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ELABORATION OF THE SYSTEM FOR OPTIMAL DEVELOPMENT MANAGEMENT IN THE PRESENCE OF UNCERTAINTIES

The paper proposes a three-level management system of multiproduct manufacturing system development that has no analogs. At the first level a single-step problem of optimal distribution of the system resources distribution between production elements is solved, the second level provides the solution of the multistep variational development problem. At the third level development strategy correction parameters are determined on the basis of the prediction system data. Algebraization of the dynamics and optimization problems is used.

Key words: *prediction, optimal development, allocation of recourses, variational problem, predictor model, composite model, generalized manufacturing system, algebraization of problems.*

Problem statement. Modern manufacturing systems are in the state of constant changes in technology and production. The idea of "cybernetic corporation" with the model of the same corporation in the contour of the automatic management system originated more than 50 years ago. Current level of the solutions to this problem has not increased. The function of transaction processing, all types of accounting and statistical analysis are realized reliably and effectively. The functions of prediction and optimal development management, however, are realized by top managers on the basis of intuition and experience. There exists robot software that can optimize portfolio of securities proceeding from the results of financial market analysis. It is difficult to evaluate the real state of this scientific field due to the absence of information (for confidentiality reasons) about the predictor programs and business analytics systems that are currently in use.

We will consider the optimal management of a distributed technological system development in a complex environment. Instead of the obsolete expression – "uncertainty" – a new one – "complex environment" – will be used. It implies globalization: environment is not a passive spontaneous element, but dozens of "close" and "distant" suppliers, consumers and competitors. Application of the "globalization" notion to this phenomenon is non-strict and fuzzy. Availability of fundamental works, many qualitative monographs create an illusion of full completeness of this scientific trend. However, new and complex technological problems are not only arising more often today, but also have an infinitely greater "weight". Ignoring a new production technology today leads to the loss of consumers, then to the loss of a segment and finally – to the destruction of a social structure and emergence of another "not a state".

The term of "development process" is of a biological origin and in application to production it means allocation of a certain optimal part of manufacturing system resources to updating of the production nomenclature, design and manufacturing technologies. The development aim is stability and higher efficiency. As to the quantitative growth, modern production always has certain limits: it is not technically difficult to satisfy the needs in communicators, notebooks and means of transport.

Problem statement: elaboration of the mathematical predictor model based on the optimal development processes according to the integral criterion. Development model must calculate the optimal process for all permissible initial conditions, functions of adaptation, manufacturing and disturbances. Integral criterion is interpreted as the effect accumulated during the planning period or as expenses accumulated under corresponding restrictions. So we have a rather complicated problem that requires a stepwise solution: we choose and develop functional modules and assemble the required model from them. And here a terminological problem arises: the assembled model cannot be classified as a management system where connections between blocks have the "input-output" character. It is not also a hierarchical system of models [1 – 3] and a meta model [4 -5]. Let us call the elaborated structure "the composite model".

Designing a composite model of the development process

Definition of the composite model will be given on the basis of UML model conception. In UML transitive diagrams of *communication and sequence* are defined. They express interaction but show it by various means and could be transformed one into another with sufficient degree of accuracy. Communication diagram (in UML 1.x – collaboration diagram) is a diagram where interactions between the composite structure parts or cooperation roles are reflected. Unlike the sequence diagram, the communication diagram clearly shows interrelations between the elements (objects), and time as a separate dimension is not used (serial numbers of the calls are used). The *Sequence diagram* is a diagram where time-ordered interactions are depicted. In particular, it depicts objects that take part in interaction and the sequence of messages they exchange.

All the presented models can be used autonomously. Functional submodels are stratified and oriented to the definite segments of production. It is obvious that characteristics of demand in tankers and pleasure boats differ not only parametrically but also structurally.

