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GENERALIZATION OF OPTIMUM AGGREGATION METHOD FOR THE CASE OF MANUFACTURING SYSTEMS WITH RANDOM STRUCTURES

Problem of elements replacement of manufacturing systems of random structures by optimum, equivalent by input-output aggregate element is set and solved. Computationally efficient models of optimal aggregation have been constructed.

***Key words**: distributed system, optimal aggregation, manufacturing function, model, manufacture, technology.*

Problem set-up. Manufacturing systems of high dimensionality require the development of optimization methods and models, enabling to optimize distributed manufacturing systems with great amount of manufacturing elements and manufactured products. In distributed systems of high dimensionality resources, loads should be constantly distributed, so that to optimize global criterion of the system, for instance, total production, total expenditures. Distributed manufacturing systems are of rather complex structure: with serial and parallel connection of manufacturing elements and feedbacks between these elements. Nowadays there exist numerous methods of such system optimization, developed for classes with certain characteristics of manufacturing elements – linear. square, logarithmic, Greater part of these classes, as well as, differences in characteristics, require the development of generalized optimization method of manufacturing systems of high dimensionality.

Fig. 1 presents the example of bioreactor systems designed and constructed with participation of one of the authors of the given paper. The problem of modernization of existing manufacturing lines is more than actual for poultry plants, hog-raising farms, oil mills and even food markets.

Fig. 1. Examples of bioreactor systems - elements of ecological networking of manufacturing systems

Nowadays, all over the world, ecological programmes of "ecological modernization of manufacturing processes" are adopted and realized. Bioreactor systems include subsystems with parallel and serial connections of functional elements, bioreactor systems is a subsystem, providing "ecological networking" of technological processes of certain manufacturing system.

This scientific direction, due to its boundary position among different subjects, does not have satisfactory mathematical models and optimization methods of processes of functioning and development of such systems.

Great number of papers on this subject proves the actuality and, at the same time, shows the complexity of the problem, dealing with optimum distribution of the resources. Close analogs, regarding the problem set-up and methodology [1 – 3], have constraint for bounded functions and criteria. Nowadays the scientific direction of alteration of optimization problems has been formed, it is based on combination of computational methods and classic methods of mathematical analysis. In the given paper basic concept for construction of the generalized method of optimization – presentation of manufacturing system elements as technological converters of resources "resource –

product or ”expenditures-output” (production functions (PF)) is described, PF of existing production elements are non-linear, non-stationary, stochastic, fuzzy. Globalization, increased production efficiency make resources and products freely convertible. In first approximation we will consider resources and products to be freely convertible. This assumption allows to consider manufacturing systems as converters of resources with scalar input and output. Thus, we can apply and develop for analysis and optimization of production systems methods, similar to the methods of equivalent transformations in the theory of automatic control (TAC). These transformations allows to replace random structure of dynamic control system by the equivalent element with ratio “input-output”.

In the theory of automatic control (TAC) basic elementary connections are parallel, serial and feedback connections. The structures of existing production systems and corresponding modeling problems are also represented in similar but not identical structures: in TAC, connections between the elements are mainly informational, for manufacturing systems connections are resources. The approach, chosen in this work, to the solution of the problem of manufacturing system presentation in the from of technological transformation of resources, differs from the analogues by the fact, that we combine classic problem of equivalent transformation of limited resources. Such combination allowed to create, non searchable, suitable for random nonlinearities, method for the case of parallel connection of production system elements – “method of optimal aggregation”.

