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ALGEBRAIC CONSTRUCTION OF THE POLYNOMIAL BASIS OF A SIX-NODE OCTAHEDRON

The paper presents second-degree polynomial bases for the finite element in the form of an octahedron constructed by means of matrix calculus. The obtained functions are harmonics according to Laplace. Geometric properties of the corresponding level surfaces are analyzed.

Key words: *six-node octahedron, basis, level surfaces.*

Introduction

For numerical solution of the problems by the method of discrete elements tetrahedrons and hexahedrons are widely used. Recently, octahedrons in combination with tetrahedrons came to be used in dimensional lattices. Consequently, it became necessary to equip the octahedron, as a final element, with its own system of basis functions.

Analysis of prior publications, the goal of the paper

The problem of building the basis of a seven-node octahedron with the nodes located in its vertexes and barycentre is solved in [1, 2]. The authors of [1] propose a piecewise-linear basis of the octahedron without giving the details of its construction and apply it for approximation of analytical functions in the volumetric visualization problems. The author of [2] builds a quadratic basis using Taylor expansion of the function of three independent variables in combination with the finite-difference schemes. Construction of a hexahedron and an inscribed octahedron is used to solve the problems of potential flow of ideal fluids in 3D. In some problems the central node is not required, e.g. in the generalized Dirichlet problem for Laplace equation on the octahedron with discretely given conditions on the boundary. Therefore, the idea emerged to build a six-node octahedron. Initial results are presented in [3]. Two bases of the six-node octahedron model (piecewise-linear and quadratic ones) are obtained from the corresponding bases of the seven-node model using the condensation procedure.

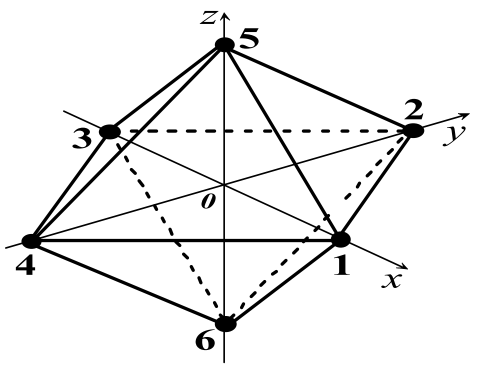
The goal of the paper is to build a six-node polynomial basis (bases) of an octahedron by means of matrix calculus and to investigate its geometrical properties.

The main part

Let us consider an octahedron with the center located in the origin of Oxyz coordinate system and vertexes (nodes) being the tangent points of the octahedron of a unit-radius circumscribed sphere. The coordinate axes are directed as shown in fig. 1. Let us assume that a certain physical quantity Φ (e.g. the temperature) is defined at each point of the octahedron and the values of function $\Phi(x, y, z)$ in the octahedron nodes are known and equal to $\Phi_i \left(i = \overline{1,6}\right)$. The problem is formulated as follows: to find basis functions N_i, $i = \overline{1,6}$ that satisfy the following conditions:

$$N_i(x_k, y_k, z_k) = \delta_{ik}, \quad i,k = \overline{1,6}, \qquad \sum_{i=1}^{6} N_i = 1, \tag{1}$$

where $\delta_{i,k}$ – Kronecker symbol, i – the number of a function, k – the number of a node.



$$1-|x|-|y|-|z|\geq 0.$$

Fig. 1. Octahedron with 6 interpolation nodes

Let us compose the interpolation polynomial for function $\Phi(x,y,z)$ in the form of a complete second-degree polynomial:

$$\Phi=\alpha_1+\alpha_2 x+\alpha_3 y+\alpha_4 z+\alpha_5 x^2+\alpha_6 y^2+\alpha_7 z^2+\alpha_8 xy+\alpha_9 yz+\alpha_{10} xz\ . \tag{2}$$