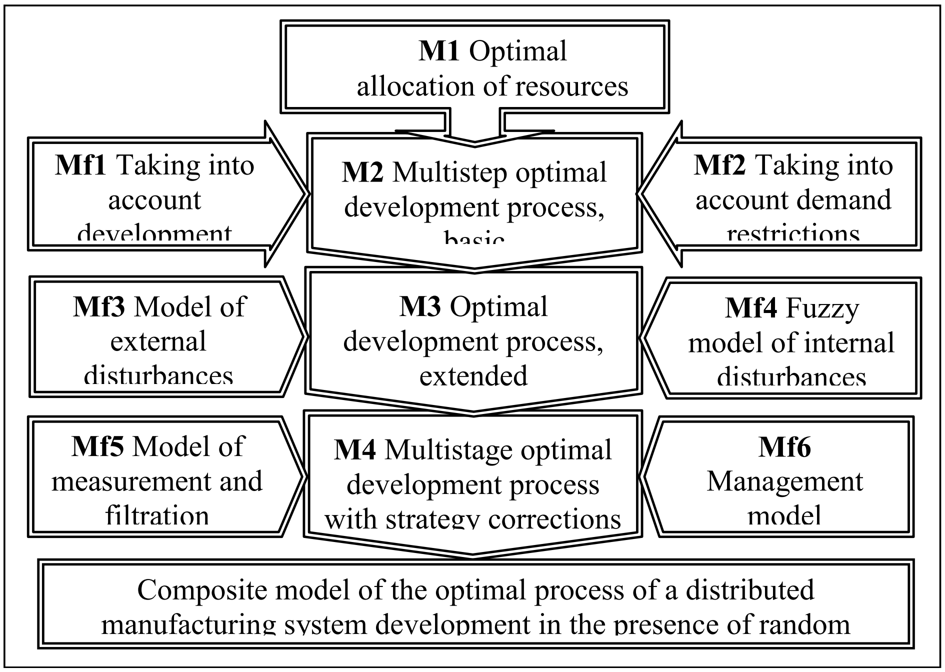

Fig. 1. Diagram of designing a composite model of the development process

Information technology of the model construction includes functional, structural and reductional decompositions, algebraization of the dynamics and optimization models and reduction of the complex multidimensional nonlinear systems to the pseudo one-dimensional form. Let us consider the sequence of development models.

Model of the optimal allocation of resources as a one-step process

For compact and integral representation of the model for optimal allocation of resources, the main results are collected in the informational block (fig. 2).

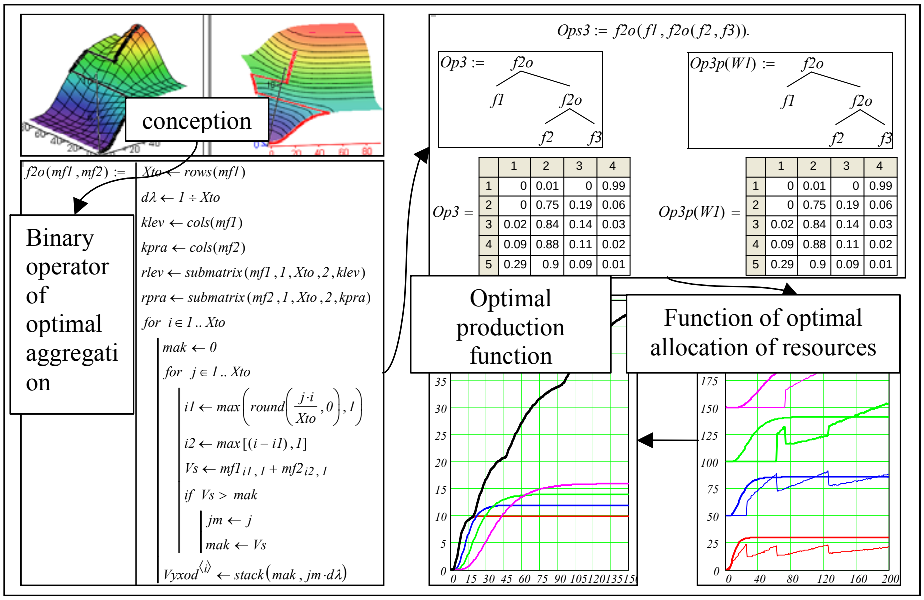

Рис. 2. M1 – model of the optimal allocation of resources

Full description and substantiation of the model is a bulky process. This model is essentially different from the analogs and is a key model for the entire development process. The essence of the optimal aggregation method is the replacement of the multidimensional nonlinear programming problem by a sequence of one-dimensional optimization problems. The result of applying this method is an equivalent optimal one-dimensional production element. An advantage of the optimal aggregation method as a method for nonlinear programming problem solution is the absence of restrictions on the form of target functions such as differentiability, convexity and reversibility of the aggregation operation. Optimal aggregation programming module implements a binary operator, associative and commutative for the objects – discrete production functions. Finally, the nonlinear programming problem is reduced to a simple algebraic problem.

