# V. Sh. Feiziev, Cand. Sc. (Eng.); V. Sh. Feiziev; G. I. Buniyatova <br> SIMULATION APPROACH TO THE SOLUTION OF THE PROBLEM OF FINDING SUBOPTIMAL STRATEGY OF THE ACCESS TO MULTI RATE QUEUING SYSTEMS 


#### Abstract

The paper considers approximated method of the solution of the problem dealing with the establishment of suboptimal strategy of access in multi rate queuing system. Depending on current situation in the system stand-by channels can be activated. The usage of these channels lead to certain economic charges. The suggested method is based on the ideas of simulation modeling. The results of numerical experiments are presented.


Key words: multi rate queue system, stand-by channels, access strategy, suboptimal strategy, simulation modeling.

## Introduction

Multi Rate Queue (MRQ) systems are mathematical models of the process of diverse type of information processing in communication networks of the last generation [1-3]. In known research papers models of such systems using simple access strategies, are investigated when we speak about simple access strategies, we mean systems where all the channels are equally used by diverse calls. At the same time, due to limited resources of the networks, as well as, due to the fact that diverse calls have different levels of importance, equal usage of system channels is not always efficient. That is why, some channels are reserved and used only in case of conflicts emergence. To solve these conflicts the most efficient means are Markov processes of decision-making (MPDM). Such an approach was already used in [4] to establish optimal access strategy. In the given research the analogous approach is used for investigation of MRQ models with reserve channels.

## Problem set-up

Let us consider Multi Rate Queue (MRQ) system, where all the channels are divided into groups: active and stand-by. Active channels are used according to complete access scheme, where as switching of stand-by channels can be controlled. The latter means, that the usage of stand-by channels is connected with determination of economic losses, that is why, at the moment of arrival of diverse calls, the decision regarding the usage of these channels is to be taken. In this case such decisions are taken only when the number of free channels is not sufficient for servicing of the obtained call. The aim of the control over switching on of the reserve channels is to minimize total economic losses per unit of time of stationary mode, connected with losses of calls and usage of reserve channels.

All $N>1$ channels of the system are divided into two groups, i.e., $N=A+R$, where $A>1$ indicates the number of reserve channels, all the channels being identical. Input stream of calls is a poisson one, with the parameter $\Lambda$, each arrived call with probability $\alpha_{i}$, requires $b_{i}$ channels simultaneously, where $\alpha_{1}+\ldots \alpha_{K}=1$, i.e., initial stream is superposition $K$ of independent poisson streams with the intensities $\lambda_{i}:=\Lambda \alpha_{i}, i=1, \ldots, K$, where calls from $i$ - ${ }^{\text {th }}$ stream require simultaneously $b_{i}$ channels, all channels start and complete queuing simultaneously. Time of $i^{\text {th }}$ type call queuing is exponentially distributed random value with average $\mu_{i}^{-1}, i=\overline{1, K}$.

If at the moment of any type of call arrival, the number of free active channels is sufficient, then the necessary amount of active channels is assigned for its queuing. In other case, free reserve channels can be used for this purpose. At the same time, if the total number of free channels (active and reserve) is not sufficient for queuing of the arrived call, then it is lost (blocked) with the probability of 1 .

Mechanism of switching on and switching off of reserve channels is the following. If at the moment of complition of any type of call queuing the number of occupied channels is not less than $A$, all free channels are switched to reserve group; in other case any channels $A$ will be included into active group, and the rest will remain in reserve group.

Let us assume, that the loss of one call of $i^{\text {th }}$ type is evaluated by a penalty of $c$ (i) conventional units, $i=1, \ldots, K$, and switching on of $j$ reserve channels per unit of time results in penalty of $d(j)$ conventional units, $j=1, \ldots, R .$. then the problem of finding of an optimal strategy of reserve channels switching on is formulated in the following way: such a strategy of reserve channels switching on is to be found that will minimize total penalties per unit of time of stationary mode, connected with the losses of various types of calls and switching on of reserve channels.

Optimal call admission control (CAC) strategy is the sequence of decisions taken at the moment of calls arrival. At each moment of calls arrival, taking into account their type and current state of system it is necessary to take one of two decisions either the arrived call is lost, or certain number of reserve channels is used for its queuing.