According to Lagrange interpolation formula, at any interior point of the octahedron the value of function Φ is given by

$$\Phi=\sum_{i=1}^{6} N_i\Phi_i, \tag{3}$$

where $N_i(x,y,z)$ are polynomial basis functions of the Lagrange type. From conditions (1), (3) it is evident that in the octahedron center the value of interpolation polynomial $\Phi(0{,}0{,}0)=\frac{1}{6}\sum_{i=1}^{6}\Phi_i$. The last equality makes it possible to define the free coefficient α_1 of formula (2) as $\alpha_1=\frac{1}{6}\sum_{i=1}^{6}\Phi_i$. The remaining nine unknowns $\alpha_i\left(i=\overline{2,10}\right)$ are found from the system of linear algebraic equations:

$$\begin{cases}\Phi_{1,3}=\frac{1}{6}\sum\limits_{i=1}^{6}\Phi_i\pm\alpha_2+\alpha_5+0\cdot\alpha_8+0\cdot\alpha_9+0\cdot\alpha_{10};\\ \Phi_{2,4}=\frac{1}{6}\sum\limits_{i=1}^{6}\Phi_i\pm\alpha_3+\alpha_6+0\cdot\alpha_8+0\cdot\alpha_9+0\cdot\alpha_{10};\\ \Phi_{5,6}=\frac{1}{6}\sum\limits_{i=1}^{6}\Phi_i\pm\alpha_4+\alpha_7+0\cdot\alpha_8+0\cdot\alpha_9+0\cdot\alpha_{10}.\end{cases} \tag{4}$$

Investigating system (4) for compatibility, we conclude that the system is uncertain and the unknowns $\alpha_i\left(i=\overline{2,7}\right)$ are uniquely determined by the system of equalities:

$$\begin{cases} \alpha_2 = \frac{1}{2}\Phi_1 - \frac{1}{2}\Phi_3; \\ \alpha_3 = \frac{1}{2}\Phi_2 - \frac{1}{2}\Phi_4; \\ \alpha_4 = \frac{1}{2}\Phi_5 - \frac{1}{2}\Phi_6; \\ \alpha_5 = \frac{1}{2}\Phi_1 + \frac{1}{2}\Phi_3 - \frac{1}{6}\sum_{i=1}^{6}\Phi_i; \\ \alpha_6 = \frac{1}{2}\Phi_2 + \frac{1}{2}\Phi_4 - \frac{1}{6}\sum_{i=1}^{6}\Phi_i; \\ \alpha_7 = \frac{1}{2}\Phi_5 + \frac{1}{2}\Phi_6 - \frac{1}{6}\sum_{i=1}^{6}\Phi_i. \end{cases} \quad (5)$$

The unknowns e $\alpha_8, \alpha_9, \alpha_{10}$ can take any arbitrary real values. Hence, interpolation polynomial (2) can be represented in the form of

$$\Phi = \sum_{i=1}^{6} N_i^0(x,y,z)\Phi_i + \alpha_8 xy + \alpha_9 yz + \alpha_{10} xz, \quad (6)$$

where functions $N_i^0\left(i=\overline{1,6}\right)$ are defined as:

$$\begin{aligned} N_{1,3}^0 &= \frac{1}{6} \pm \frac{1}{2}x + \frac{1}{3}x^2 - \frac{1}{6}y^2 - \frac{1}{6}z^2; \\ N_{2,4}^0 &= \frac{1}{6} \pm \frac{1}{2}y + \frac{1}{3}y^2 - \frac{1}{6}x^2 - \frac{1}{6}z^2; \\ N_{5,6}^0 &= \frac{1}{6} \pm \frac{1}{2}z + \frac{1}{3}z^2 - \frac{1}{6}x^2 - \frac{1}{6}y^2. \end{aligned} \quad (7)$$

As the equality $\alpha_8 xy + \alpha_9 yz + \alpha_{10} xz = 0$ is valid at any node i of the octahedron and does not depend on the coefficients $\alpha_8, \alpha_9, \alpha_{10}$, then assuming $\alpha_j = \sum_{i=1}^{6}\alpha_j^{(i)}\Phi_i$ $(j = 8,9,10)$, where $\alpha_j^{(i)}\left(i=\overline{1,6}\right)$ – arbitrary real values, interpolation polynomial (6) can be represented in the form of (3), where

$$\begin{aligned} N_{1,3} &= N_{1,3}^0 + \alpha_8^{(1),(3)}xy + \alpha_9^{(1),(3)}yz + \alpha_{10}^{(1),(3)}xz; \\ N_{2,4} &= N_{2,4}^0 + \alpha_8^{(2),(4)}xy + \alpha_9^{(2),(4)}yz + \alpha_{10}^{(2),(4)}xz; \\ N_{5,6} &= N_{5,6}^0 + \alpha_8^{(5),(6)}xy + \alpha_9^{(5),(6)}yz + \alpha_{10}^{(5),(6)}xz. \end{aligned} \quad (8)$$

