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PHASE PLANE RESEARCH OF THE PROCESS OF MASTERING THE CURRICULUM BY PROMISING STUDENT

With the help of mathematical models, synthesized on the phase plane, there had been conducted the research of the process of mastering the curriculum by promising student.

There had been built the phase trajectory for the process of mastering the curriculum by promising student assisted by a lecturer and independently.

Key words: *student, subject, phase plane, mathematical model.*

This work goes on researching the process of mastering the curriculum by promising student on the phase plane within the vicinity of the found singular point [1 – 3], but by the promising student, that is the student for whom the conditions leading to the appearance of the second singular point within the allowed values of phase coordinates on the phase plane come true.

Task setting and initial preconditions

Paper [1] synthesized the mathematical models of the process of mastering the curriculum by promising student on the phase plane in the kind

$$\begin{aligned} \frac{dx_1}{dt} &= -\alpha_{11}x_1 + \alpha_{12}x_2x_1, \\ \frac{dx_2}{dt} &= -\alpha_{22}x_2 + \alpha_{21}x_1x_2 \end{aligned} \tag{1}$$

for the i-th time half-interval $[t^{(i)})$, within which the student does not master the subject, which looks like

$$\begin{aligned} \frac{dx_1}{dt} &= -\alpha_{11}x_1 + \alpha_{12}x_2x_1 + \beta_{11}x_1, \\ \frac{dx_2}{dt} &= -\alpha_{22}x_2 + \alpha_{21}x_1x_2 \end{aligned} \tag{2}$$

for the j-th time half-interval $[t_1^{(j)})$, within which the student gains additional knowledge communicating with lecturer, which looks like

$$\begin{aligned} \frac{dx_1}{dt} &= -\alpha_{11}x_1 + \alpha_{12}x_2x_1, \\ \frac{dx_2}{dt} &= -\alpha_{22}x_2 + \alpha_{21}x_1x_2 + \beta_{22}x_2 \end{aligned} \tag{3}$$

for the k-th time half-interval $[t_2^{(k)})$, within which the student gains additional knowledge independently.

In the mathematical models (1), (2), (3) α_{11}, α_{22} – factors, characterizing the degree of students forgetting material of the subject, previously learned with the lecturer and independently, α_{12}, α_{21} – factors, characterizing synergetic influence of the components of the learning process with lecturer and independently, β_{11}, β_{22} – factors, characterizing the degree of learning new knowledge during classes with lecturer and independently, and x_1, x_2 – phase coordinates, setting in the relative unit the degree of learning the curriculum by the student during classes with lecturer and independently, for which the following conditions are satisfied:

$$x_1 = \frac{X_1}{X}, x_2 = \frac{X_2}{X}, \tag{4}$$

$$\begin{aligned} &x_1 \le 1, \\ &x_2 \le 1, \\ &x_1 + x_2 \le 1, \end{aligned} \tag{5}$$

where X – the volume of knowledge the student may have upon learning all the sections of specific subject within the specific time T, X_1 the volume of knowledge on the specific subject, which the student gets from the lecturer during classes, and X_2 – the volume of knowledge on the specific subject which the student learns independently by studying specific sections of the subject.

In work [1], assuming that the half intervals $[t^{(i)}), [t_1^{(j)}), [t_2^{(k)})$ follow each other and marking the end points of these half intervals by symbols $t_к^{(i)}$, $t_{1к}^{(j)}$, $t_{2к}^{(k)}$, we determine the initial conditions necessary for unique solution of the system of differential equations (1), (2), (3), in the kind:

For system of equations (1) –

$$\begin{aligned} x_{1n}^{(i)} &= x_1(t_{2к}^{(k-1)}), \\ x_{2n}^{(i)} &= x_2(t_{2к}^{(k-1)}), \end{aligned} \tag{6}$$

For system of equations (2) –

$$\begin{aligned} x_{1n}^{(j)} &= x_1(t_к^{(i)}), \\ x_{2n}^{(j)} &= x_2(t_к^{(i)}), \end{aligned} \tag{7}$$

For system of equations (3) –

$$\begin{aligned} x_{1n}^{(k)} &= x_1(t_{1к}^{(j)}), \\ x_{2n}^{(k)} &= x_2(t_{1к}^{(j)}). \end{aligned} \tag{8}$$

The work [2] started researches of the process of mastering the curriculum by student on the phase plane using the synthesised mathematical models on the set half intervals $[t^{(i)}), [t_1^{(j)}), [t_2^{(k)})$ and there had been determined the singular points $O_l(x_{1o}, x_{2o}), l = 1,2$ of the phase plane.