New models of production systems have been obtained – *equivalent, optimally aggregate*. Method of optimal aggregation for parallely connected elements (metallurgical aggregates, bioreactors, boilers) is theoretically substantiated and realized on programming level [4 – 8]. In the given paper the problem of development of the method, intended for presentation of distributed production system with random structure, by one element – technological transformation of input resources in final products, equivalent by ratio “input-output” of primary distributed system is put forward. Fig. 2 contains diagram of the system of problems, dealing with the development of models of this work, as a process of construction of aggregated models system. Novelty element in the paper - *algebraization of optimization problems and their incorporation in the models of functioning and development of production systems.*

Let us define the notion “production systems with random structures”, we will consider three types of basic connections of production (technological) elements: parallel, serial and feedback. The simplest example of feedback is synthesis of styrene: at the output of the reactor, the components, that did not enter the reaction are taken form the product, and send them back at the input. The structures, consisting of a random number of subsystems, elements of which are connected by basic band, are considered to be random. For investigation we take those random structures that are found in real production systems.

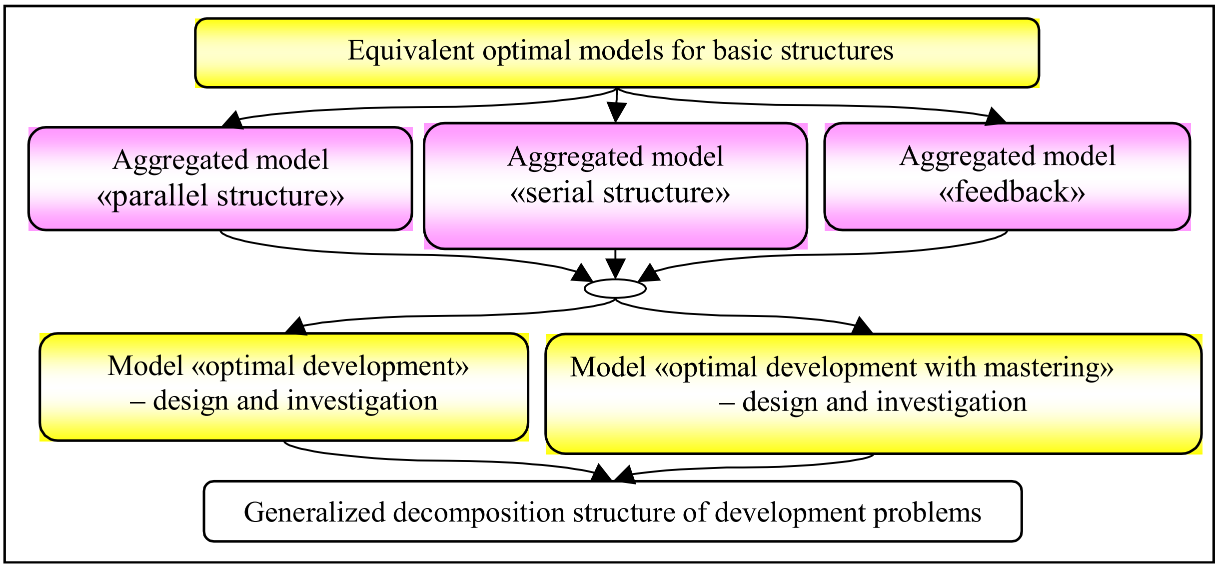

Fig. 2. Diagram of models development tasks system

Mathematical model of equivalent optimally aggregated production system with parallely operating elements. Let us consider the direct problem – maximization of total production in case of limited resources. The system of N production elements, using certain resources x_i and manufacturing production in quantity: $v_i = f_i(x_i);\ i = 1, \ldots, N,$ where x_i – is the volume of resource, allocated to *i-th* element. Resource R must be distributed, so that to maximize the total production:

$$F(x_1, x_2, \ldots, x_N) = \sum_{i=1}^{N} f_i(x_i);\ \text{if}\ G(x_1, x_2, \ldots, x_N) = \sum_{i=1}^{N} x_i - R = 0. \qquad (1)$$

Formally, this problem can always be solved by the method of direct exhaustive search, computational constrain – the number of parallely operating element should not exceed 5 – 7. Method of optimal aggregation [4] allows to perform decomposition of the problem of the extremum search of the function of *variables* by the sequence of $(N-1)$ problems of extremum function search by one variable [4, 5].

