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MARKOV MODEL APPLICATION FOR ANALYZING CYCLICITY INFLUENCE ON THE BRANCHED TECHNOLOGICAL PROCESS MANAGEMENT

The paper presents the obtained inhomogeneous Markov model that makes it possible to estimate the risk of the variants of branched manufacturing process realization and to select the realization variant that has minimal risk. On the basis of the obtained model, prediction as to the expedient number of cyclic technological process repetitions has been made. The developed model is applied to the software testing technological process.

Key words: *cyclic technological process, Markov model, risk.*

Introduction

Complex branched cyclic technological processes (CBTP) have found wide application in many industries [1].

Cyclicity is CBTP property of repeatability of an operation (or of several operations). In this case it is important to take the decision to stop cyclicity [1]. Having the alternative of repeating both a separate subprocess, a group of subprocesses or a process as a whole is a characteristic feature of cyclic processes.

In modern literature simulation of cyclic economic processes occupies an important place [2]. Building adequate mathematical models of economic cyclic processes determines development and application of the latest information technologies for the problems of computer-aided analysis and prediction of economic cycles. Among the founders of the mathematical modeling and analysis of economic cycles there are such famous scholars as Slutskiy, Frisch, Louçã. A considerable number of scientific works deal with these problems, in particular [3, 4].

However, the problems of managing branched technological processes with repeatability (cyclicity) have not been sufficiently investigated yet.

The task of cyclic BTP simulation is complicated by the uncertainty of the number of cycle repetitions as well as by the dependence of the parameters of subsequent cycles on the parameters and characteristics of the previous ones. Hence, the task of solving the problem of cyclic BTP simulation is of **current importance.**

Running of separate subprocesses and their results depend on the input parameters of the production object and do not depend on how and by what means the production object parameters have been obtained. In cyclic BTP Running of one flow of operations could be influenced by cyclic flow of operations. Therefore, it could be assumed that it is possible to describe a cyclic BTP by the inhomogeneous Markov model [5, 6].

The model of a cyclic technological process management is based on inhomogeneous cyclic graphs and inhomogeneous Markov chains [6, 7].

The work **is aimed** at building an inhomogeneous Markov model for cyclic BTP and using this model for analyzing the influence of cyclicity on branched technological process management.

The algorithm of transforming the graph of a cyclic BTP into acyclic form

In order to simplify application of Markov model to cyclic processes, we propose to transform the cyclic graph of performing BCTP operations into the equivalent acyclic graph. A scheme of such transformation is shown in Fig. 1.

The transformation consists in representation of each of the cycle realizations as a branched part of the process (subprocess), transition to which is made as a result of taking decision at the end of the previous realization. Dotted arrows in Fig 1 show the influence of the operations of the previous cycle realization on the operations of subsequent realization.