It had been shown that the three mathematical models have one common singular point $O_1(0,0)$ and each model has one more singular point $O_2(x_{1o}, x_{2o})$, and for the model (1) – this point $O_2(\frac{\alpha_{22}}{\alpha_{21}}, \frac{\alpha_{11}}{\alpha_{12}})$, for the model (2) – this point $O_2(\frac{\alpha_{22}}{\alpha_{21}}, \frac{\alpha_{11} - \beta_{11}}{\alpha_{12}})$, and for the model (3) – this point $O_2(\frac{\alpha_{22} - \beta_{22}}{\alpha_{21}}, \frac{\alpha_{11}}{\alpha_{12}})$. This paper [2] also states, that talking about student with weak logics and memory, that is, about the student with average abilities, than his singular point $O_1(0,0)$ is placed within the sphere of allowed values of phase coordinates, determined by correlations (5), since for this case the conditions are satisfied

$$\alpha_{11} > \alpha_{12}, \alpha_{22} > \alpha_{21}. \tag{9}$$

It had also been ascertained that talking about student with nice logics and memory, it is necessary o note that in the shaded region of the phase plane, which is allowable following conditions of our task, there may be a singular point $O_1(0{,}0)$, and a singular point $O_2(x_{1o}, x_{2o})$, since for this case the conditions are satisfied

$$\alpha_{11} < \alpha_{12}, \alpha_{22} < \alpha_{21}. \qquad (10)$$

The paper [3] researches the characteristics of the singular points on the phase plane and determines the character of the phase trajectories in the vicinity of these singular point for the process of learning the curriculum by student with average abilities.

In this work we suggest the research of the process of mastering curriculum by the student on the phase plane using models (1), (2), (3) in the vicinity of the found singular points and build the phase trajectories for this process, but now for the promising student, that is the student, for whom the conditions (10) are satisfied, which lead to the appearance of the second singular point within the allowed values of phase coordinates on the phase plane.

Determination of the Singular Points Characteristics of Promising Student Models and Building Phase Trajectories in its Vicinity

Since within each of half-intervals $[t^{(i)}), [t_1^{(j)}), [t_2^{(k)})$ the process of mastering the curriculum by promising student is described with different models from the set (1), (2), (3), trajectory character on the phase plane within the allowed values of phase coordinates and singular points characteristics $O_1(0{,}0)$, $O_2(x_{1o}, x_{2o})$ will also differ.

Since differential equations of linear approximation within singular point vicinity $O_1(0{,}0)$ for all models from the set (1), (2), (3) for promising student will be the same as for the student with average abilities, therefore for promising student within observation of the process of mastering the curriculum on the phase plane relatively to singular point $O_1(0{,}0)$ and phase trajectories' characteristics within its vicinity all those results, that we obtained in the work [3] for a student with average abilities, would be correct. Thus, those computations, conducted in the paper [3], relatively to singular point $O_1(0{,}0)$, will not be re-conducted in this paper, only its results will be used and the attention will be paid to identification of the singular point characteristics $O_2(x_{1o}, x_{2o})$ and building both, a phase trajectory within its vicinity and a phase portrait of the process of mastering the curriculum by promising student within every half-interval, mentioned above, as a whole.

Before considering singular point characteristics $O_2(x_{1o}, x_{2o})$ and building phase portraits within every half-interval from $[t^{(i)}), [t_1^{(j)}), [t_2^{(k)})$ as a whole, let us pay attention to one common feature of these characteristics, which means that in paper [3] during the consideration of singular point $O_1(0{,}0)$, and synthesizing linearized equations for models (1), (2), (3), the non-linear components $\alpha_{12}x_2x_1$ and $\alpha_{21}x_1x_2$ were dropped because the linear parts of its expanding in formal power series in this point were equal to zero. But expanding these non-linear components in the vicinity of the singular point $O_2(x_{1o}, x_{2o})$ in formal power series, linear parts would not be equal to zero, therefore they should be taken into consideration during linearization of the models (1), (2), (3). We will make it by expanding the whole right part of each equation in the models (1), (2), (3) in formal power series.