## Computation of model characteristic

The state of the given system at random moment of time can be described by $K$-dimensional vector $n=\left(n_{l}, \ldots, n_{K}\right)$, where $n_{i}$ indicates the number of calls of $i$ - $^{\text {th }}$ type in the system. Since each call of $i^{\text {th }}$ type simultaneously requires $b_{i}$ channels of the system, then maximum amount of calls in the system is limited by the value $\left[N / b_{i}\right]$, where $[x]$ denotes the whole part of $x, i=\overline{1, K}$. Total number of occupied channels in the system in state $n$, is determined as a scalar product of vectors $n$ and $b=\left(b_{l}, \ldots, b_{K}\right)$. Thus, the set of possible states of the system is determined as:

$$
\begin{equation*}
S:=\left\{\boldsymbol{n}: n_{i}=0,\left[N / b_{i}\right], i=\overline{1, k},(\boldsymbol{n}, \boldsymbol{b}) \leq N\right\} . \tag{1}
\end{equation*}
$$

For description of strategies class, where the optimal strategy of reserve channels switching on is, we will consider the moment of calls arrival.

Let at the moment of $i^{-{ }^{\text {th }}}$ type call arrival, the system be in state $n \in S$. The number of free active channels in this sate is determined as $f(n)=A$ - $(n, b)$, if $\quad f(n) \geq 0$. Thus, if $f(n)<0$, it means that in the state $n$ the number of reserve channels used equals $-f(n)$. We draw a conclusions, that the value $f(n)+R$ indicates total number of free active and reserve channels in the state $n \in S$, if $f(n)>0$ and in case $f(n) \leq 0$ this value means the number of free reserve channels in state $n \in S$.

Since active channels of the system are used in accordance with complete access scheme, then at the moment of $i^{\text {th }}$ type call arrival the system is in the state $n \in S$, in which $f(n) \geq b_{i}$ then arrived call will be received with the probability of 1 , and for its queuing $b_{i}$ free active channels are provided. If at that moment the components of state vector $n$ satisfy the inequality $b i>f(n)+R$, then the arrived call of $i^{- \text {th }}$ type is lost with probability 1 , as at that moment the number of free channels (active and reserve) is not sufficient for servicing the arrived call. Alternative solutions are possible at the moment of $i^{\text {th }}$ type call arrival, if at these moments the system is in one of subclasses of possible states set:

$$
\begin{gather*}
S_{i}^{*}:=\left\{\boldsymbol{n} \in S: f(\boldsymbol{n}) \leq 0, b_{i} \leq f(\boldsymbol{n})+R\right\} ;  \tag{2}\\
S_{i}^{* * *}:=\left\{\boldsymbol{n} \in S: f(\boldsymbol{n})>0, f(\boldsymbol{n})<b_{i} \leq f(\boldsymbol{n})+R\right\} . \tag{3}
\end{gather*}
$$

In both subclasses of states (2) and (3) the following decisions are possible: $\mathrm{d} 1-$ the arrived call is lost and d 2 - reserve channels are used for servicing the arrived call of $i^{\text {th }}$ type. Note, that if decision d 2 is taken, then in subclass (2) $b_{i}$ reserve channels are used, and in subclass (3) the number of reserve channels, allocated for servicing the arrived call of $i^{\text {th }}$ type, equals $b_{i}-f(n)$. Probabilities of taking the decisions d 1 and d 2 are denoted by $\alpha_{i}^{-}(\boldsymbol{n})$ and $\alpha_{i}^{+}(\boldsymbol{n})$, correspondingly. Since theses probabilities make up the complete group, then we have:

$$
\begin{equation*}
\alpha_{i}^{-}(\boldsymbol{n})+\alpha_{i}^{+}(\boldsymbol{n})=1 \text { for all } \boldsymbol{n} \in S_{i}, \tag{4}
\end{equation*}
$$

where $S_{i}=S_{i}^{*} \bigcup S_{i}^{* *}, i=\overline{1, K}$.
Now let us consider the moments of calls sending by the system. Let immediately before sending of $i^{\text {th }}$ type call from the system it were in state $n$, where $n_{i}>0$. Then at the moment of call sending, the following state of the system will be $n$ - $e_{i}$, where $e_{i}-K$-dimensional vector, all the components except $i^{\text {th }}$ being equal 0 , and $i^{\text {th }}$ component equals 1 . If $\left(\boldsymbol{n}-\boldsymbol{e}_{i} \boldsymbol{b}\right) \geq A$, then all free $b_{i}$ channels become reserve:; in other case, any $A$ channels become active, and the rest of the channels are switched into reserve group. After sending from the system of $i^{- \text {th }}$ type call, the system is transfered into $\boldsymbol{n}$ - $\boldsymbol{e}_{i}$ state, with the intensity $n_{i} \mu_{I}, i=1, \ldots, K$.