Verification of condition $\sum_{i=1}^{6} N_i = 1$ shows that equality

$$\sum_{i=1}^{6}\left(\alpha_8^{(i)}xy + \alpha_9^{(i)}yz + \alpha_{10}^{(i)}xz\right) = 0 \quad (9)$$

is satisfied at any interior point (x,y,z) of the octahedron. And this is possible if

$$\sum_{i=1}^{6}\alpha_8^{(i)} = \sum_{i=1}^{6}\alpha_9^{(i)} = \sum_{i=1}^{6}\alpha_{10}^{(i)} = 0. \quad (10)$$

The obtained functions (8) are harmonic according to Laplace and coincide with the quadratic basis of the six-node octahedron [3] if $\alpha_8^{(i)} = \alpha_9^{(i)} = \alpha_{10}^{(i)} = 0, i = \overline{1,6}$.

Taking into account geometric isotropy of the octahedron, depending on the type of symmetry different means to choose the coefficients $\alpha_8^{(i)}$, $\alpha_9^{(i)}$, $\alpha_{10}^{(i)}$, $i=\overline{1,6}$ of basis functions could be suggested for both the nodes located on the same coordinate axis and for the nodes located on different coordinate axes – one per each axis. If dissymmetry, axial and central symmetries are used for the construction of form functions, we obtain four rules:

Table 1

(I): $\begin{cases} N_3(x,y,z)=N_1(-x,y,z); \\ N_4(x,y,z)=N_2(x,-y,z); \\ N_6(x,y,z)=N_5(x,y,-z). \end{cases}$	(II): $\begin{cases} N_3(x,y,z)=N_1(-x,-y,z); \\ N_4(x,y,z)=N_2(x,-y,-z); \\ N_6(x,y,z)=N_5(-x,y,-z). \end{cases}$
(III): $\begin{cases} N_3(x,y,z)=N_1(-x,y,-z); \\ N_4(x,y,z)=N_2(-x,-y,z); \\ N_6(x,y,z)=N_5(x,-y,-z). \end{cases}$	(IV): $\begin{cases} N_3(x,y,z)=N_1(-x,-y,-z); \\ N_4(x,y,z)=N_2(-x,-y,-z); \\ N_6(x,y,z)=N_5(-x,-y,-z). \end{cases}$

For neighboring nodes that belong to one (any) of the octahedron faces there are four different variants of the permutation of coordinates in transition from one function to another, which are realized by means of affine transformations of Oxyz system and namely:

Table 2

1: $(x,y,z)\to(y,z,x)$	2: $(x,y,z)\to(-y,z,x)$	3: $(x,y,z)\to(y,-z,x)$	4: $(x,y,z)\to(y,z,-x)$

It is evident that for any set of rules (one from each table 1 and 2) the number of free coefficients $\alpha_8^{(i)}$, $\alpha_9^{(i)}$, $\alpha_{10}^{(i)}$, $i=\overline{1,6}$ can be reduced to three β_i, $i=\overline{1,3}$,where $\beta_1=\alpha_8^{(1)}$, $\beta_2=\alpha_9^{(1)}$, $\beta_3=\alpha_{10}^{(1)}$.

Table 3 presents six-node polynomial bases of the octahedron obtained by combining the rules from tables 1 and 2:

Table 3

Configuration of the rules	Basis functions	Restrictions for	№ of the formula
(I,1)	$N_{1,3}=\frac{1}{6}+\frac{1}{3}x^2\pm\frac{1}{2}x-\frac{1}{6}y^2-\frac{1}{6}z^2\pm\beta_1xy\pm\beta_2yz\pm\beta_3xz;$ $N_{2,4}=\frac{1}{6}+\frac{1}{3}y^2\pm\frac{1}{2}y-\frac{1}{6}z^2-\frac{1}{6}x^2\pm\beta_3xy\pm\beta_1yz\pm\beta_2xz;$ $N_{5,6}=\frac{1}{6}+\frac{1}{3}z^2\pm\frac{1}{2}z-\frac{1}{6}x^2-\frac{1}{6}y^2\pm\beta_2xy\pm\beta_3yz\pm\beta_1xz.$	$\beta_i\in R$	(11)
(I,2)	$N_{1,3}=\frac{1}{6}+\frac{1}{3}x^2\pm\frac{1}{2}x-\frac{1}{6}y^2-\frac{1}{6}z^2\pm\beta_1xy\pm\beta_2yz\pm\beta_3xz;$ $N_{2,4}=\frac{1}{6}+\frac{1}{3}y^2\pm\frac{1}{2}y-\frac{1}{6}z^2-\frac{1}{6}x^2\mp\beta_3xy\mp\beta_1yz\pm\beta_2xz;$ $N_{5,6}=\frac{1}{6}+\frac{1}{3}z^2\pm\frac{1}{2}z-\frac{1}{6}x^2-\frac{1}{6}y^2\mp\beta_2xy\pm\beta_3yz\mp\beta_1xz.$	$\beta_i\in R$	(12)
(I,3)	$N_{1,3}=\frac{1}{6}+\frac{1}{3}x^2\pm\frac{1}{2}x-\frac{1}{6}y^2-\frac{1}{6}z^2\pm\beta_1xy\pm\beta_2yz\pm\beta_3xz;$ $N_{2,4}=\frac{1}{6}+\frac{1}{3}y^2\pm\frac{1}{2}y-\frac{1}{6}z^2-\frac{1}{6}x^2\pm\beta_3xy\mp\beta_1yz\mp\beta_2xz;$	$\beta_i\in R$	(13)