Let's analyze the process of mastering the curriculum by promising student on the phase plane within time half-interval $[t^{(i)})$, within which a student does not learn this subject. In this case the phase plane points $O_1(0{,}0)$ and $O_2(\frac{\alpha_{22}}{\alpha_{21}}, \frac{\alpha_{11}}{\alpha_{12}})$, are within the allowed domain (5), would be the singular points, as it is presented in the paper [2].

Characteristic of the singular point $O_1(0{,}0)$ and character of the phase trajectories in its vicinity is identified in paper [3]: they look like it is presented in fig. 1a.

Now let's define the characteristic of the singular point and character of the phase trajectories in its vicinity. To do this we need to linearize the right part of the equations (1) in the singular point vicinity, as it is presented in the paper [4]. Therefore, expanding the right part of the equation system (1) in the vicinity of the singular point $O_2(x_{1o}, x_{2o})$ in formal power series and accepting only linear components we will receive:

$$\begin{aligned}
&-\alpha_{11}x_1 + \alpha_{12}x_2x_1 = -\alpha_{11}x_{1o} + \alpha_{12}x_{2o}x_{1o} + (-\alpha_{11} + \alpha_{12}x_{2o})(x_1 - x_{1o}) + \alpha_{12}x_{1o}(x_2 - x_{2o}),\\
&-\alpha_{22}x_2 + \alpha_{21}x_1x_2 = -\alpha_{22}x_{2o} + \alpha_{21}x_{1o}x_{2o} + \alpha_{21}x_{2o}(x_1 - x_{1o}) + (-\alpha_{22} + \alpha_{21}x_{1o})(x_2 - x_{2o}).
\end{aligned} \tag{11}$$

Considering that

$$x_{1o} = \frac{\alpha_{22}}{\alpha_{21}}, x_{2o} = \frac{\alpha_{11}}{\alpha_{12}}, \tag{12}$$

correlation (11) may be written as follows:

$$\begin{aligned}
-\alpha_{11}x_1 + \alpha_{12}x_2x_1 &= \alpha_{12}\frac{\alpha_{22}}{\alpha_{21}}(x_2 - x_{2o}),\\
-\alpha_{22}x_2 + \alpha_{21}x_1x_2 &= \alpha_{21}\frac{\alpha_{11}}{\alpha_{12}}(x_1 - x_{1o}).
\end{aligned} \tag{13}$$

Proceeding to the co-ordinates

$$z_1 = x_1 - x_{1o}, z_2 = x_2 - x_{20}, \tag{14}$$

That is, having transferred the beginning of the co-ordinates to the singular point $O_2(\frac{\alpha_{22}}{\alpha_{21}}, \frac{\alpha_{11}}{\alpha_{12}})$ and considering correlation (13), the linearized system of equation for the mathematical model (1) in the vicinity of the singular point may be written as:

$$\begin{aligned}
\frac{dz_1}{dt} &= \alpha_{12}\frac{\alpha_{22}}{\alpha_{21}}z_2,\\
\frac{dz_2}{dt} &= \alpha_{21}\frac{\alpha_{11}}{\alpha_{12}}z_1.
\end{aligned} \tag{15}$$

Matrix of coefficients of the system of differential equations (15) will look like

$$A = \begin{bmatrix} 0 \ldots\ldots\ldots\ldots \alpha_{12}\dfrac{\alpha_{22}}{\alpha_{21}} \\ \\ \alpha_{21}\dfrac{\alpha_{11}}{\alpha_{12}} \ldots\ldots\ldots 0 \end{bmatrix}, \tag{16}$$

And the characteristic equation –

$$\begin{vmatrix} 0-\lambda \ldots\ldots\ldots \alpha_{12}\dfrac{\alpha_{22}}{\alpha_{21}} \\ \\ \alpha_{21}\dfrac{\alpha_{11}}{\alpha_{12}} \ldots\ldots 0-\lambda \end{vmatrix} = 0, \tag{17}$$

or

$$\lambda^2 - \alpha_{11}\alpha_{22} = 0. \tag{18}$$

Roots of the characteristic equation (18) are

$$\lambda_1 = \sqrt{\alpha_{11}\alpha_{22}}, \qquad \lambda_2 = -\sqrt{\alpha_{11}\alpha_{22}}. \tag{19}$$

Since they are real numbers of different signs, the singular point $O_2(\frac{\alpha_{22}}{\alpha_{21}}, \frac{\alpha_{11}}{\alpha_{12}})$ – is a «saddle».