Fig. 3 presents the scheme of the problem of system aggregation from parallel elements. In the upper part – initial (primary state) system, in the lower part – the scheme of equivalent optimum element. Programming module calculates equivalent optimum element, using total resource and returns total optimum product output Yop .

Lower branch of the scheme presents the built-in module of optimum aggregation, which, for the given constrains by the resources Xs, calculates optimum distribution of this resources among the elements of production system.

Diagram in Fig. 3, 4, 5 – are conceptual, it is an important stage of mathematical models construction. Fig. 3 presents optimum aggregation of the system with parallel elements.

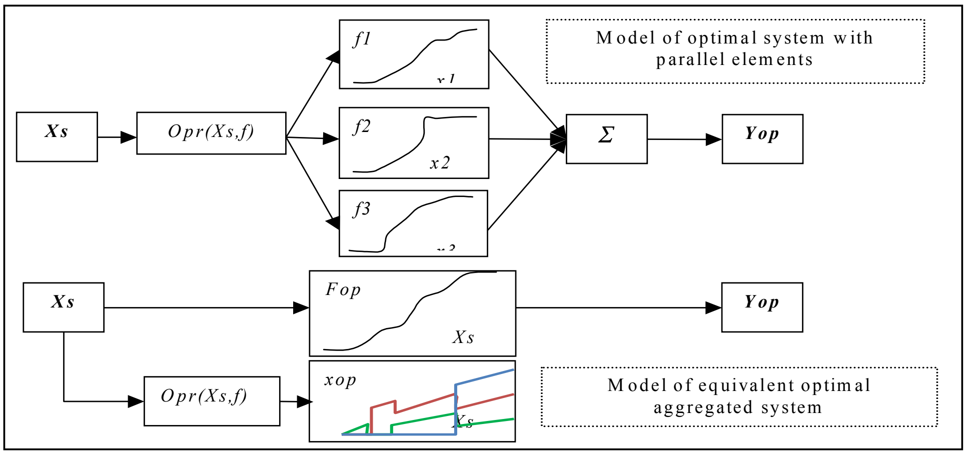

Fig. 3. Optimum aggregation of the system with parallel elements

Mathematical model of equivalent optimally aggregated production system with serially functioning elements. The system with serially connected elements considerably differs from the system with parallely operating elements. Optimization task is conjugated relatively the task of total output [4, 5]: minimization of total expenditures while constrain on production rate. Abstract models of production systems can not be considered separately from content interpretations, more exactly – from practical tasks. Interpretation of the task – vertically production system with distributed technological process, into technological and organizational in the sequence of subprocesses.

The product at the output can appear only after carrying out operations at all stages of manufacturing process, that is , the first condition of satisfactory functioning of such systems – is matching of carrying capacity (production capacity) of elements. Fig. 4 shows the scheme of system model aggregation with serial connection of elements. Unlike previous problem, control variables are volumes of resources for creation and support (operational expenditures) production capacities of system elements.

Resources, converted by manufacturing system into the product, are preset. Fig. 4 presents optimum aggregation of the system with serial elements. The line of technological transformation of the resources X in product Yop is marked out. For control the resource Rp is used.

Formally this problem can be solved applying the method of direct exhaustive search can applied to production system in case of serial connection, of global criterion is multiplicative. For vertically integrated systems, reliability of technological line operation, which, in accordance with the theory of reliability equals the product of elements reliabilities is main criterion. According to the principle of optimality, performed for this problem, irrespective of the volume of resources allocated for production development, this resource must be distributed optimally. The chain of elements of vertically integrated system are replace by equivalent and optimal by “input-output” reflection element.

