# P. D. Lezhniuk, Dc. Sc. (Eng.), Prof.; M. V. Kutina <br> MATHEMATICAL MODEL FOR DETERMINATION OF CONTACT RESISTANCE IN PLACE OF WIRE BREAKAGE OF OVER-HEAD TRANSMISSION LINES 

The paper contains the analysis of factors, influencing the value of contact resistance in the place of overhead transmission line wire fall on the ground. Mathematical model intended for determination of contact resistance in the place of wire fall on the ground is suggested.

Key words: wire breakage of over-head transmission line, contact resistance.

## Introduction

The most typical faults ( $60-70 \%$ of the total number of faults) in distributive networks with overhead transmission lines of $6-35 \mathrm{KV}$ are single-phase partial ground faults (SPPG) [1, 2]. The reasons of partial ground faults are various: electric break-down of insulation due to the impact of internal and atmospheric over -voltages [1, 2], mechanical damages of phase insulation, breakage and fall of the wire of over-head transmission line or structure grounding; contact of people or animals with non-insulated current-carrying parts, fautly actions of the staff. Main features of SPPG revealing is the usage of current and zero sequence voltage or the value of the impedance or active resistance of network phases insulation relatively the ground at overlapping on the network of independent power supply [3]. Electric characteristics of single-phase faults of electric installations with insulated voltage neutral of $6-35 \mathrm{KV}$ do not depend on specific designation of electric installation. However, greater portion of emerging SPPG in various situations and minor variations of transmission line electric parameters, as compared with normal operation mode complicate the choice of optimal methods of prevention, revealing and elimination of SPPG.

Analysis of previous research. Mathematical dependences of electric values in case of SPPG on the parameters of electric installation can be divided into two groups - arcless and arcing. Arcless short circuits emerge in case of reliable conductive coupling of damaged phase with the ground. The form of current curve of arcless SPPG depends considerably on the active resistance of wire contact with the ground $R_{c}$ and resistance of current spreading $R_{s}$.

The results of research [1] showed that stable burning of the arc in over-head transmission lines of $6-35 \mathrm{kV}$ is possible only in case of insulator break-down on wooden pole-arms of over-head line support, if contact resistance is several tens of kOhms .

The most dangerous is SPPG in case of wire breakage of over-head line from the side of power supply [3]. In the place of fall great contact resistance may emerge, it considerably reduces the value of fault-to earth current, and protective devices in greater part of cases do not react.

The faulty impression about the lack of damage appears.
Besides, on L.V. side of the transformer supplied from damaged line, interphase voltages are distorted, and consumers pass to non-symmetrical operation mode, dangerous from the point of view of overheating of transformer windings and motors.

The aim of research is evaluation of contact resistance in place of SPPG by means of construction of mathematical model, that corresponds to real process of wire breakage for account of its value while calculation SPPG current.

It is known, that contact resistance, in the location of wire fall $R_{\text {con }}$ includes serial coupling of contact resistance of conductor contact with the earth $R_{c}$ and spreading resistance of fault-to-earth current $R_{s}$, that is [4]

$$
\begin{equation*}
R_{c o n}=R_{c}+R_{s} . \tag{1}
\end{equation*}
$$

Contact resistance $R_{c}$ is determined by the resistance in the location of narrowed sections of the
wire $R_{n e r}$ where current flows to short-circuit and film resistance $R_{f}$ places on the surface of the conductor [4]

$$
\begin{equation*}
R_{c}=R_{\text {nar }}+R_{f} . \tag{2}
\end{equation*}
$$

Film resistance is characterized by the dependence[4]

$$
\begin{equation*}
R_{f}=\rho_{e q} \Delta / S, \tag{3}
\end{equation*}
$$

where $\rho_{e q}$ - is mean specific resistance of conductive material film ( $\mathrm{Ohm} * \mathrm{~m}$ ); $\Delta$ - is film thickness, $\mathrm{m} ; S$ - contact surface, $\mathrm{m}^{2}$.

In greater part of cases [4] films on contact surfaces are formed under the influence of oxygen, present in the air, ozone, nitrogen and other chemical reagents. Their thickness is $\Delta \approx 10^{-8} \mathrm{~m}$ and specific electric resistance $\rho \approx 10^{3} \mathrm{Ohm} * \mathrm{~m}$ [4].

To determine contact resistance in the location of wire contact with the ground the most suitable is elliptic model of the contact, then $R_{\text {nar }}$ [4,5] can be defined as

$$
\begin{equation*}
R_{n a r}=\frac{2 \rho}{\pi d} \ln \left[\frac{l^{2} \sqrt{E}}{1,5 \sqrt{F_{c} R^{2}}}\right], \tag{4}
\end{equation*}
$$

where $l$ - is length of the wire, contacting the ground; $R$ - is the radius of surface curvature; $E$ - is modulus of ground elasticity; $F_{c}$ - is the force, contact presses upon the ground, H.