To determine the character of the phase trajectory in the vicinity of this singular point, we divide the second equation of the system (15) by the first one. In the result we get

$$\frac{dz_2}{dz_1} = \frac{\alpha_{11}}{\alpha_{22}} \frac{z_1}{z_2}, \tag{20}$$

or

$$z_2 dz_2 = \frac{\alpha_{11}}{\alpha_{22}} z_1 dz_1. \tag{21}$$

Integrating the equation (21), we get

$$z_2^2 = \frac{\alpha_{11}}{\alpha_{22}} z_1^2 + 2C, \tag{22}$$

where C – constant of integration, numerical value of which shall be found proceeding from the initial conditions (6).

Curve of the equation (22) is a hyperbola [5], to build which, this equation is better to rewrite as

$$z_2 = \pm\sqrt{\frac{\alpha_{11}}{\alpha_{22}} z_1^2 + 2C}. \tag{23}$$

From equation (23) it is seen, that the hyperbola has two vertexes with coordinates $(0, \sqrt{2C})$, $(0, -\sqrt{2C})$ and « antenna» in the kind of two straight lines:

$$z_2 = \sqrt{\frac{\alpha_{11}}{\alpha_{22}}} z_1, \tag{24}$$

$$z_2 = -\sqrt{\frac{\alpha_{11}}{\alpha_{22}}} z_1. \tag{25}$$

To determine the direction of movement of the point on the hyperbola phase trajectories, we go back to the differential equations (15). Substituting the equation (24), we get

$$\begin{aligned} \frac{dz_1}{dt} &= \alpha_{12} \frac{\alpha_{22}}{\alpha_{21}} \sqrt{\frac{\alpha_{11}}{\alpha_{22}}} z_1, \\ \frac{dz_2}{dt} &= \alpha_{21} \frac{\alpha_{11}}{\alpha_{12}} \sqrt{\frac{\alpha_{22}}{\alpha_{11}}} z_2, \end{aligned} \tag{26}$$

or

$$\begin{aligned} \frac{dz_1}{z_1} &= \frac{\alpha_{12}}{\alpha_{21}} \sqrt{\alpha_{11}\alpha_{22}}\, dt, \\ \frac{dz_2}{z_2} &= \frac{\alpha_{21}}{\alpha_{12}} \sqrt{\alpha_{11}\alpha_{22}}\, dt. \end{aligned} \tag{27}$$

Integrating the equation (26), (27), we receive

$$\ln z_1 = \frac{\alpha_{12}}{\alpha_{21}} \sqrt{\alpha_{11}\alpha_{22}}\, t + \ln C_1, \tag{28}$$

$$\ln z_2 = \frac{\alpha_{21}}{\alpha_{12}} \sqrt{\alpha_{11}\alpha_{22}}\, t + \ln C_2, \tag{29}$$

where $C_1, C_2 -$ constant of integration, numerical value of which shall be determined by the initial conditions (6), and in the spheres of positive values of phase coordinates z_1, z_2 these constants will have signs «plus», and in the spheres of negative values – signs «minus».

Equation (28), (29) may also be writes like:

$$z_1 = C_1 e^{\frac{\alpha_{12}}{\alpha_{21}}\sqrt{\alpha_{11}\alpha_{22}}\, t}, \tag{30}$$

$$z_2 = C_2 e^{\frac{\alpha_{21}}{\alpha_{12}}\sqrt{\alpha_{11}\alpha_{22}}\, t}. \tag{31}$$

Expressions (30), (31) show that the increase in values of time *t* causes the increase in numerical value of phase coordinates on the absolute value, which, in turn means, that on the «antenna» (24) the phase point moves in the direction form the singular point $O_2(\frac{\alpha_{22}}{\alpha_{21}}, \frac{\alpha_{11}}{\alpha_{12}})$ both ways.

Having made the same computations with the « antenna » (25), we obtain that:

$$z_1 = C_1 e^{-\frac{\alpha_{12}}{\alpha_{21}}\sqrt{\alpha_{11}\alpha_{22}}\, t}, \tag{32}$$

$$z_2 = C_2 e^{-\frac{\alpha_{21}}{\alpha_{12}}\sqrt{\alpha_{11}\alpha_{22}}\, t}. \tag{33}$$

Expressions (32), (33) show that, the increase in values of time *t* causes the decrease in numerical value of phase coordinates on the absolute value, which, in turn means, that on the « antenna »(25), the phase point moves in the direction form the singular point $O_2(\frac{\alpha_{22}}{\alpha_{21}}, \frac{\alpha_{11}}{\alpha_{12}})$ from both ends.