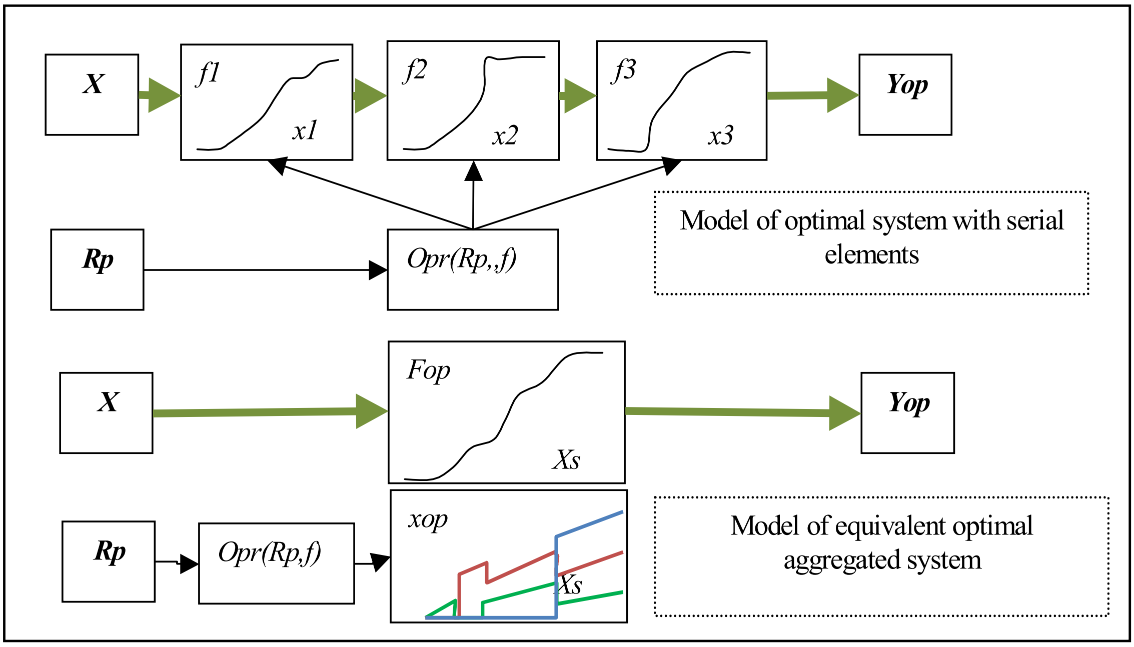

Fig. 4. Optimal aggregation of the system with serial elements

Let us modify this method for multiplicative global criterion in the following way:

– assign models of production functions of elements, local criteria and models parameters ;

– write equivalent additive model of optimization problem;

– define for the equivalent additive problem optimal production function and vector-function of optimal resource distribution ;

– calculate final results for multiplicative problem.

We perform the transition to additive form of optimization criterion. It should be noted, that for any algebraic we can find transformation, that converts them in additive form. Typical mathematical operations – multiplication, division, differentiation, integration can be reduced to additive from by means of certain transformations. They are logarithmic transformations, Laplace transformations and Z – transformations. For further expansion of criteria and constrains set, for which transformations in additive form exist, we use methodology of geometric programming [3], where basic model of the criterion is posinom, generalized by the polynomial of the type:

$$g(x)=\sum_{i=1}^{n}u_i(x)=\sum_{i=1}^{n}c_i\cdot\prod_{j=1}^{m}x_j^{a_{i,j}},\quad x_j\succ 0,\ c_i\succ 0,\ a_{i,j}\in\Re, \tag{2}$$

where $u_i(x),\ i=\overline{1,n}$ – is monom.

The properties of posinom are: is $g(x)$ – is a posinom, $\lambda > 0$ – constanta, then $\lambda g(x)$ – is a posinom; if $g(x)$ – posinom, $f(x)$ – posinom, then $g(x) + f(x)$ – posinom, is $g(x)$ – is a posinom, $u(x)$ – posinom, then $u(x)+g(x)$ – posinom, if $g(x)$ – posinom, $u(x)$ – is a monom , then $g(x)\ /\ u(x)$ – posinom.