Model of the optimal allocation of resources as a multistep process

Discrete development process can be represented as a multistep process of making decisions that optimize a certain criterion – a sum of functions of a dynamic system state at each step. The method of L. Pontriagin maximum principle makes it possible to replace a single problem of finding the functional extremum by a sequence of problems of finding the maximum of Hamilton function at each step of making decisions. For an aggregated model it is a function of one or two variables of management.

Fig. 3 presents the informational block of the development model M2. Let us pay attention to the three-dimensional plot constructed from Hamilton functions for each step of the process modeling. The trajectory of Hamilton function maximums is superimposed on this plot. Top view (optimal strategy) is a proportion of the system resources distribution between accumulation and development. The optimal strategy is a complex non-smooth, non-monotonic function of time.

Finally, variational problem for a multiproduct system is reduced to the equivalent one-dimensional variational problem that consequently has taken the form of the “optimal development process” (a parameterized one).

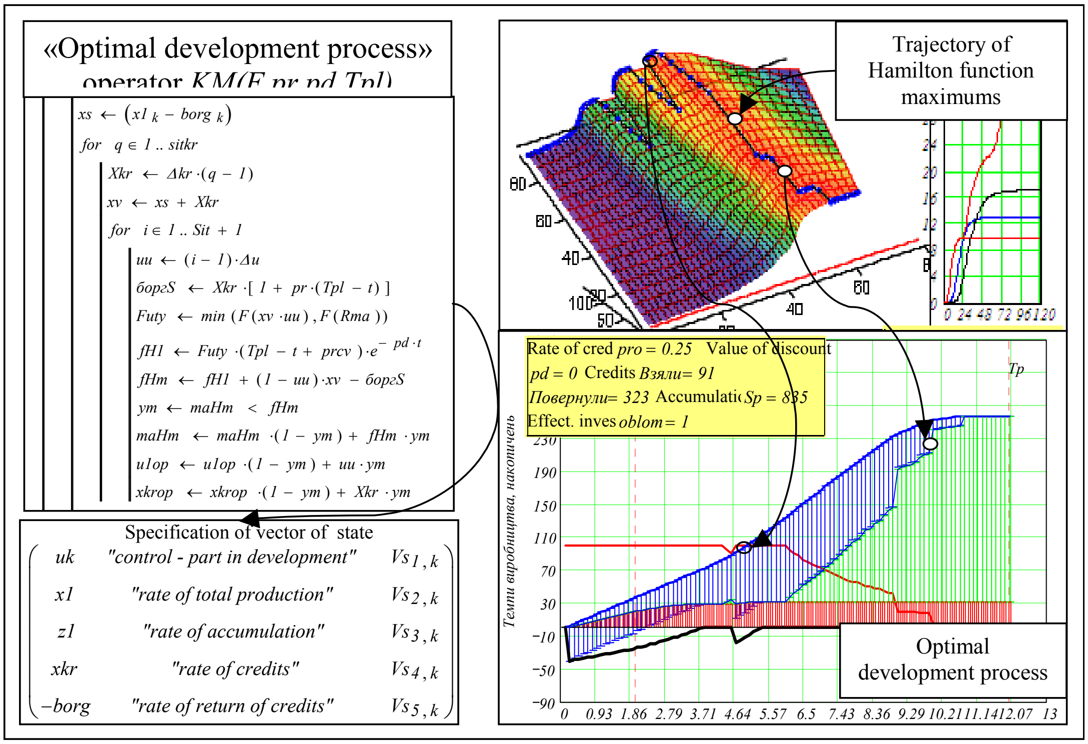

Fig. 3. M2 – model of the optimal allocation of resources in the development process

An extended model of the optimal allocation of resources as a multistep process

One of the essential advantages of the basic development model is the simplicity of modification. Initial changes are included into the extended system of dynamics equations and comprise equations for the criterion. Then these equations undergo the procedures of formation and solution of the system of differential equations for conjugated functions and for obtaining the expression for Hamilton function. We circumvent the problems that cannot be solved analytically – solutions of the nonlinear differential equation system, finding maximum of Hamilton function and use numerical methods for obtaining the corresponding parameterized expressions. Proceeding from the analysis of these accurate solutions, effective approximations can be obtained (not in the space of Hamilton functions but in the space of strategies). As a result, a variational problem solution for the modified model becomes, actually, "a designing process".