For determination of recess surface curvature radius, formed by the broken wire in the ground, we will make use of the method of hardness determination, based on the measurements of indentations dimensions, obtained as a result of forcing steel spheres, diamond cones or prism into the surface of the investigated material [6] (Brinell hardness, Rockwell hardness, Vinser hardness). For illustration we will consider the method of Brinell hardness determination, according to this method, hardness determination is performed by forcing steel hardened sphere into the surface of the investigated body under the action of certain load. The diameter of the formed indentation $d$ is measured. If the diameter of the sphere is $D$, and the load is $P$, then the measure of hardness is the value $E$, determined by the formula[6]

$$
\begin{equation*}
E=\frac{2 P}{\pi D\left(D-\sqrt{D^{2}-d^{2}}\right)} \tag{5}
\end{equation*}
$$

where $P$ - is measured in $\mathrm{N} ; D$ and $d$ in $\mathrm{m} ; E-\mathrm{in} \mathrm{N} / \mathrm{m}^{2}$.
From (5) the diameter of the formed indentation is determined

$$
\begin{equation*}
d=\sqrt{D^{2}-\left(D-\frac{2 P}{E \pi D}\right)^{2}} . \tag{6}
\end{equation*}
$$

Assuming that $d=2 R, P$ presents the mass of broken wire in $\mathrm{kg} / \mathrm{m} ; D$ the diameter of broken wire, taking into account (4) and (6) $R_{n e r}$ can be determined in the following way

$$
\begin{equation*}
R_{n a r}=\frac{2 \rho}{\pi l} \ln \left[\frac{l^{2} \sqrt{E}}{1,5 \sqrt{0,5 P\left(D^{2}-\left(D-\frac{2 P}{E \pi D}\right)^{2}\right)}}\right] \tag{7}
\end{equation*}
$$

Proceeding from [7] and (6) the resistance of spreading current can be determined

$$
\begin{equation*}
R_{s}=\frac{\rho}{\pi l} \ln \frac{2 l}{d}=\frac{\rho}{\pi l} \ln \frac{2 l}{\sqrt{D^{2}-\left(D-\frac{2 P}{E \pi D}\right)^{2}}} \tag{8}
\end{equation*}
$$

Contact resistance in the location of wire fall can be determined by the formulas (7), (8)

$$
\begin{equation*}
R_{\text {con }}=\frac{\rho}{\pi l} \ln \left[\frac{1,778 l^{5} E}{P\left(D^{2}-\left(D-\frac{2 P}{E \pi D}\right)^{2}\right)^{\frac{3}{2}}}\right]+\rho_{e q} \Delta / S \tag{9}
\end{equation*}
$$

Specific resistance of the soil is substituted in the formula (9) instead of $\rho$.
AC current flow in the contact produces additional increase of contact resistance caused by surface effect. Fault-to-earth currents from SPPG do not exceed ten amperes, that is why, the impact of surface effect can be neglected.

While computation of contact resistance, applying equation (9) it is necessary to take into account the dependence of specific resistance of contact material on the temperature. The temperature of the contact is not identical on all of its length. In approximated calculations [4] $\rho$ can be determined for average temperature in the area of contact narrowing. In [4] it is shown that average temperature approximately equals $2 / 3$ of maximum temperature $T_{M}$, relatively the surface of touch. While flow of fault to -earth current of up to 30 A , soil temperature can not exceed $T_{M C}=80 \div 100^{\circ} \mathrm{C}$. When such temperature is reached sintering of the soil occurs and dependence of specific resistance of the soil on the temperature within such limits of its variation corresponds to linear dependence

$$
\begin{equation*}
\rho=\rho_{0}\left(1+\frac{\rho_{T}-\rho_{0}}{\rho_{0}\left(T-T_{0}\right)}\right) \tag{10}
\end{equation*}
$$

where $\rho_{0}$ - is specific resistance of the soil at ambient temperature $T_{0} ; \rho_{T}$ - is specific resistance at heating to temperature $T$.


Fig. 1. Scheme for calculation of the temperature of broken wire contact with the ground
Let us consider the process of contact heating in the location between the broken wire and the
ground. Electric current, flowing in the contact, generates in its contact resistance $R_{\text {cur }}$ heat power $R_{\text {cur }} I^{2}$. Temperature $T$ in the location of wire contact with the earth will be increased as compared with the temperature of wire and the ground. To determine the dependence of the temperature on contact parameters and the environment we will make use calculated circuit, shown in Fig. 1.