Posinom - is a basic notion in geometric programming. By means of posinoms problems wide sphere of mathematics are described and solved, in particular: optimal planning, optimal control, design problems and risks calculation. In geometric programming non- linear functional dependencies are classified by topological features (monotonically increasing and monotonically decreasing), analyze and arrange the results of algebraic transformations of monotonic functions, for instance: $f1+f2,\ f1\cdot f2,\ f1^{f2}$ and more generalized transformation $f1(f2)$.

Main difference of the generalized method of optimal programming from the method of geometrical programming is in further usage of the properties of algebraic operations over monotonic functions:

– in the method of optimum aggregation they use for obtaining equivalent one-dimensional function ”input-output” for technological system;

– in the method of geometric programming they use for the analysis of properties of multidimensional efficiency function, presented by posinom .

Let us consider the problem of optimal aggregation for multiplicative criterion, take the logarithm of the expression by symbolic processor:

$$\ln\left(\prod_{i=1}^{N1} f(x_i)_i\right) \ \exp and,\ f_i \to \sum_{i=1}^{N1} \ln\left(f(x_i)_i\right). \tag{3}$$

For certain classes production systems: reactors of chemical synthesis, bioreactors, metallurgic plants, manufacturing of various tanks of plastics and composites – technological functions belong to certain parametric classes, in this case we obtain the sum of values of single class function, but with various parameters.

$$\ln\left(\prod_{i=1}^{N1} f4(R\cdot\alpha_i,\, P_i)\right) \ \exp and,\ f_i \to \sum_{i=1}^{N1} \ln\left(f4(R\cdot\alpha_i,\, P_i)\right), \tag{4}$$

where R – is constrain value by the resource, α_i – is dimensionless variable – part of the resource for the *i-th* element, P_i – is vector of parameters of *i-th* function, $f4$ – is function a certain class (in the given case – *S*-function [4]), $f4\lg$ – is logarithmic form of the function.

Let us introduce the notation

$$F4\lg(R,\alpha,P) = \ln\left(f4(R,\alpha,P)\right) = \sum_{i=1}^{N1} f4\lg(R\cdot\alpha_i,\, P_i). \tag{5}$$

We obtain criterial function of the system (5) in the required form for application of the method of optimal aggregation. Monotonicity of the function logarithm provides that the points of extremu7m of multiplicative efficiency and its additive form coincide.

We will formulate optimization problem: for each assigned R – constraint of the resources for the system – find distribution of this resource: $\alpha_1, \alpha_2, ..., \alpha_N$, $\sum_{i=1}^{N} \alpha_i = 1$, maximizing value of the criterion (5). The statement of conjugated optimization problem is possible constraint is the set production output, and criterion – total expenditures for creation and support of required level of production capacities. The aim of optimization – minimum of total expenditures.

In lower part of Fug. 4 the scheme of equivalent optimal element is presented. Programming module, realizing equivalent optimal element, uses resources *Rp* and returns corresponding product output *Yop*. Lower branch of the diagram presents the built-in module of optimal aggregation, which calculates optimal distribution of *Rp* resources among serial elements of production system.

Mathematical model of equivalent optimally aggregated production system with random structure. Let us consider the problem on the example of the structure “feedback”. Aggregation of serial and parallel connections allows reduce random structure, having feedbacks, to single – contour system. Taking into account non-linearity of elements characteristics, the obtaining of analytical expression for production function of equivalent optimal production element, as a rule, is insoluble problem. However, development of the operator of optimal aggregation for the system with feedback in program environment of modeling package can always be performed during finite time.

Content interpretation of the feedback: along with the useful product, the production system generates certain amount of by-products with negative value – waste. Main drawback of modern sate of environmental protection is that the stages of new products development and development of means of safe utilization of waste are divided in time and space. We mean typical situations: means of utilization we start to develop, when the manufacturing of the product has already begun, the environment is polluted, outside organization develops technologies for utilization of the waste of basic products.