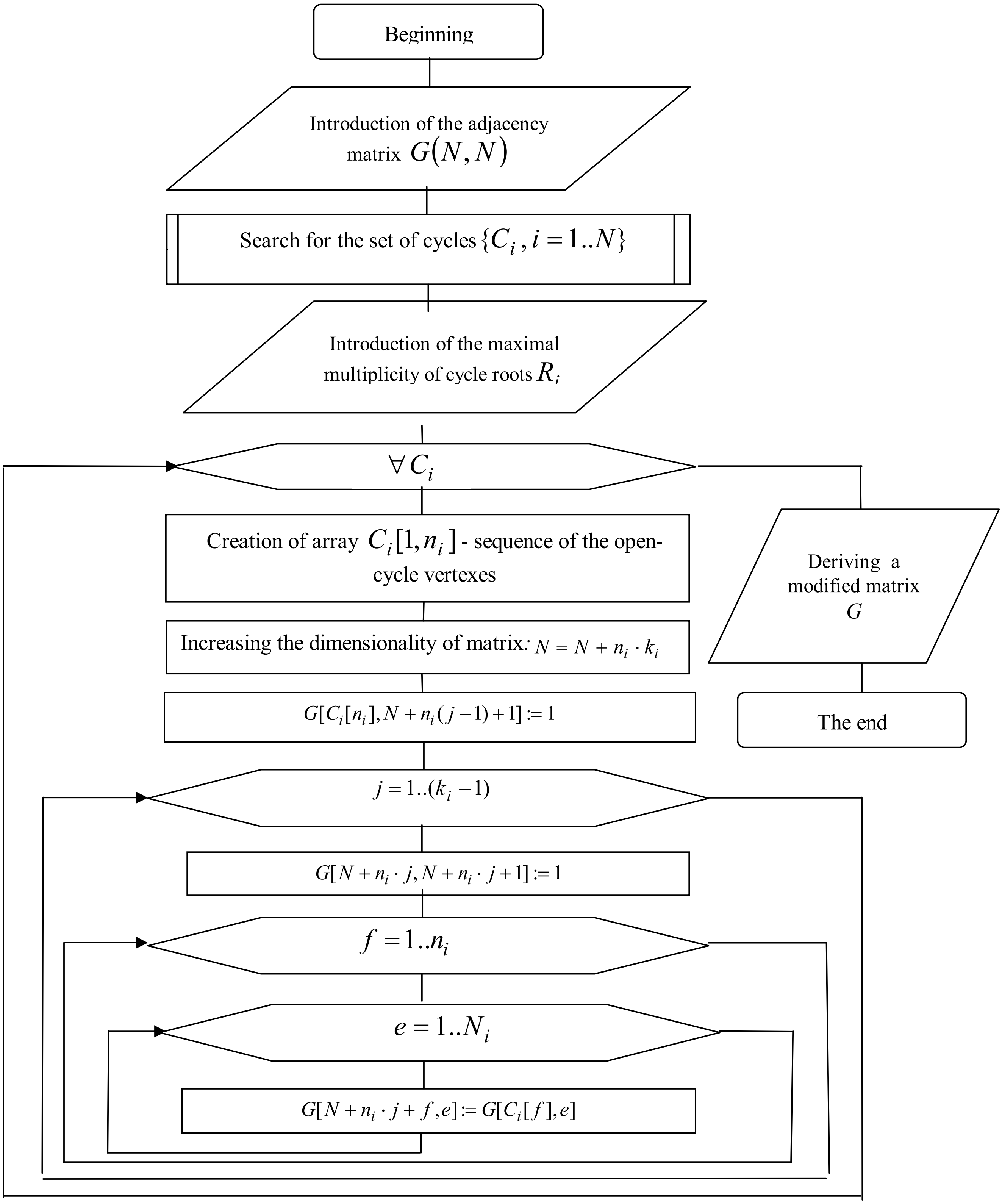


Fig. 2. The algorithm of transforming the BTP graph into acyclic form taking into account the permissible multiplicity of execution of subprocesses

On the basis of Fig.1 a method is developed for transforming the graph of a branched technological process into acyclic form taking into account the permissible multiplicity of performing subprocesses. The method makes it possible to build a structure of the branched technological process in the conditions of uncertainty of input parameters by means of using oriented graphs and taking into account multiplicity of the cycles. This results in the reduction of losses and increased efficiency of the technological process management.

The algorithm of a cyclic BCTP graph transformation is presented in Fig. 2 where the following designations are adopted: G – adjacency matrix of the graph; N – the number of vertexes; C_i – cycle; $\{ C_i \}$ – set of cycles; m – the number of cycles; n_i – the number of vertexes in the i-th cycle; k_i – maximal multiplicity of the i-th cycle repetitions.

Application of Markov BTP model to cyclic processes

Let us present application of Markov BTP model to cyclic processes taking into account the cyclic graph transformation into an equivalent acyclic form.

We present Markov model of the technological process as a set of Markov models of operations and subprocesses P_i. Inhomogeneous Markov model is taken as a basis [8].