	$N_{5,6}=\frac{1}{6}+\frac{1}{3}z^2\pm\frac{1}{2}z-\frac{1}{6}x^2-\frac{1}{6}y^2\mp\beta_2xy\mp\beta_3yz\pm\beta_1xz.$		
(I,4)	$N_{1,3}=\frac{1}{6}+\frac{1}{3}x^2\pm\frac{1}{2}x-\frac{1}{6}y^2-\frac{1}{6}z^2\pm\beta_1xy\pm\beta_2yz\pm\beta_3xz;$ $N_{2,4}=\frac{1}{6}+\frac{1}{3}y^2\pm\frac{1}{2}y-\frac{1}{6}z^2-\frac{1}{6}x^2\mp\beta_3xy\pm\beta_1yz\mp\beta_2xz;$ $N_{5,6}=\frac{1}{6}+\frac{1}{3}z^2\pm\frac{1}{2}z-\frac{1}{6}x^2-\frac{1}{6}y^2\pm\beta_2xy\mp\beta_3yz\mp\beta_1xz.$	$\beta_i\in R$	(14)
(II,1)	$N_{1,3}=\frac{1}{6}+\frac{1}{3}x^2\pm\frac{1}{2}x-\frac{1}{6}y^2-\frac{1}{6}z^2\pm\beta_2yz\pm\beta_3xz;$ $N_{2,4}=\frac{1}{6}+\frac{1}{3}y^2\pm\frac{1}{2}y-\frac{1}{6}z^2-\frac{1}{6}x^2\pm\beta_3xy\pm\beta_2xz;$ $N_{5,6}=\frac{1}{6}+\frac{1}{3}z^2\pm\frac{1}{2}z-\frac{1}{6}x^2-\frac{1}{6}y^2\pm\beta_2xy\pm\beta_3yz.$	$\beta_1=0,$ $\beta_2,\beta_3\in R$	(15)
(II,2)	$N_{1,3}=\frac{1}{6}+\frac{1}{3}x^2\pm\frac{1}{2}x-\frac{1}{6}y^2-\frac{1}{6}z^2\pm\beta_2yz\pm\beta_3xz;$ $N_{2,4}=\frac{1}{6}+\frac{1}{3}y^2\pm\frac{1}{2}y-\frac{1}{6}z^2-\frac{1}{6}x^2\mp\beta_3xy\pm\beta_2xz;$ $N_{5,6}=\frac{1}{6}+\frac{1}{3}z^2\pm\frac{1}{2}z-\frac{1}{6}x^2-\frac{1}{6}y^2\mp\beta_2xy\pm\beta_3yz.$	$\beta_1=0,$ $\beta_2,\beta_3\in R$	(16)
(II,3)	$N_{1,3}=\frac{1}{6}+\frac{1}{3}x^2\pm\frac{1}{2}x-\frac{1}{6}y^2-\frac{1}{6}z^2\pm\beta_2yz\pm\beta_3xz;$ $N_{2,4}=\frac{1}{6}+\frac{1}{3}y^2\pm\frac{1}{2}y-\frac{1}{6}z^2-\frac{1}{6}x^2\pm\beta_3xy\mp\beta_2xz;$ $N_{5,6}=\frac{1}{6}+\frac{1}{3}z^2\pm\frac{1}{2}z-\frac{1}{6}x^2-\frac{1}{6}y^2\mp\beta_2xy\mp\beta_3yz.$	$\beta_1=0,$ $\beta_2,\beta_3\in R$	(17)
(II,4)	$N_{1,3}=\frac{1}{6}+\frac{1}{3}x^2\pm\frac{1}{2}x-\frac{1}{6}y^2-\frac{1}{6}z^2\pm\beta_2yz\pm\beta_3xz;$ $N_{2,4}=\frac{1}{6}+\frac{1}{3}y^2\pm\frac{1}{2}y-\frac{1}{6}z^2-\frac{1}{6}x^2\mp\beta_3xy\mp\beta_2xz;$ $N_{5,6}=\frac{1}{6}+\frac{1}{3}z^2\pm\frac{1}{2}z-\frac{1}{6}x^2-\frac{1}{6}y^2\pm\beta_2xy\mp\beta_3yz.$	$\beta_1=0,$ $\beta_2,\beta_3\in R$	(18)
(III,1)	$N_{1,3}=\frac{1}{6}+\frac{1}{3}x^2\pm\frac{1}{2}x-\frac{1}{6}y^2-\frac{1}{6}z^2\pm\beta_1xy\pm\beta_2yz;$ $N_{2,4}=\frac{1}{6}+\frac{1}{3}y^2\pm\frac{1}{2}y-\frac{1}{6}z^2-\frac{1}{6}x^2\pm\beta_1yz\pm\beta_2xz;$ $N_{5,6}=\frac{1}{6}+\frac{1}{3}z^2\pm\frac{1}{2}z-\frac{1}{6}x^2-\frac{1}{6}y^2\pm\beta_2xy\pm\beta_1xz.$	$\beta_3=0,$ $\beta_1,\beta_2\in R$	(19)
(III,2)	$N_{1,3}=\frac{1}{6}+\frac{1}{3}x^2\pm\frac{1}{2}x-\frac{1}{6}y^2-\frac{1}{6}z^2\pm\beta_1xy\pm\beta_2yz;$ $N_{2,4}=\frac{1}{6}+\frac{1}{3}y^2\pm\frac{1}{2}y-\frac{1}{6}z^2-\frac{1}{6}x^2\mp\beta_1yz\pm\beta_2xz;$ $N_{5,6}=\frac{1}{6}+\frac{1}{3}z^2\pm\frac{1}{2}z-\frac{1}{6}x^2-\frac{1}{6}y^2\mp\beta_2xy\mp\beta_1xz.$	$\beta_3=0,$ $\beta_1,\beta_2\in R$	(20)