Nowadays, modeling is widely used in the processes of products design, development of technologies and technological processes. Problems of ecological safety and environmental protection must be included at early stages of innovations realization – at the stage of modeling. However, without development and testing of basically new class of integrated models of functioning and development of production systems, taking account such aspects as preservation and protection of the environment, all this will remain only declaration.

Today main strategic goal of innovation development is networking and insulation of all technological cycles within the limits of this very production. For planning of innovation development operating production models with reflection of feedbacks in technological processes are required. In accordance with classic paradigm, model is the reflection of important for user, properties of real object. In the situations of innovation development, reference object is missing, that is why, it is necessary to construct models, that are complicated and specified, and each previous model is the source of information for construction of the next model.

The given paper contains the results of development and investigation of the models of first approximation, which require minimum of empiric data for construction on the basis of fundamental laws, taking place in technological transformations. Fig. 5 shows the scheme of the model with feedback. Feedback in the model can reflect such constructive – technological solutions:

–counter-current heat exchangers for heat recovery in technological cycle;

– circulating water supply- purification and recovery of water and solutions in technological cycle;

–expansion of the spectrum of useful products on the base of deeper recycling;

– recycling of organic waste on the base of ratification ecological systems – bioreactors [2].

Fig. 5 shows the scheme of the technique of optimal aggregation of systems with random structures, that does not differ from the scheme in Fig. 4. The difference is in the operator of optimal aggregation, that takes limitation of the resource *Rp* for creation of corresponding production capacities and operation expenditures, generalized production functions of elements and returns total optimal product output *Yop*. Fig. 5 conventionally shows non-linear functions of system elements – technological converters. All these functions are determined in the first quadrant, they are limited, non- strictly monotonic.

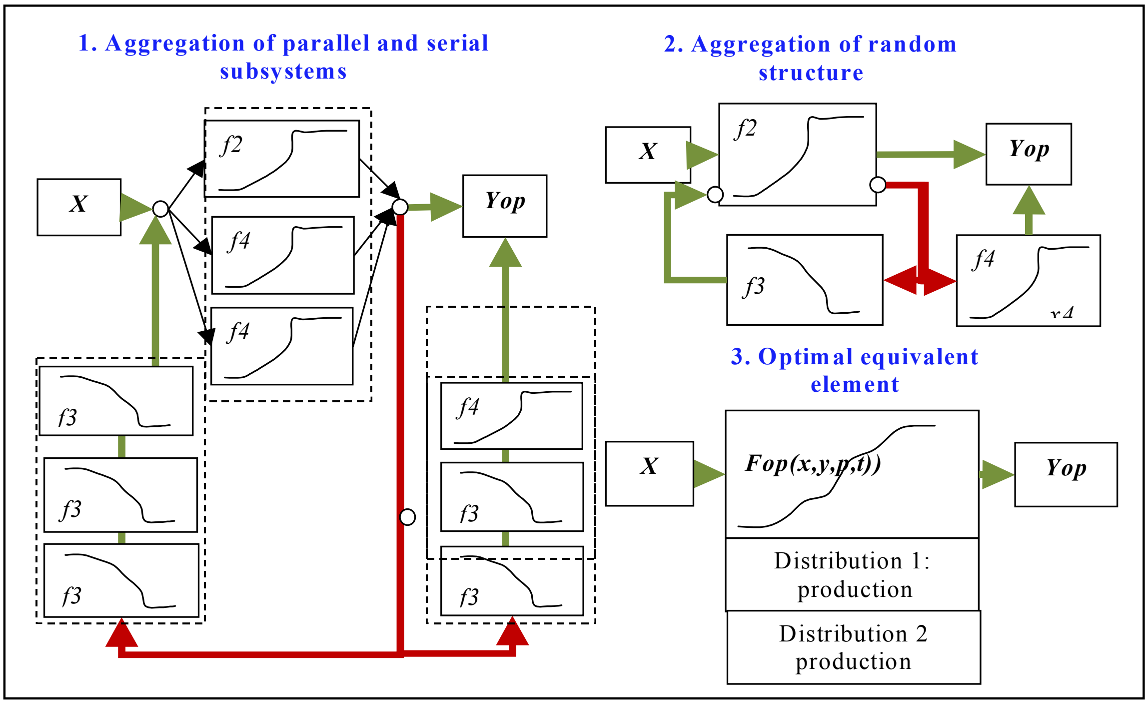

Fig. 5. Scheme of the technique of optimal aggregation of systems with random structures