Fig. 3 shows an example of the graph of operation state variations. The “state” is understood as a set of values of the operation parameters.

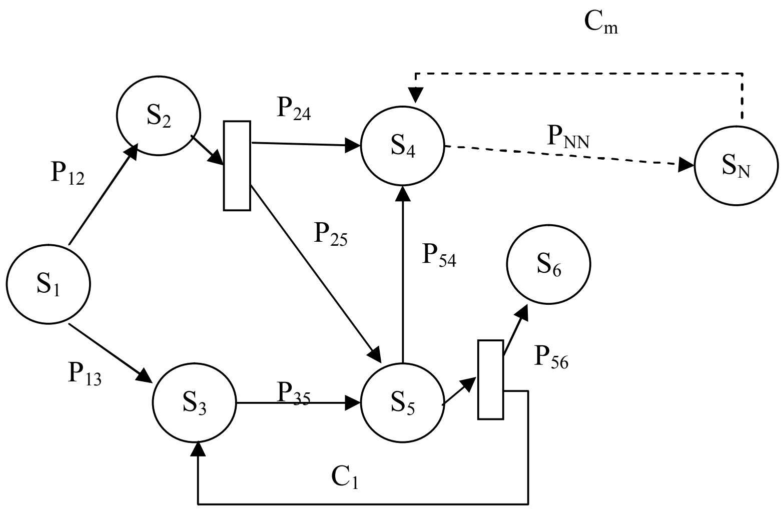


Fig. 3. The graph of operation state changes

Let m be a maximal number of states of cyclic BTP operation. Let us designate the probability of operation transition from state S_i to state S_j as b_{ij}. Then probability of the operation transition from one state to another will be described by the matrix

$$B = \begin{pmatrix} b_{11} & b_{12} & ... & b_{1m} \\ b_{21} & b_{22} & ... & b_{2m} \\ ... & ... & ... & ... \\ b_{m1} & b_{m2} & .... & b_{mm} \end{pmatrix}. \quad (1)$$

The scheme of the relationships between the states of the operations and the meaning of designations is presented in Fig. 4.

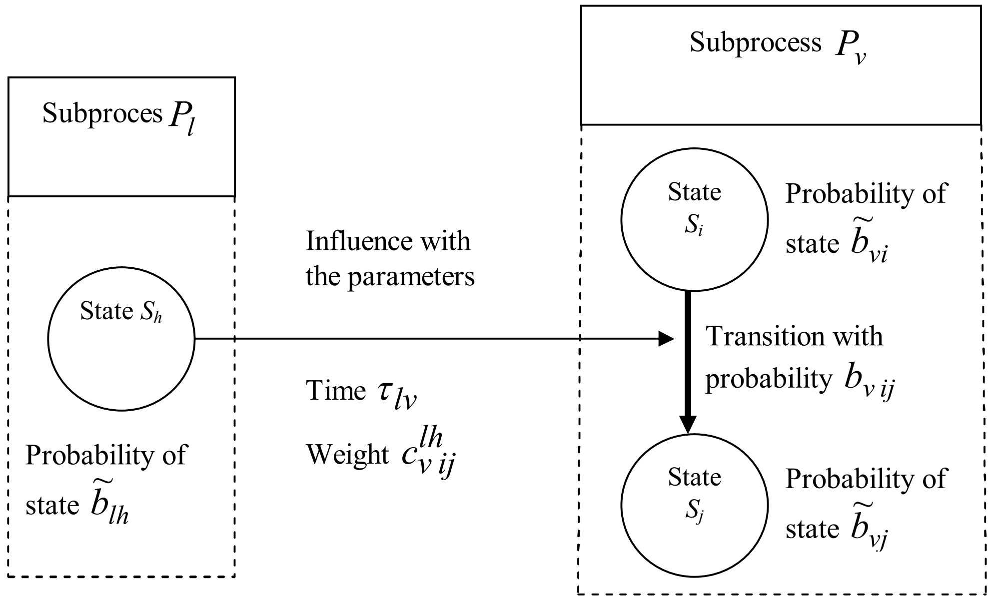


Fig. 4. The scheme of the relationships between Markov model parameters of BTP operations

Since transition probabilities are variables, which is determined by a mutual influence of operations of two sequential subprocesses, Markov model of cyclic BTP is inhomogeneous.

