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ELASTIC-PLASTIC MODELING OF A FOUNDATION PLATE GEOTECHNICAL PROCESS

The paper illustrates application of the boundary element method for analyzing the stress-strain state of the school building subgrade using mathematical modeling of the foundation structure work under load.

Key words: *boundary element method, stress-strain state, foundation structure, active zone discretization.*

Introduction

Current tendency towards the increase of construction volumes placed a number of requirements to the designers concerning the necessity to analyze the stress-strain state (SSS) of bases of buildings and structures. Obviously, conventional engineering methods do not make it possible to achieve sufficient accuracy in the estimation of SSS of bases without taking into account irreversibility of their deformations and to adopt effective design solutions.

Development of non-linear soil mechanics and creation of the powerful computer base in design and research organizations stimulated elaboration of software complexes where mathematical models of the soils taking into account their elastic plastic behavior are implemented.

Calculations of the bases of structures are conducted for two not interconnected groups of limit states: deformations (in all cases) and bearing capacity (in special cases). When the bearing capacity is determined, deformation is not estimated. When the settlement is estimated, the stress is limited to the value that does not correspond to the bearing capacity of the soil. Modern base design problems require the base SSS analysis for the entire "load – settlement" range.

Problem statement. Determination of the relationship

While designing building foundations, experimental substantiation of the foundation design variant is required, which essentially increases its cost. Under such conditions it will be expedient to use mathematical modeling of the process of the foundation structure work under load. In this paper modeling of the process of the school building foundation plate settlement is performed using the boundary element method (fig. 1).

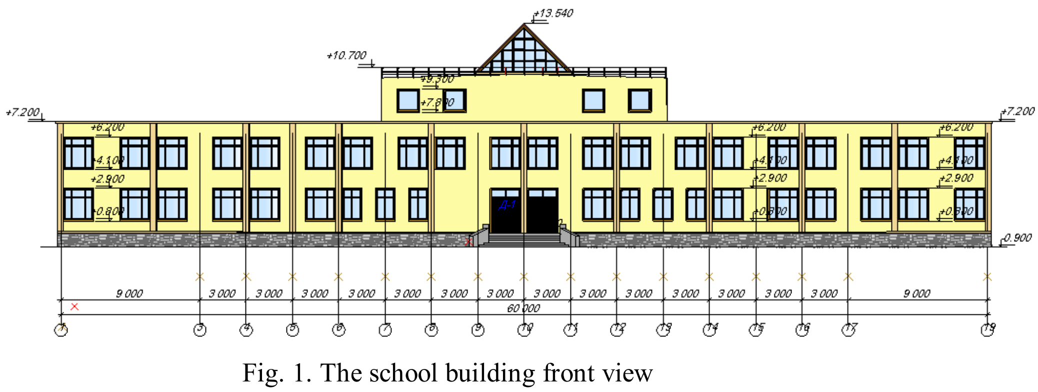

Fig. 1. The school building front view

The greatest difficulties involved in the calculation of foundation construction are connected with strong physical nonlinearity of the structure subgrade (mainly, with its compressibility).

To evaluate SSS of the soil mass during foundation construction and further work, a method of the problem solution in physically nonlinear statement is used with the application of plasticity theory in combination with the numerical boundary element method (BEM) and analog-iteration procedure. The model takes into account dilatancy and simultaneous presence of both limit- and pre-limit stress zones in the soil

Inelastic dilatancy model makes it possible to consider the limit state of the base for the two groups (bearing capacity and deformation) within a single computational model (scheme) of the soil.

Input parameters of the soil model include geometrical dimensions of the foundation plate and physical natural characteristics of the soil taken as weighted averages of the four engineering-geological elements:

$$E = 24224\,kPa,\ \nu = 0{,}33,\ \rho = 1{,}89\ t/m^3,$$

$$\rho^{\min} = 1{,}55\ t/m^3,\ \rho^{\max} = 2{,}036\ t/m^3,\ \varphi = 0{,}329\,radian,\ c = 32{,}4\ kPa$$

Normative values of the physico-mechanical properties are adopted in accordance with the data of the technical report on engineering-geological surveys. Fig. 2 gives dimensions of the school building foundation plate (fig. 1) and discretization of the active zone around the foundation subgrade consisting from 218 triangular cells.