It may be shown, that penalties per unit of time are calculated as:

$$
\begin{align*}
& G\left(p(\boldsymbol{n}), \alpha_{i}^{ \pm}(\boldsymbol{n})\right):=\sum_{\boldsymbol{n} \in S} \sum_{i=I}^{K}\left(\lambda _ { i } p ( \boldsymbol { n } ) \left(c(i)\left(I\left(b_{i}>f(\boldsymbol{n})+R\right)+I\left(\boldsymbol{n} \in S_{i}\right) \alpha_{i}^{-}(\boldsymbol{n})\right)+\right.\right.  \tag{5}\\
& \left.\left.+\left(d\left(b_{i}\right) I\left(\boldsymbol{n} \in S_{i}^{*}\right)+d\left(b_{i}-f(\boldsymbol{n})\right) I\left(\boldsymbol{n} \in S_{i}^{* *}\right)\right) \alpha_{i}^{+}(\boldsymbol{n})\right)\right),
\end{align*}
$$

where $p(\boldsymbol{n})$ means stationary probabilities of $\boldsymbol{n} \in S$ state.
Proceeding from the above-mentioned, we make a conclusion, that the aim of the given system study is the solution of the following problem:

$$
\begin{equation*}
G\left(p(n), \alpha_{i}^{ \pm}(n)\right) \xrightarrow[\alpha_{i}^{ \pm}(n)]{ } \min \tag{6}
\end{equation*}
$$

The limitations of this problem is the system of balancing equations. Hence, the problem of determination of optimal strategy of reserve channels switching on is reduced to certain problem of Markov's programming. It has non-randomized optimal solution. For determining optimal strategy of reserve channels switching on at small values of $N$ and $K$ method of linear programming can be used. At great values of these parameters approximation methods can be applied (4).

At the same time, in practice, especially while investigating MRQ models with great number of call types, the state of the system is not observed completely, i.e. partial information regarding its state is available, namely, only total number of occupied (free) channels is observed. That is why, CAC in question must take decisions, based on such incomplete information. Optimal CAC, based only on information about the number of occupied (free) channels we will call suboptimal.

Let active channels, as before, be used according to complete access scheme, and the number of reserve channels that can be used for servicing of $i^{- \text {th }}$ type calls, will be limited by the value $r_{i}$, and $r_{1}+\ldots+r_{K} \geq R$. The task of system optimization is to find values of $r_{i}, i=1, \ldots, K$, to minimize total penalties (6).

It should be noted, that suboptimal strategy will not be better, than optimal strategy. This can be explained by the fact that optimal CAC, while decision-making takes into consideration the detailed information regarding system state (i.e., only information about the total number of occupied channels). In other words, two different states, in which the number of occupied channels is equal, are considered as one state from the point of view of suboptimal strategy of reserve channels switching on.

The program of simulation modeling is developed and applied for finding suboptimal strategy in MRQ model, with the parameters $A=20, R=10, K=2, b 1=1, b 2=6, \quad c(1)=c(2)=1, d(i)=$ $\mathrm{I}, \mathrm{i}=1, \ldots, 6$. While each run of simulation program 100.000 calls were used, that completely finished queuing. The number of repetitions of each experiment is 5 , and their mean value was selected as basic indexes of QoS system. As a result of experiments suboptimal strategy of reserve channels switching was found. Corresponding results are shown in Table 1.

It should be noted, that in order to reduce the number of switching of various suboptimal strategies, the strategies, marked by asterisk may be replaced by the strategy $(10,1)$ as maximum difference between minimal values of corresponding efficiency functions does not exceed $0.5 \%$. For
instance, Table 3. 3 shows that if optimal suboptimal strategy $(\rho 1, \rho 2)=(4,4)$ and $(\rho 1, \rho 2)=(4,6)$ and if $(\mathrm{r} 1, \mathrm{r} 2)=(10,1)$, corresponding strategy is defined as $(\mathrm{r} 1, \mathrm{r} 2)=(9,1)$. The latter means, that when $\left(\rho_{l}, \rho_{2}\right)=(4,5)$ and $\left(r_{1}, r_{2}\right)=(10,1)$, is taken as suboptimal strategy, then the number of switching of various suboptimal strategies is considerably reduced, and great errors are not made (as the difference does not exceed $0,5 \%$ ).

Table 1

## Suboptimal strategy of reserve channels switching



The research suggests simulation modeling approach for solution of the problem dealing with finding of suboptimal strategy of reserve channels switching in multirate systems, when accurate solution of the problem of optimal strategy finding is not possible due to large dimensionality of initial task. The investigated model is widely used in modern communication networks where various information is processed.

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