In [8] a linearized model is also presented for estimating probability of the state of two subprocesses.

$$\widetilde{b}_{vj} = \sum_{i=1}^{m} \left\{ \widetilde{b}_{vi} \cdot \left[c_{vij}^{00} + \sum_{l=1}^{n} \sum_{h=1}^{m} \left(c_{vij}^{lh} \cdot \widetilde{b}_{lh} \right) \right] \right\}, v = 1...n, \quad i, j = 1...m, \tag{2}$$

where $\widetilde{B}^{(k)}$ is a probability matrix of the states of subprocesses; C_v – four-dimensional array of weight coefficients $[n, m, n+1, m+1]$.

The matrix element $c_{vij}^{lh} \in C_v$ determines the influence of the *h*-th state of the *l*-th subprocess on the vector of transisiton probabilities of the *v*-th subprocess. Element c_{vij}^{00} is the probability of the *v*-th subprocess transition from the i-th to the j-th state without taking into account the influence of other subprocesses.

Taking into account a determinate character of the sequence of BCTP operations and the fact that the influence of the previous subprocess operation on the similar operation of the next subprocess is considered, i.e. $l = v - 1$, model (2) is simplified as:

$$\widetilde{b}_{vj} = \sum_{i=1}^{m} \left\{ \widetilde{b}_{vi} \cdot \left[c_{vij}^{00} + \sum_{h=1}^{m} \left(c_{vij}^{(v-1),h} \cdot \widetilde{b}_{(v-1),h} \right) \right] \right\}.$$

Let us consider the number of the operation state as a vector of parameters *X*. Then model (2) will have the form of

$$\widetilde{b}_v\left(X_j\right) = \sum_{i=1}^{m} \left\{ \widetilde{b}_v\left(X_i\right) \cdot \left[c_v^{00}\left(X_i, X_j\right) + \sum_{h=1}^{m} \left(c_v^{v-1,h}\left(X_i, X_j\right) \cdot \widetilde{b}_{v-1}\left(X_h\right) \right) \right] \right\}. \tag{3}$$

If the influence of subprocess v-2 is taken into account, then

$$\widetilde{b}_{v-1}(X_h)=\sum_{i=1}^{m}\left\{\widetilde{b}_{v-1}(X_i)\cdot\left[c_{v-1}^{00}(X_i,X_h)+\sum_{r=1}^{m}\left(c_{v-1}^{v-2,r}(X_i,X_h)\cdot\widetilde{b}_{v-2}(X_r)\right)\right]\right\}. \tag{4}$$

If the relations (3) and (4) are the models of different realizationsя of the same subprocess, we can write

$$c_v^{00}\left(X_i,X_j\right)=c_{v-1}^{00}\left(X_i,X_j\right)=c_1;$$

$$c_v^{v-1,h}\left(X_i,X_j\right)=c_{v-1}^{v-2,h}\left(X_i,X_j\right)=c_2. \tag{5}$$

Substituting (4) into (3) and taking into account (5), we obtain

$$\widetilde{b}_v\left(X_j\right)=\sum_{i=1}^{m}\left\{\widetilde{b}_v\cdot\left[c_1+\sum_{h=1}^{m}\left(c_2\cdot\sum_{i=1}^{m}\{\widetilde{b}_{v-1}\cdot[c_1+\sum_{r=1}^{m}(c_2\cdot\sum_{i=1}^{m}\{\widetilde{b}_{v-2}\cdot[c_1+\sum_{r=1}^{m}(c_2\cdot\widetilde{b}_{v-3})]\})]\}\right)\right]\right\}. \tag{6}$$