Let us consider typical model extensions at the level of Hamilton approximation function formation. In the criterion we take into account the cost of the production capacity at the end of the planning period. We write the expressions for the modified criterion (1) and two expressions for Hamilton function – the initial and the modified ones (2).

$$JN = \int_0^T [x(t)\cdot(1-u(t)) + fin(x(t)\cdot u(t))\cdot pri\,\mathrm{cov}\,ar]dt; \quad (1)$$

$$\begin{aligned} H(x,u) &= x(t)\cdot(1-u(t)) + fin(x(t)\cdot u(t)\cdot(T-t) \\ H(x,u) &= x(t)\cdot(1-u(t)) + fin(x(t)\cdot u(t)\cdot(T-t+pri\,\mathrm{cov}\,ar). \end{aligned} \quad (2)$$

In the expression for the criterion an additional component appears under the integral – increase of production capacities multiplied by *pricover* – a reduced coefficient of the fund costs.

Taking into account the use of external resources. In this case we use meaningful interpretation of Hamilton function, which is often forgotten by theorists-Scholastics: external resources, taken at the given moment, reduce the criterion value through repayment of the debt and increase it by introduction and implementation of new technologies, product brands and production capacities.

$$H(x,u,xkr) = xs(1-u) + fin(xs\cdot u, p)\cdot(T - t + prcv) - xkr\cdot[1 + prc(T-t)], \tag{3}$$

where $xs(t) = x(t) + xkr(t)$ – total current resources; $x(t)$ – current production capacities; $u(t)$ – current part of the resources for development; $xkr(t)$ – current value of the external resources; prc – price of the resource (particularly, credit rate); $fin(\cdot)$ – investment return function; T – planning period; $prcv$ - reduced coefficient of the final cost of funds.

Taking into account discounting of the flow of resources. It is not difficult to take into account the resources flow discounting in our model. We modify the optimization criterion for the manufacturing system development.

$$JN = \int_0^T xs(t)\cdot unak(t)\cdot e^{-pt}dt. \tag{4}$$

Taking into account inertia and time-lags in the manufacturing system. Usually investments give return in the form of a certain rate of the new product manufacture with a certain time-lag caused by deliveries, construction, installation of the equipment, keeping products at the warehouse for too long. There is also an inertial time-lags in return of the development costs. This is reflected in the model by modifying the equation of the production capacity dynamics.

$$\frac{d}{dt}x(t) = fin(y(t)) \Rightarrow \frac{d}{dt}xp(t) = fin(y(t-\tau)); \quad \frac{d}{dt}xd(t) = Kob(xp(t) - xd(t)), \tag{5}$$

where t – time-lag, xp – nominal (potential increase of the funds), xd – actual current increase of the funds, Kob – the coefficient. In the standard form the inertial time-lag model is given by:

$$Tos\cdot\frac{d}{dt}xd(t) + xd(t) = xp(t),\ Tos = \frac{1}{Kob},$$

where Tos – time constant.

Taking into account production development effects. In this model we distinguished subsystems and the models of production as such, of production extension and development. Actual processes can be non-strictly divided into: investment development (a relatively fast process of the installed equipment development), production development (a long-term process within the entire life-cycle of improvement of the product and production). In this paper the effects of production scale and development are taken into account by the following model:

$$xc(t) = fct\left(xd(t), t, \int_0^t xd(t)dt\right). \tag{6}$$

Verbal interpretation of the expression (6): the reduced rate $xc(t)$ of the resource creation is equal to a certain function from the current production rate, time and accumulated output.

Taking into account the demand and competition. It is not difficult to create the model of the consumption of copybooks, books, batteries, notebooks, prediction programs. Transition from a single product and producer to several ones requires totally different approaches to modeling and the models themselves become complex – stochastic and fuzzy. In the framework of this paper the following modification of the production model is chosen (6):

$$\frac{d}{dt}x(t) = fin(y(t)) \Rightarrow \frac{d}{dt}x(t) = ef\cdot x(t)\left(\frac{Ryn(t) - Sus(t,x(t))}{Ryn(t)}\right)\cdot y(t), \tag{7}$$

where $x(t)$ – product manufacture rate, $y(t)$ – investment rate, ef – index of the efficiency of investments into development (it depends on energy and capital intensity of the production, experience and quality of the personnel, investment climate); $Ryn(t)$ – needs (the demand); $Sus(t,x(t))$

– satisfying the demand (filling the market). The first three factors are coefficients that characterize current efficiency of the investment conversion $y(t)$ into the capacity growth rate.