The above as for the process of mastering the curriculum by the promising student is graphically presented on fig. 1б. in the time half interval $[t^{(i)})$.

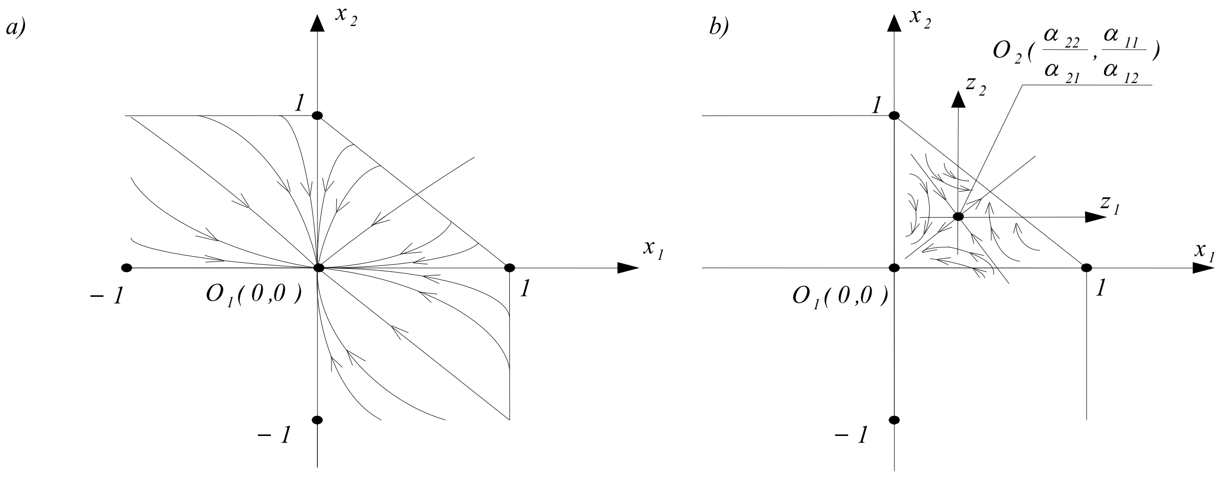

Fig. 1. Phase trajectories of the process of learning the curriculum by the promising student on the period of time, when the student does not work over the subject neither with lecturer nor independently, in the vicinity of the singular point $O_1(0,0)$ and in the vicinity of the singular point

$$O_2\left(\frac{\alpha_{22}}{\alpha_{21}},\frac{\alpha_{11}}{\alpha_{12}}\right)$$

Fig. 2 presents the phase portrait of the process of learning the curriculum by the promising student on the period of time, when the student does not work over the subject neither with lecturer nor independently, «compiled » from the phase trajectories (fig. 1).