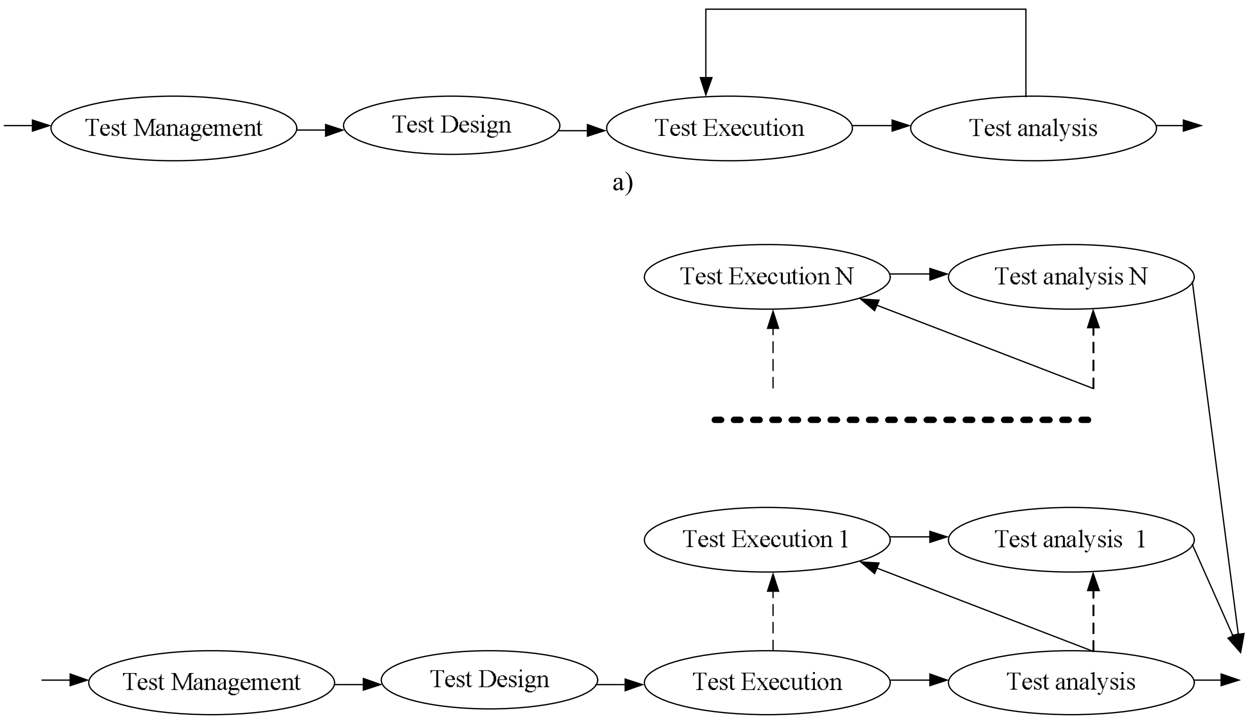
After generalization of (6), we obtain

$$\widetilde{b}_v\left(X_j\right)=\sum_{i=1}^{m}\sum_{h=1}^{v+1}\left\{\overbrace{\sum\cdots\sum}^{(h-1)}c_1c_2^{h-1}\prod_{r=0}^{h-1}\widetilde{b}_{v-h}\left(X_i\right)\right\}.$$

On the basis of Markov model the risk of cyclic BTP realization could be determined, which will make it possible to improve the quality of taking decisions connected with the management of branched cyclic technological processes.

Markov model application to the software testing process

The proposed model has been applied to the software testing process as a branched cyclic technological process. The scheme of the process is presented as a graph in Fig. 5.



b)

Fig. 5. Software testing process:
a) cyclic graph of the process;
b) cyclic graph transformation into acyclic form

Testing is one of the quality control techniques that includes:

– Test Management;
– Test Design;
– Test Execution;
– Test Analysis [9].