Let the start of dependence $T=f(l)$ coordinates is located at the beginning of the contact with the ground. At the distance $l$ from the beginning of coordinates we will allocate element of $d l$ length. Power balance in this element will be written in the form of expression $P_{1}+P_{3}=P_{2}+P_{4}$, where $P_{1}, P_{3}$ - are powers feeding the contact; $P_{2}, P_{4}$ - are powers, removed from the contact (Fig. 1).

Powers we will determine applying heat equations [8]

$$
P_{1}=-\lambda S \frac{d T}{d l} ; \quad P_{2}=-\lambda S \frac{\partial}{\partial l}\left(T+\frac{\partial T}{\partial l} d l\right) ; P_{3}=\rho \frac{I^{2}}{S} d l ; P_{4}=k_{T}\left(T-T_{0}\right) p d l
$$

where $\lambda$ - is heat conduction factor; $p$ and $S$ - are perimeter and cross-section area of the conductor; $k_{T}$ - is the coefficient of convective heat exchange from the surface of the conductor; $T_{0}$ - is the temperature of ambient air after transition from partial derivatives to normal derivatives we obtain different equation;

$$
\begin{equation*}
\lambda S d^{2} T / d l^{2}-k_{T} p\left(T-T_{0}\right)+\rho I^{2} / S=0 . \tag{11}
\end{equation*}
$$

The solution of the equation is

$$
T=A_{1} e^{-k l}+A_{2} e^{k l}+T_{s t},
$$

where ${ }^{A_{1}}, A_{2_{-}}$are integration constants; $k=\sqrt{k_{T} \rho / \lambda S}$.
The established value of the temperature of conductor heating the current can be determined form the equation (11). When the first member of the equation is converted into zero, then

$$
\begin{equation*}
T_{s t}=T_{0}+\rho I^{2} / p S \tag{12}
\end{equation*}
$$

Proceeding form the condition, that from the location of the contact beginning $l=0$ in one direction approximately half of the power in the contact $P_{d} \approx \frac{1}{2} R_{n a r} I^{2}$ is spreading at corresponding limiting conditions we find the expression for the temperature along the conductor

$$
\begin{equation*}
T=T_{0}+\frac{\rho I^{2}}{k_{T} p S}+\frac{R_{c} I^{2}}{2 \sqrt{\lambda k_{T} p S}} \exp \left(-\frac{\sqrt{k_{T} p}}{\lambda S} l\right) \tag{13}
\end{equation*}
$$

For elliptic model of the contact from (14)

$$
\begin{equation*}
T_{M}=\left(T_{0}+\rho \frac{I^{2}}{k_{T} p S}\right) /\left(\cos \left(\frac{I \sqrt{L}}{4 \lambda R}\right)\right) \tag{14}
\end{equation*}
$$

where $R$ - is the radius of surface element narrowing, determined from (6) as $R=d / 2 ; L$ - is Lorentz constant, determined for aluminium and steel conductors from the Table [9].

For verification of adequacy of the mathematical model, intended for determination of contact resistance in the location of wire fall on the ground a number experimental research of contact resistance in the place of wire fall on the ground were, carried out. For experimental determination Наукові праці ВНТУ, 2011, № 2
of soil specific resistance the method of vertical electric probing and MC-08 device were used. For determination of electric resistance along the wire, lying on the ground, the expressions for calculation of horizontally located groundings [10] are used. Type of wire, used in the experiment is AC-35, length of wire -5 m and 10 m . The experiment was carried out in autumn, ambient temperature was $10^{\circ} \mathrm{C}$ for three types of soil. The results of the experiments are presented in Table 1.

Table 1
The results of experiment

| Type of <br> soil | Length of the <br> wire, lying on <br> the ground, m | Specific <br> resistance <br> of the <br> soil, <br> Ohm*m | Calculated value of <br> the resistance in the <br> place of conductor <br> contact with ground <br> $R_{s}$, Ohm | Experimentally <br> determined value of the <br> resistance in the place of <br> conductor contact with | Relative error, <br> the ground $R_{d}$ <br> (Ohm |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Glay | 5 | ${\multirow{4}-{R_{\text {cal }} 100 \%}R _ { \text { cal } } 1 0 0 \%}{R_{d}} }$ |  |  |  |

Experimental research carried out for verification of model adequacy, showed, that the error of the model does not exceed $\pm 20 \%$.

## Conclusions

Creation of mathematical model for determination of contact resistance in the location of breakage of over-head transmission line wire is one of the stages of SPPG revealing problem solution.

Model enables to calculate with high accuracy the contact resistance in the location of conductor fall on the ground, taking into account the conditions of the environment.

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