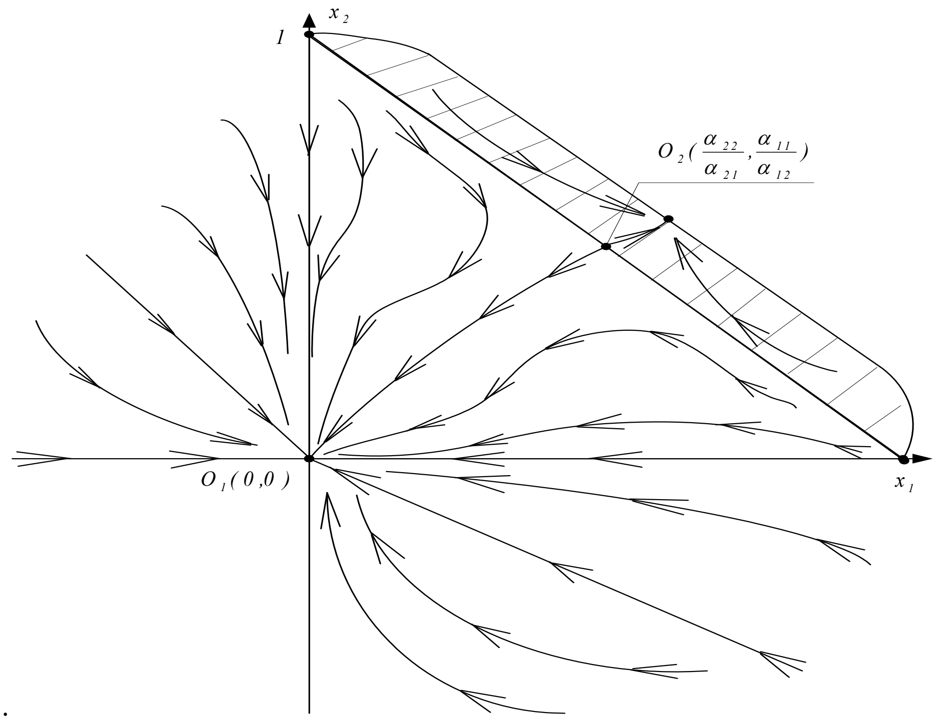

Fig. 2. Phase portrait of the process of learning the curriculum by the promising student on the period of time, when the student does not work over the subject neither with lecturer nor independently

On the phase portrait, presented in fig. 2, when the phase point falls into crosshatched region there takes place the process, which qualitatively differs from that, which takes place when the phase point falls into any other region of the phase plane, since this is the region, where even without further learning with lecturer or independently a student due to synergetic effect caused by ental subconscious brain work entirely masters the curriculum. We will call this region a genius region, since it takes place for a very small number of promising students.

Let’s to analyze the process of mastering the curriculum by promising student on the phase plane within the time half-interval $[t_1^{(j)})$, within which a student gains additional knowledge communicating with lecturer. In this case the phase plane points $O_1(0,0)$ and $O_2(\frac{\alpha_{22}}{\alpha_{21}},\frac{\alpha_{11}-\beta_{11}}{\alpha_{12}})$, lying within the allowed region (5), will be the singular points of the process as it is presented in the work [2].

The characteristics of the singular point $O_1(0,0)$ as well as the character of phase trajectories in its vicinity are determined in [3]: they look as shown in fig. 3a for the case, when the student is «smarter» than the lecturer ($\beta_{11} < \alpha_{11}$), and on fig. 3б for the case, when the lecturer is «smarter» than the student ($\beta_{11} > \alpha_{11}$).

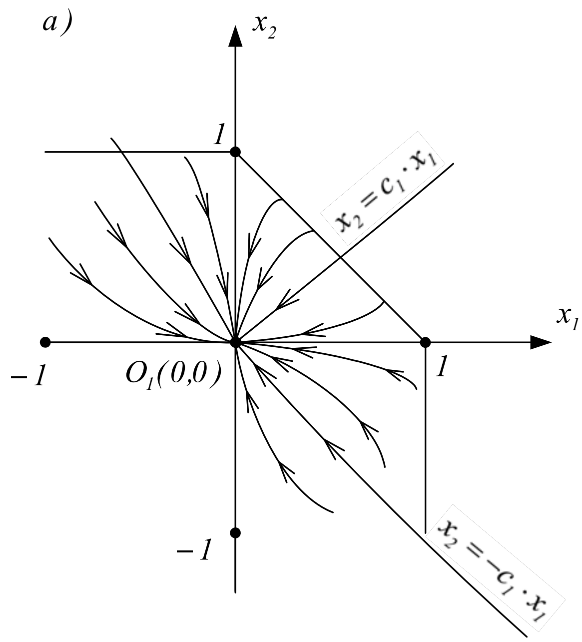

Fig. 3. Phase trajectories of the process of mastering the curriculum by the promising student in the vicinity of the singular point $O_1(0,0)$ in the period of time, when the student works over the subject with lecturer and the student is «smarter» than a lecturer (a), and when the lecturer is «smarter» than a student (b)

Let's define the characteristic of the singular point $O_2(\frac{\alpha_{22}}{\alpha_{21}}, \frac{\alpha_{11}-\beta_{11}}{\alpha_{12}})$ and phase trajectories' character in its vicinity. To do it, as well as in the previous case, we need to linearize the right part of the equations (2) in the vicinity of this singular point. Therefore, expanding right part of the equation system (2) in the vicinity of the singular point in formal power series and accepting only linear components we will obtain:

$$\begin{aligned}&(-\alpha_{11}+\beta_{11})x_1+\alpha_{12}x_2x_1=(-\alpha_{11}+\beta_{11})x_{1o}+\alpha_{12}x_{2o}x_{1o}+(-\alpha_{11}+\beta_{11}+\alpha_{12}x_{2o})(x_1-x_{1o})+\\&+\alpha_{12}x_{1o}(x_2-x_{2o}),\\&-\alpha_{22}x_2+\alpha_{21}x_1x_2=-\alpha_{22}x_{2o}+\alpha_{21}x_{1o}x_{2o}+\alpha_{21}x_{2o}(x_1-x_{1o})+(-\alpha_{22}+\alpha_{21}x_{1o})(x_2-x_{2o}).\end{aligned} \quad (34)$$