These four stages are operations of the software testing process. Each operation contains states that vary depending on the input parameters and the parameters of the previous operation execution. In particular, "Test Management" contains the following parameters that influence the state of the operation:

– information about software structure or system in documentation ("white box");
– test data sets for verifying proper operation of the components and the system as a whole without the knowledge of their structure ("black box");
– limit values, decision taking tables, data flows, statistics of failures, etc.;
– block-diagrams of building programs and test suites for covering the system with these tests, etc.

_"Test execution" - "Test Analysis " subprocess is executed cyclically until the minimal number of defects is revealed. In this case, after the first cycle of the subprocess execution "Test execution" operation will be influenced by the previous cycle result, i. e. the result of "Test Analysis" operation. The input data for the next cycle execution will vary since they depend on the results of the previous cycle process execution.

Depending on how the defect is found, the state of the defect revealing operation will vary:

1) we learn (or have already learned) the expected result;
2) we learn (or have already learned) the actual result;
3) we equalize points 1 and 2.

The state of the operation will also be determined by test data which are used for checking the system operation and are derived in different ways: by test data generator, by the design group on the basis of the documents or the existing files, by the user from the requirement specification, etc.

On the basis of the known scheme of the relationships between Markov model of BTP parameters, we will give an example of calculating the system subprocess state probability for several operations.

We have 2 subprocesses P_1 and P_2. Let subprocess P_1 has one state – S_1 with probability $\widetilde{b}_{11}$ and subprocess P_2 has two states – S_1, S_2 with respective probabilities $\widetilde{b}_{21}$ and $\widetilde{b}_{22}$. State S_1 of subprocess P_2 passes to state S_2 with probability $b_{2\,12}$. State S_1 of subprocess P_1 influences the transition between two states S_1 - S_2 of subprocess P_2: the influence with time τ_{12}, the ifluence with weight $c^{11}_{2\,12}$. Let us compose a table of the values (Table 1).

Table 1

Parameters of the Markov model of software testing BCTP

Cycle	Operation	State	State probability	Influence		Transition with the probability
				time	weight	
P_1	Operation 1	S_1	$b_{11} = 0{,}2$	$\tau_{12} = 2$	$c^{11}_{2\,12} = 0{,}3$	$b_{2\,12} = 0{,}7$
P_2	Operation 1	S_1	$b_{21} = 0{,}4$			
		S_2	$b_{22} = 0{,}6$			

Let us calculate the probability of the second operation (P_2) being in the second state (S_2) after execution of 4 cycles:

$$\widetilde{b}_{22}^{(4)} = \widetilde{b}_{21}^{(4-1)} \cdot \left[b_{11} + \left(c^{12}_{2\,12} \cdot \widetilde{b}_{11}^{(4-2)}\right)\right];$$

$$\tilde{b}_{22}^{(4)} = (0,4)^3 \cdot (0,2 + 0,3 \cdot 0,2^2) = 0,013568 .$$

After 3 cycles:

$$\tilde{b}_{22}^{(3)} = (0,4)^2 \cdot (0,2 + 0,3 \cdot 0,2^1) = 0,0416 .$$

After 2 cycles:

$$\tilde{b}_{22}^{(2)} = (0,4)^1 \cdot (0,2 + 0,3 \cdot 0,2^0) = 0,2 .$$

Let us show that probability of the state of operation P_2 depends on the number of execution cycles (Fig. 6).

