was chosen according to the direction of the load from the node. For the currents of all phases to be oriented relative to vector $\dot{U}_A$, the current vector expression is multiplied by vector $\mathbf{s}_1$ on the right.

In the system of symmetrical components

$$\mathbf{J}_s = \frac{\sqrt{3}}{U_{\text{н}}}\mathbf{s}^{-1}\underline{\hat{\mathbf{S}}}_{\text{Y д}}\mathbf{s}_1 . \tag{13}$$

In the case when the phase loads are connected into a "triangle", the vector of currents in the lines is formed taking into account the phase shifts of voltages at the load phases

$$\mathbf{J} = \frac{e^{j30^\circ}}{U_{\text{н}}}\begin{bmatrix} \hat{S}_{AB} - a\hat{S}_{CA} \\ a^2\hat{S}_{BC} - \hat{S}_{AB} \\ a\hat{S}_{CA} - a^2\hat{S}_{BC} \end{bmatrix},$$

Therefore, the vector of phase currents [4]

$$\mathbf{J} = \frac{e^{j30^\circ}}{U_{\text{н}}}\mathbf{m}\underline{\hat{\mathbf{S}}}_{\Delta \text{д}}\mathbf{s}_1 , \tag{14}$$

where $\mathbf{m} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$ – the matrix of transition from the load phase currents (powers), connected into a "triangle", to linear currents. The vector of symmetrical components of the currents can be written in the following form:

$$\mathbf{J}_s = \frac{e^{j30^\circ}}{U_{\text{н}}}\mathbf{s}^{-1}\mathbf{m}\underline{\hat{\mathbf{S}}}_{\Delta \text{д}}\mathbf{s}_1 . \tag{15}$$

When asymmetric load connected into a "star" is activated in a three-phase network, the vector of symmetric components of the current is given by

$$\mathbf{J}_s = \frac{\sqrt{3}}{U_{\text{н}}}\mathbf{s}^{-1}\underline{\hat{\mathbf{S}}}_{\text{Y д}}\mathbf{s}_1 = \frac{1}{\sqrt{3}\cdot U_{\text{н}}}\begin{bmatrix} \underline{\hat{S}}_A + \underline{\hat{S}}_B + \underline{\hat{S}}_C \\ \underline{\hat{S}}_A + a\underline{\hat{S}}_B + a^2\underline{\hat{S}}_C \\ \underline{\hat{S}}_A + a^2\underline{\hat{S}}_B + a\underline{\hat{S}}_C \end{bmatrix}. \tag{16}$$

When asymmetric load connected into a "triangle" is activated in a three-phase network, the vector of symmetric components of the current

$$\mathbf{J}_s = \frac{e^{j30^\circ}}{U_{\text{н}}}\mathbf{s}^{-1}\mathbf{m}\underline{\hat{\mathbf{S}}}_{\Delta \text{д}}\mathbf{s}_1 = \frac{1}{\sqrt{3}\cdot U_{\text{н}}}\begin{bmatrix} \underline{\hat{S}}_{AB} + \underline{\hat{S}}_{BC} + \underline{\hat{S}}_{CA} \\ e^{j60^\circ}\underline{\hat{S}}_{AB} - \underline{\hat{S}}_{BC} + e^{-j60^\circ}\underline{\hat{S}}_{CA} \\ 0 \end{bmatrix}. \tag{17}$$

Taking into account that the product of matrices

$$\mathbf{p}^{\text{т}}\cdot\mathbf{s} = \begin{bmatrix} \sqrt{\frac{3}{2}} & \sqrt{\frac{3}{2}} & 0 \\ -j\sqrt{\frac{3}{2}} & j\sqrt{\frac{3}{2}} & 0 \\ 0 & 0 & \sqrt{3} \end{bmatrix}, \tag{18}$$

the vector of orthogonal components of the current, when asymmetric load connected into a “star” is activated in a three-phase network, will be given by:

$$\mathbf{J}_p = \frac{\sqrt{3}}{U_н}\mathbf{p}^т\underline{\hat{\mathbf{S}}}_{\mathrm{Y}\,д}\mathbf{s}_1 = \frac{1}{U_н}\begin{bmatrix} \sqrt{\frac{1}{2}}\left(2\underline{\hat{S}}_A + e^{j60^\circ}\underline{\hat{S}}_B + e^{-j60^\circ}\underline{\hat{S}}_C\right) \\ \sqrt{\frac{3}{2}}\cdot e^{-j90^\circ}\left(e^{-j30^\circ}\underline{\hat{S}}_B + e^{j30^\circ}\underline{\hat{S}}_C\right) \\ \left(\underline{\hat{S}}_A + e^{-j120^\circ}\underline{\hat{S}}_B + e^{j120^\circ}\underline{\hat{S}}_C\right) \end{bmatrix}. \qquad (19)$$

When asymmetrical load, connected into a “triangle”, is activated in a three-phase network, the vector of the current orthogonal components

$$\mathbf{J}_p = \frac{e^{j30^\circ}}{U_н}\mathbf{p}^т\mathbf{m}\underline{\hat{\mathbf{S}}}_{\Delta\,д}\mathbf{s}_1 = \frac{1}{U_н}\begin{bmatrix} \sqrt{\frac{3}{2}}\left(e^{j30^\circ}\underline{\hat{S}}_{AB} + e^{-j30^\circ}\underline{\hat{S}}_{CA}\right) \\ \sqrt{\frac{1}{2}}\cdot e^{-j90^\circ}\left(e^{-j60^\circ}\underline{\hat{S}}_{AB} + 2\underline{\hat{S}}_{BC} + e^{j60^\circ}\underline{\hat{S}}_{CA}\right) \\ 0 \end{bmatrix}. \qquad (20)$$

Hence, it is equally simple to represent driving currents of asymmetric load in a system of orthogonal and symmetric coordinates, which is an essential prerequisite for the analysis of transient and quasi-stationary modes of electrical networks in the presence of nodes of the loads with powerful synchronic and asynchronous machines.

Conclusions

Thus, the method is proposed for calculating the mode of asymmetrical load distribution network, based on the use of orthogonal coordinate system.

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