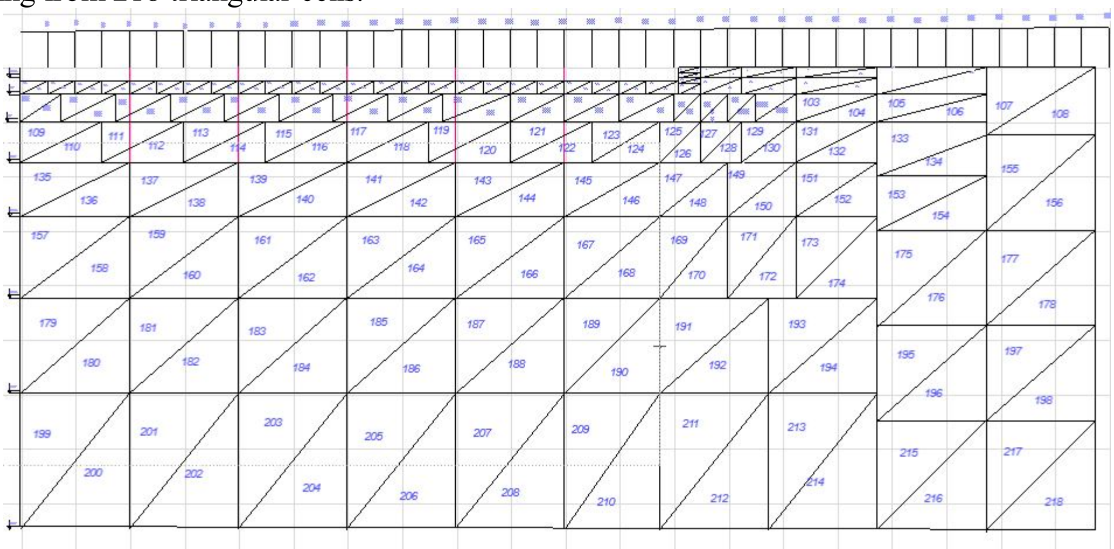

Fig. 2. Discretization of the soil zone near the foundation plate

The problem of finding the subgrade SSS and bearing capacity of the foundation plate involves determination of 15 functions σ_{ij}, ε_{ij}, u_i that satisfy:

• three equilibrium equations, that can be written in the form of Laplace differential equations

$$\sigma_{ij,j} + b_j = 0, \tag{1}$$

where $\sigma_{ij,\,j}$ are derivatives with respect to spatial coordinates of the stress tensor (expression (1) in the symbols of Einstein); b_j – volumetric load components.

• six relationships between loads and deformations (physical equation)

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl}\,. \tag{2}$$

• six relationships between deformations and displacements (geometrical equation)

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{3}$$

and boundary conditions in displacements or loads.

This boundary problem was solved using the boundary element method of Brebbia [1]. The system of 15 differential equations in partial derivatives is reduced to the limit integral equation

$$C_{ij}(\xi)u_j(\xi) + \int_{\Gamma} p_{ij}^*(\xi,x)u_j(x)d\Gamma(x) = \int_{\Gamma} u_{ij}^*(\xi,x)p_j(x)d\Gamma(x), \tag{4}$$

where u is a given vector of displacement velocities at the boundary of the pile; $\dot{u}^*, p^*, \sigma^*$ – kernels of the limit equation – fundamental solutions of P. Mindlin for an elastic half-plane; Γ, ξ, X – the boundary, the point of disturbance, the point of observation respectively.