(III,3)	$N_{1,3}=\frac{1}{6}+\frac{1}{3}x^2\pm\frac{1}{2}x-\frac{1}{6}y^2-\frac{1}{6}z^2\pm\beta_1 xy\pm\beta_2 yz;$ $N_{2,4}=\frac{1}{6}+\frac{1}{3}y^2\pm\frac{1}{2}y-\frac{1}{6}z^2-\frac{1}{6}x^2\mp\beta_1 yz\mp\beta_2 xz;$ $N_{5,6}=\frac{1}{6}+\frac{1}{3}z^2\pm\frac{1}{2}z-\frac{1}{6}x^2-\frac{1}{6}y^2\mp\beta_2 xy\pm\beta_1 xz.$	$\beta_3=0,$ $\beta_1,\beta_2\in R$	(21)
(III,4)	$N_{1,3}=\frac{1}{6}+\frac{1}{3}x^2\pm\frac{1}{2}x-\frac{1}{6}y^2-\frac{1}{6}z^2\pm\beta_1 xy\pm\beta_2 yz;$ $N_{2,4}=\frac{1}{6}+\frac{1}{3}y^2\pm\frac{1}{2}y-\frac{1}{6}z^2-\frac{1}{6}x^2\pm\beta_1 yz\mp\beta_2 xz;$ $N_{5,6}=\frac{1}{6}+\frac{1}{3}z^2\pm\frac{1}{2}z-\frac{1}{6}x^2-\frac{1}{6}y^2\pm\beta_2 xy\mp\beta_1 xz.$	$\beta_3=0,$ $\beta_1,\beta_2\in R$	(22)
(IV,1)	$N_{1,3}=\frac{1}{6}+\frac{1}{3}x^2\pm\frac{1}{2}x-\frac{1}{6}y^2-\frac{1}{6}z^2+\beta_1 xy+\beta_2 yz+\beta_3 xz;$ $N_{2,4}=\frac{1}{6}+\frac{1}{3}y^2\pm\frac{1}{2}y-\frac{1}{6}z^2-\frac{1}{6}x^2+\beta_3 xy+\beta_1 yz+\beta_2 xz;$ $N_{5,6}=\frac{1}{6}+\frac{1}{3}z^2\pm\frac{1}{2}z-\frac{1}{6}x^2-\frac{1}{6}y^2+\beta_2 xy+\beta_3 yz+\beta_1 xz.$	$\beta_1+\beta_2+\beta_3=0,$ $\beta_i\in R$	(23)
(IV,2)	$N_{1,3}=\frac{1}{6}+\frac{1}{3}x^2\pm\frac{1}{2}x-\frac{1}{6}y^2-\frac{1}{6}z^2+\beta_1 xy+\beta_2 yz+\beta_3 xz;$ $N_{2,4}=\frac{1}{6}+\frac{1}{3}y^2\pm\frac{1}{2}y-\frac{1}{6}z^2-\frac{1}{6}x^2-\beta_3 xy-\beta_1 yz+\beta_2 xz;$ $N_{5,6}=\frac{1}{6}+\frac{1}{3}z^2\pm\frac{1}{2}z-\frac{1}{6}x^2-\frac{1}{6}y^2-\beta_2 xy+\beta_3 yz-\beta_1 xz.$	$\beta_1-\beta_2-\beta_3=0,$ $\beta_i\in R$	(24)
(IV,3)	$N_{1,3}=\frac{1}{6}+\frac{1}{3}x^2\pm\frac{1}{2}x-\frac{1}{6}y^2-\frac{1}{6}z^2+\beta_1 xy+\beta_2 yz+\beta_3 xz;$ $N_{2,4}=\frac{1}{6}+\frac{1}{3}y^2\pm\frac{1}{2}y-\frac{1}{6}z^2-\frac{1}{6}x^2+\beta_3 xy-\beta_1 yz-\beta_2 xz;$ $N_{5,6}=\frac{1}{6}+\frac{1}{3}z^2\pm\frac{1}{2}z-\frac{1}{6}x^2-\frac{1}{6}y^2-\beta_2 xy-\beta_3 yz+\beta_1 xz.$	$\beta_1-\beta_2+\beta_3=0,$ $\beta_i\in R$	(25)