Number of cycles	Probability
1	0.2
2	0.2
3	0.0416
4	0.01
5	0.00518144
6	0.002052915
7	0.000819593
8	0.000327711

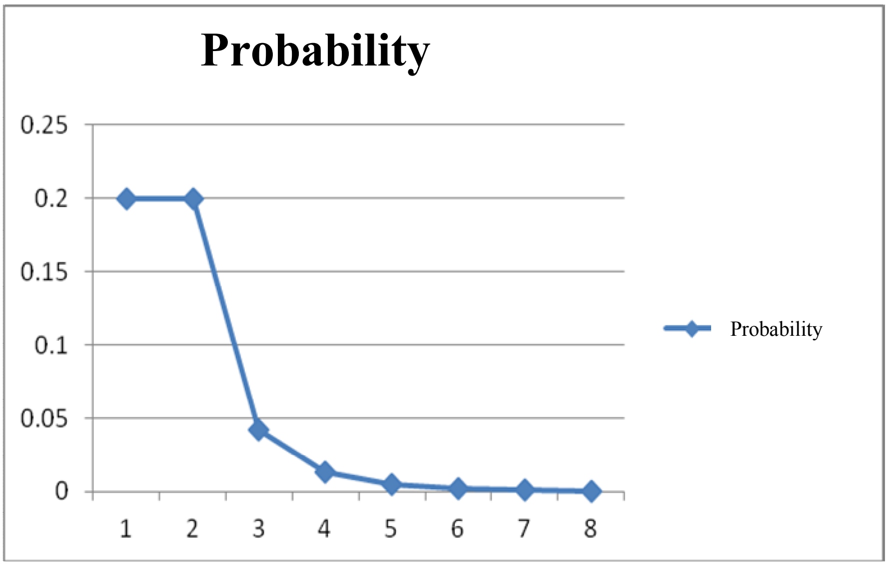


Fig. 6. Dependence of the probability of subprocess P_2 state on the number of execution operations

Let us build graphs for different Markov model parameters and analyze their nature.

E. g., we shall vary the Markov model parameter, the influence with weight $c_{2\,12}^{11}$ (Fig. 7).

$c_{2\,12}^{11}=0,5$

Number of cycles	Probability
1	0.2
2	0.28
3	0.048
4	0.01
5	0.0052224
6	0.002056192
7	0.000819855
8	0.000327732

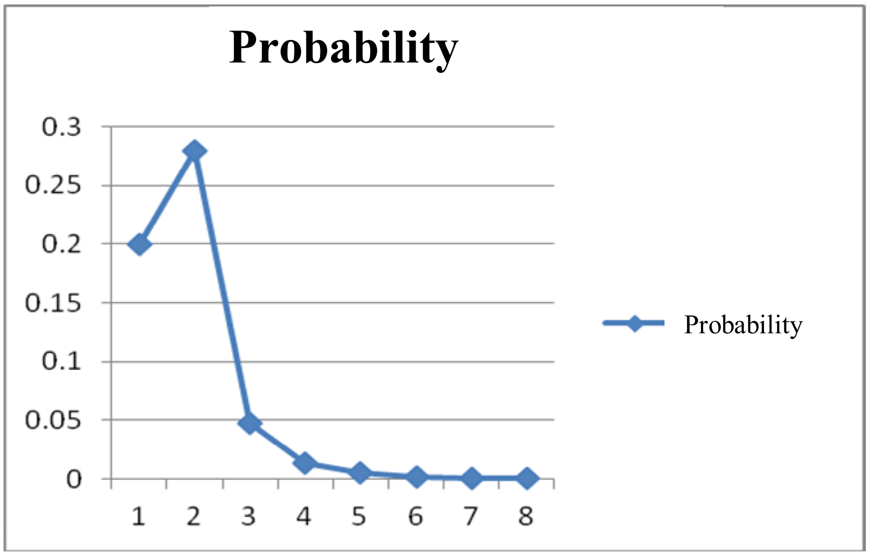
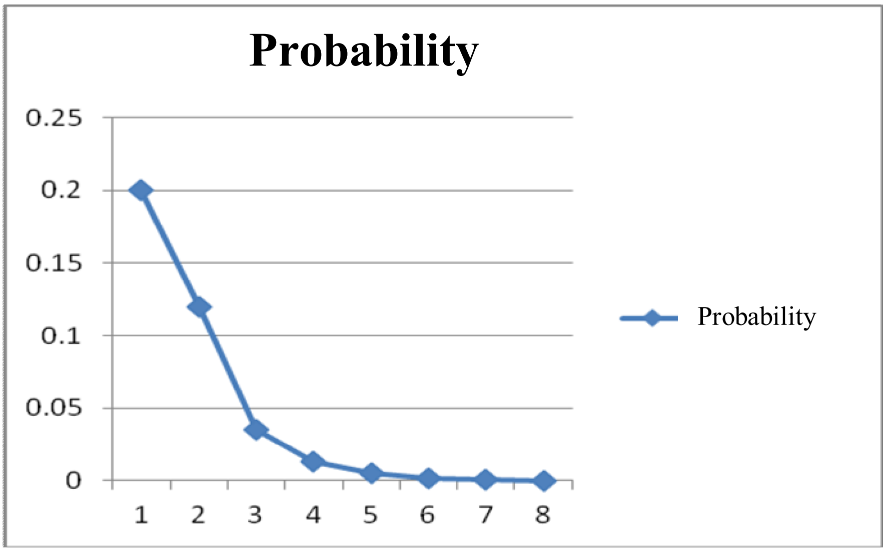