Technique of obtaining of equivalent characteristic of non-linear system with feedback. Let us consider, non-standard element of the method of optimal aggregation of random structures – replacement of the system with feedback by the equivalent optimal element. The analogue is the technique of obtaining of transfer function of linear system with feedback: $W(s) = \dfrac{W1(s)}{1 + W1(s) \cdot W2(s)}$.

We will introduce similar expression for the system with static non-linearities:

$$Y(t) = Y1(t), \tag{6}$$

$$Y2(t) = F2(Y1(t)), \tag{7}$$

$$Y1(t) = F1(X(t) - Y2(t), b1). \tag{8}$$

We search: $Y(t) = dependence(X(t))$. We exclude unnecessary variables from the equations (7) – (8), and substitute (8) into (6):

$$Y(t) = F1(X(\mathrm{t}) - Y2(t), b1), \tag{9}$$

$$Y(t) = dependence(X(t));\ Y1(t) = F1(X(t) - Y2(t), b1).$$

We substitute (7) into (9) and solve relatively $Y(t)$

$$Y(t) = F1(X(t) - F2(Y(t)), b1). \tag{10}$$

Linear function is taken as an example to show the characteristic features of technology – the necessity to create specific modules for certain classes on non-linearities. We divide the solution into two parts “x less constrains” and “x more constrains: if $|X| \le b$, then $Y(x) := X$, then $Ys = Xs - Xs^3\ solve, Ys \rightarrow (symbolic \cdot expressions)$ (Xs – variable “without value” for symbolic computations); if $|X| > b$, then $Y1(x) := k1 \cdot b1$, then $Ys = k1 \cdot b1\, solve, Ys \rightarrow (symbolic \cdot expression)$.

We obtain characteristic of equivalent element as the function of resource of production X, function of resource constrain on the development Rp and control variables $x1, x2, x3$:

$Yn(X) \Rightarrow Yn(X, x1, x2, x3, Rp)$. We perform substitution and solve optimization problem:

$Opr(Rp, f1, f2, f3) = \max(Yn(X, x1, x2, x3, Rp))$ at constrains $x1 + x2 + x3 = Rp$.

Fig. 6 constrains the results of modeling for three values of feedback parameter, applying the technique of obtaining the equivalent characteristic of non-linear system.