For numerical simulation of three-axis SSS of the subgrade Cauchy tensor of small deformations is used:

$$\varepsilon_{i,j} = \varepsilon_{i,j}^{e} + \varepsilon_{i,j}^{p}, \tag{5}$$

where $\vec{\varepsilon}_{i,j}$ is a vector of residual deformations, $\vec{\varepsilon}_{i,j}^{e}$ – elastic deformation of the soil; $\vec{\varepsilon}_{i,j}^{p}$ – plastic deformation of the soil.

The vector of plastic deformations was determined by the formula:

$$\vec{\varepsilon}_{i,j} = \vec{\varepsilon}_{i,j}^{e} + \sum \vec{\varepsilon}_{i,j}^{p} + d\vec{\varepsilon}_{i,j}^{p}\delta_{i,j}. \tag{6}$$

In order to take into account the influence of deviator and hydrostatic components on plastic deformation of the soil, in the elaborated remote model these parts were separated:

$$\sigma_{i,j} = S_{i,j} + \delta_{i,j}\sigma, \tag{7}$$

where $\sigma_{i,j}$ are components of the stress tensor T_σ; $\delta_{i,j}$ – Kronecker delta, σ – spherical part of T_σ; $S_{i,j}$ – deviator part of T_σ.

Relationship between the velocities of plastic deformations and stresses (physical equations of state for the soil work in plastic phase) was determined using the non-associated law of plastic flow:

$$d\vec{\varepsilon}_{ij}^{p} = d\lambda \frac{dF}{d\sigma_{ij}}, \quad F \neq f \tag{8}$$

where F is a plastic potential, dissipative function of the porous medium of the soil; $d\lambda$ – scalar factor of the simple load; f – criterion of transition to the limit state (9); $d\vec{\varepsilon}_{ij}^{p}$ – vector of plastic deformations.

When the building is constructed, the soil of its base will be under the action of compressing loads transferred to it from the weight of the structure. The subgrade will be compacted. At the same time tight contacts between mineral particles of soil will be broken, which will lead to rearrangement of the soil particles to a more compact arrangement. Just from these considerations, for calculation of the building settlement we used the dilatancy theory of dispersed media and classical concepts of the mechanism of the medium boundary drag formation formulated by Terzaghi. To simulate complex deformation processes, in the model of this paper the numerical method of boundary elements is used.

Destruction of the multilayer soil base begins when the limit state condition is realized in one of the layers.

Destruction of the dispersed soil medium occurs due to the plastic (residual) strain accumulation. The influence of plastic strain is manifested in the development and growth of displacements, rearrangement of the internal forces.

In this paper the yield criterion of Mises – Schleicher – Botkin is responsible for the criterion of the soil transition to plastic state for $\sigma_m \geq \rho_0$ and $\sigma_m < \rho_0$ respectively (fig. 3):

$$f = \begin{cases} T + \sigma_m tg\psi - \tau_3 = 0 \\ T + \rho_0 tg\psi - \tau_3 = 0 \end{cases}, \tag{9}$$

where σ_m is the stress on the deviator plane, T – the intensity of tangential stresses, σ_m – hydrostatic pressure, a normal component of the stresses on the plane of limit equilibrium, ρ_0 – the hydrostatic pressure level when the soil works as a continuous medium (the boundary of transition from the cone to the cylinder in fig. 3), ψ – friction angle on the octahedral plane.

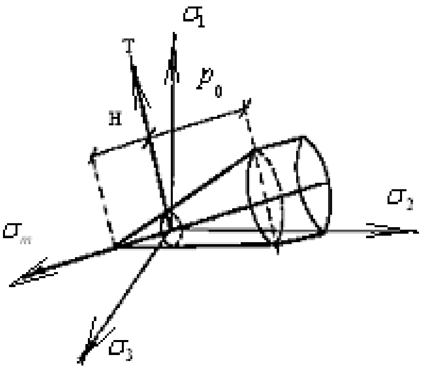

Fig. 3. Yield surface of Mises – Schleicher – Botkin in the coordinates of main stresses

Data on the calculation of BEM zones of the foundation plate bearing capacity are presented in fig. 4.