Model of the optimal management in the presence of uncertainties as a multistep process

For manufacturing systems there exists a "prediction horizon", determined by inertial character and correlations of uncertainties. Spending time and resources on receiving information can increase the «prediction horizon». Proceeding from this, a method of management in the presence of uncertainties is proposed. It consists in periodical correction of the optimal management. On the whole, the process is divided into intervals – the steps with fixed parameters of the optimal strategy. Fig. 4 shows the scheme of the development process division.

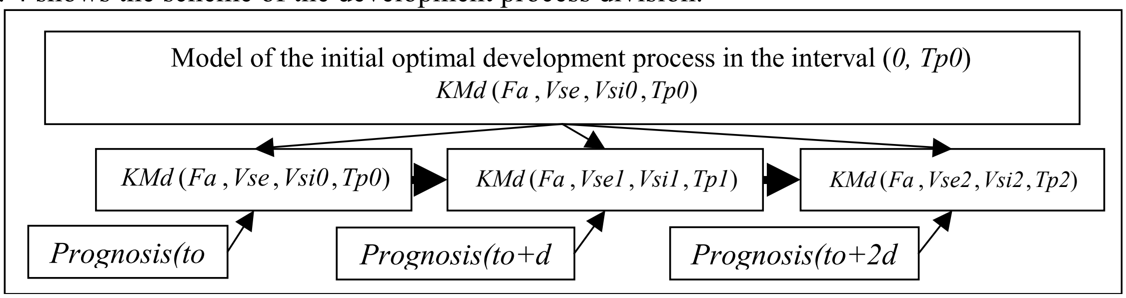

Fig. 4 The scheme of the optimal development process division into a multistep process.

Theoretical basis for the process division into the sequence of steps is the mathematical model representation in the form of algebraic object $KMd(Fa, Vse, Vsi0, Tp0)$ with division and composition operations being indicated. General solution of the strategy correlation problem goes beyond the limits of this paper.

An example of the optimal development process in the presence of uncertainties. Fig. 5 presents an example of the development process where, for clarity, the process steps are shown in the expanded form as if each step were divided to the end of the planning period. The arrows show the beginning and the end of a step with the fixed strategy. The final state of the previous step becomes the beginning of the next one.

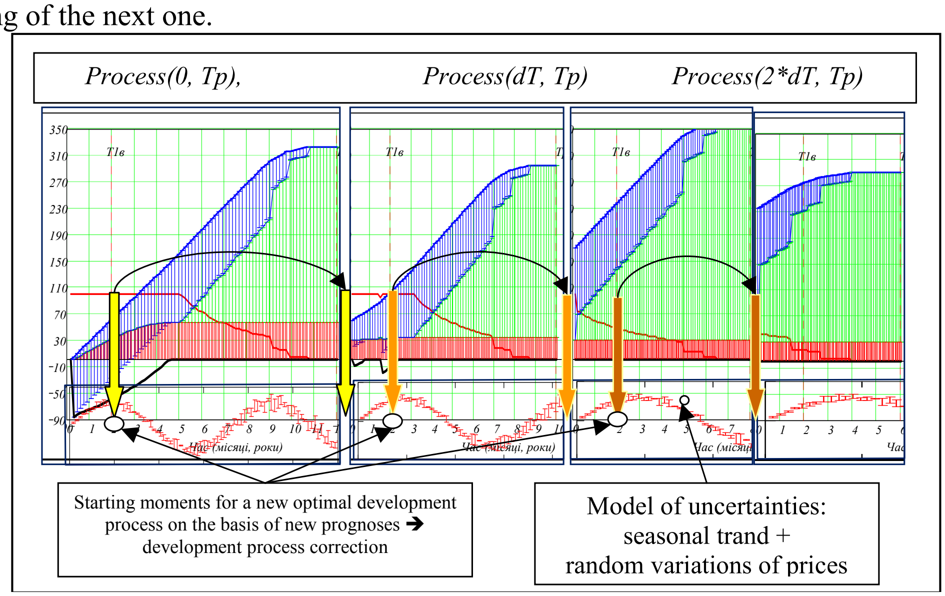

Fig. 5. An example of the multistep optimal development process under uncertainty conditions

Conclusions. A working composite model of the manufacturing system development process is built. A two-level system of the optimal development process management is proposed, where at the first level the optimal development management is determined for a deterministic problem and at the second level the moment and the value of the optimal strategy correction are chosen on the basis of the analysis of predicted uncertainties.

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