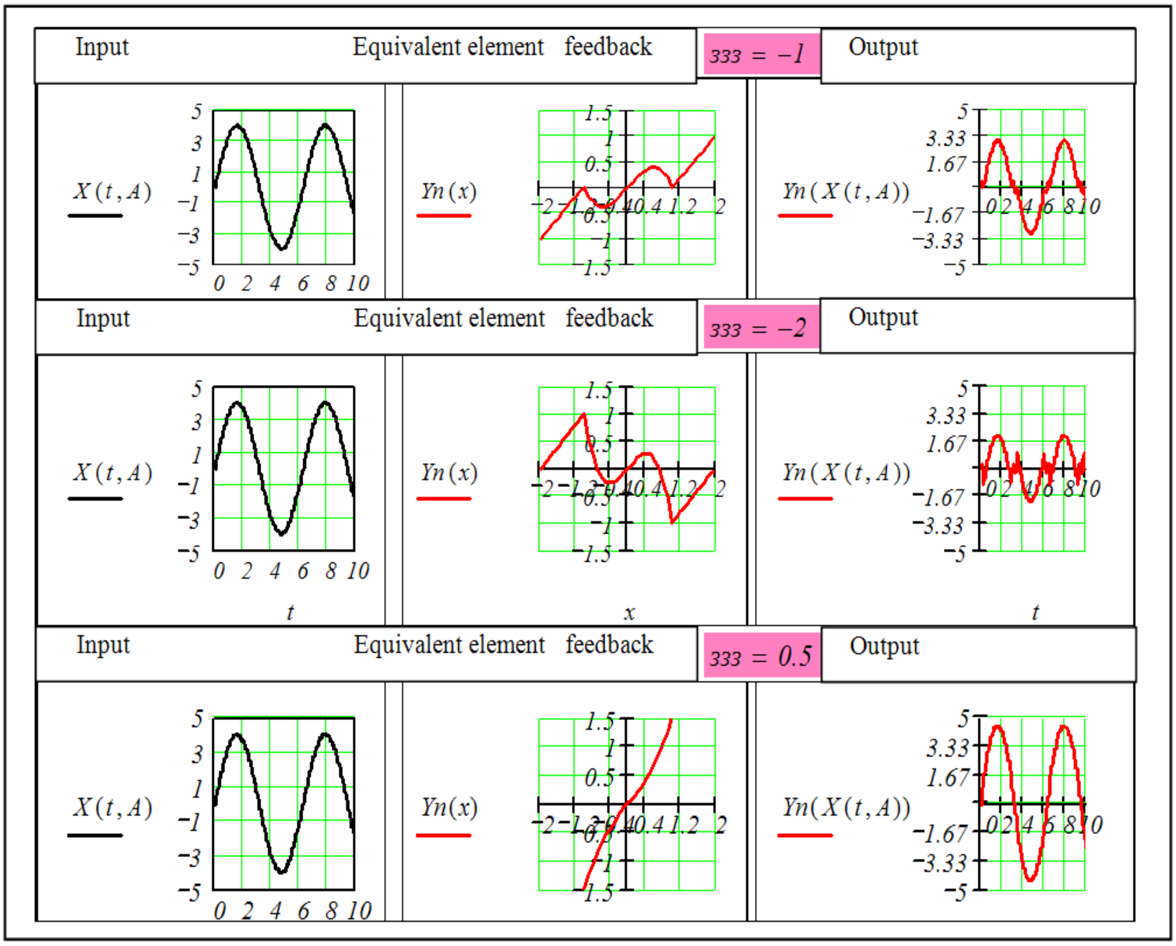

Fig. 6. Technique of obtaining equivalent characteristic of non-linear system with feedback

Generalized theoretical models are realized in mathematical package environment. We observe complex character and dynamics of equivalent characteristic $Yn(x)$ (middle column of the graphs).

In suggested models and methods we will underline novelty. The novelty of the research is efficient method of multidimensional optimization.

New class of the methods of production systems optimization – are methods of optimal aggregation. It is show, that for the systems with parallel and serial connection of production elements, the problem of optimization of the system with additive and multiplicative criteria, multidimensional problem can be reduced to the sequence of on-dimensional optimization problems. Te problem of multidimensional optimization task is reduced to algebraic problem -application of binary operator of optimal aggregation, that does not have limitations on the type of production function. For each type of global criterion of production system optimality is necessary to construct operator of optimal aggregation.

New class of production systems models are optimal equivalent one-dimensional models. Operator of optimal aggregation takes a pair of production functions and return the object of the same class optimal equivalent production function. Optimally aggregated models contain information regarding characteristics of aggregated elements. Optimally aggregated models allow to apply new approaches to the solution of the problem of production systems optimal control. Much attention is paid to examples, because the usage of the package possibilities for modeling is necessary condition for obtaining new results,

Conclusions. Generalization of the method of optimal aggregation on production systems with random structures is carried out on theoretical and applied levels. Technologies of construction of production systems working models are based on non-standard integration of the possibilities of mathematic packages and classic methods – variational computation, theory of dynamic systems – allow to obtain efficient optimization models of multi dimensional systems and solve problems of ecological safety and environmental protection at early stages of innovation – at the stage of modeling.

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