Fig. 7. Dependence of the probability of P_2 operation state on the number of execution cycles for the variable influence weight parameter

$c_{2\,12}^{11}$ =0,1

Number of cycles	Probability
1	0.2
2	0.12
3	0.0352
4	0.01
5	0.00514048
6	0.002049638
7	0.000819331
8	0.00032769



$c_{2\,12}^{11}$ =0,9

Number of cycles	Probability
1	0.2
2	0.44
3	0.0608
4	0.02
5	0.00530432
6	0.002062746
7	0.00082038
8	0.000327774

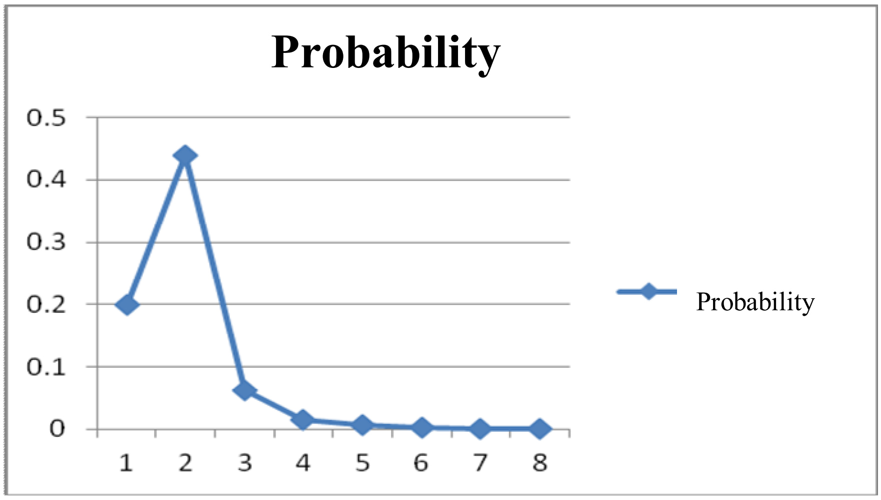


Fig. 7. Dependence of the probability of P_2 operation state on the number of cycles for the variable influence weight parameter (continuation)

Hence, with growing number of subprocess execution cycles, the influence on probability of the state increases. It is determined by the mutual influence of subprocesses, which causes inhomogeneity of Markov chain.

For this case state S_1 is desirable for operation P_2 and state S_2 is undesirable. The probability of undesirable state S_2 reduces as the number of subprocess execution cycles grows.

Conclusions

Inhomogeneous Markov model for cyclic BTP has been built. This model is proposed to be used for analyzing cyclicity influence on the management of branched technological processes. The obtained acyclic graph together with the inhomogeneous Markov model makes it possible to evaluate the risk of cyclic RTP realization variants and to select the variant having minimal risk.

Application of such models under uncertainty conditions is prospective for wide-class BCTP management.

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