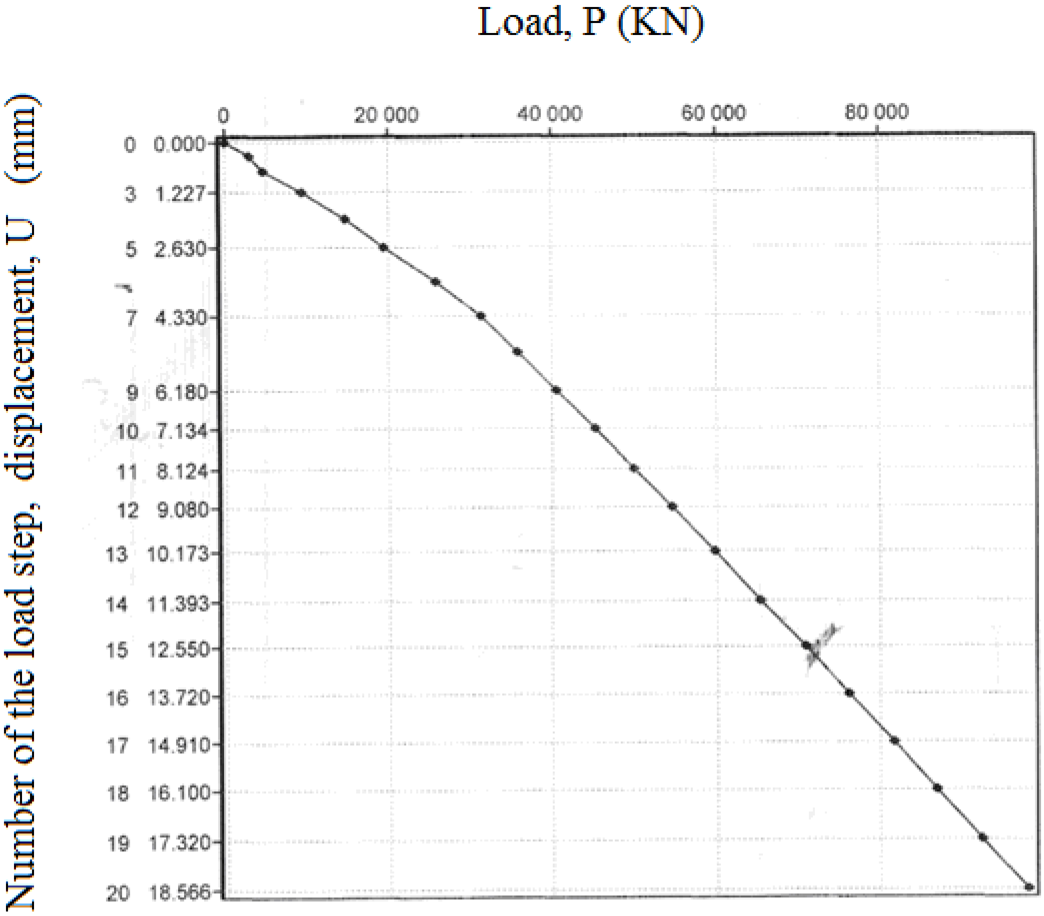

Fig. 4. The plot of load-settlement relationship

If the weight of the school structure is 60000 KN, then its settlement (for the foundation plate width of 0,3 м and physical-mechanical properties of the soil in accordance with the above-mentioned geological survey) will be S = 10,137 mm, which is less than the value permissible for the structures of this type in accordance with ДБН В.2.1.9-2009 (state norms of the construction industry):

S = 1,0137 см < S_u = 10 sm.

Conclusions

1. Thus, it is proposed to adopt a foundation plate with the width of h = 0,3 м and the size of 60 x 24 m as the foundation construction.

2. Elastic-plastic calculation of bases according to the proposed program enables much more qualitative estimation of SSS of the bases as compared with the engineering methods as well as taking effective design solutions.

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