(IV,4)	$N_{1,3}=\frac{1}{6}+\frac{1}{3}x^2\pm\frac{1}{2}x-\frac{1}{6}y^2-\frac{1}{6}z^2+\beta_1 xy+\beta_2 yz+\beta_3 xz;$ $N_{2,4}=\frac{1}{6}+\frac{1}{3}y^2\pm\frac{1}{2}y-\frac{1}{6}z^2-\frac{1}{6}x^2-\beta_3 xy+\beta_1 yz-\beta_2 xz;$ $N_{5,6}=\frac{1}{6}+\frac{1}{3}z^2\pm\frac{1}{2}z-\frac{1}{6}x^2-\frac{1}{6}y^2+\beta_2 xy-\beta_3 yz-\beta_1 xz.$	$\beta_1+\beta_2-\beta_3=0,$ $\beta_i\in R$	(26)

It is evident that basis functions in table 3 as particular cases of functions (8) are harmonic according to Laplace. Functions (23) – (26) satisfy the derivative constancy criterion for any values of coefficients $\beta_i, i=\overline{1,3}$ that do not contradict the conditions of table 3. Functions (11) – (22) satisfy the derivative constancy criterion if $\beta_1=\beta_2=\beta_3=0$, i.e. in the case when these functions coincide with the quadratic basis of the octahedron [3].

Investigation of equations (8)) using the invariants has shown that level surfaces corresponding to the basis functions could be functions of the hyperbolic (a hyperboloid, cone, a two-sheet hyperboloid) and parabolic (hyperbolic cylinder or a pair of intersecting surfaces) types, level surfaces being the surfaces of rotation only for the quadratic basis of the six-node octahedron [3].

Conclusions, prospects

The paper presents second-degree polynomial bases of the six-node octahedron obtained by means of the matrix calculus. Basis functions are harmonic according to Laplace. Geometrical properties of the level surfaces corresponding to the basis functions have been studied. We are planning to continue research on the interpolation properties of the obtained octahedron bases and have every reason to believe that it is possible to obtain alternative bases (probably, non-linear ones) of the six-node octahedron.

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