Considering that

$$x_{1o}=\frac{\alpha_{22}}{\alpha_{21}}, x_{2o}=\frac{\alpha_{11}-\beta_{11}}{\alpha_{12}}, \quad (35)$$

correlation (34) may be written as follows:

$$\begin{aligned}&-\alpha_{11}x_1+\alpha_{12}x_2x_1=\alpha_{12}\frac{\alpha_{22}}{\alpha_{21}}(x_2-x_{2o}),\\&-\alpha_{22}x_2+\alpha_{21}x_1x_2=\alpha_{21}\frac{\alpha_{11}}{\alpha_{12}}(x_1-x_{1o}).\end{aligned} \quad (36)$$

Proceeding to the co-ordinates z_1, z_2, following the equations (14), that is having transferred the beginning of the co-ordinates to the singular point $O_2(\frac{\alpha_{22}}{\alpha_{21}}, \frac{\alpha_{11}-\beta_{11}}{\alpha_{12}})$ and considering correlation (36), the linearized system of equation for the mathematical model (2) in the vicinity of the singular point we get the same as in the previous case, i.e. it looks like (15). And this means that for this case all calculations ranging from the matrix (16) to correlations for phase coordinates (32), (33) are also correct.

Therefore for this case all results correspondent to the characteristic of the singular point

$O_2(\frac{\alpha_{22}}{\alpha_{21}},\frac{\alpha_{11}-\beta_{11}}{\alpha_{12}})$ and phase trajectories' character in its vicinity are equivalent to the previous ones, that means that fig. 1b is true in this case.

"Compiling" phase trajectories, presented in fig. 3a and fig. 1b, we will get the phase portrait of the process of mastering the curriculum by promising student in the period of time, when a student works over the subject with a lecturer and when a student is «smarter» than a lecturer (fig. 4a), and "compiling" phase trajectories, presented in fig. 3b and fig. 1b, we will get the phase portrait of the equivalent process when a lecturer is «smarter» than a student (fig. 4b).

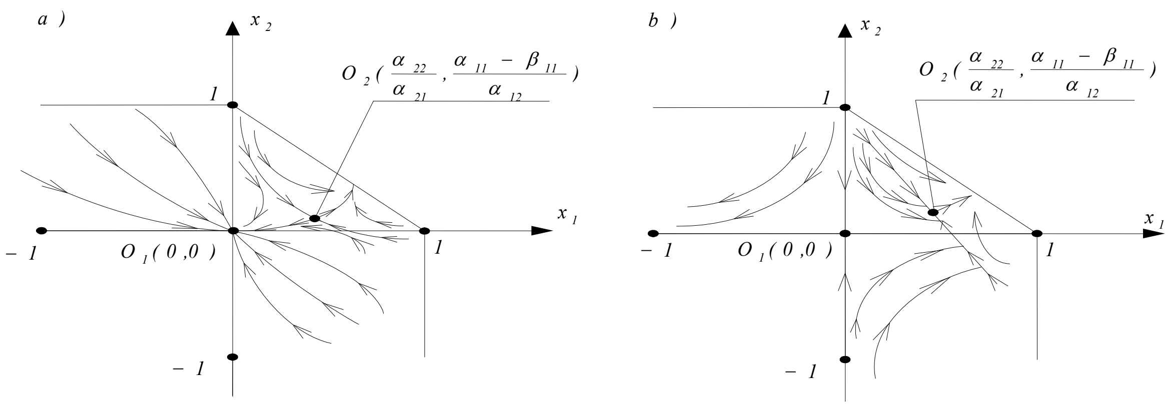

Fig. 4. Phase trajectories of the process of mastering the curriculum by promising student in the period of time, when a student works over the subject with a lecturer and a student is «smarter» than a lecturer (a), and when a lecturer is «smarter» than a student (b)

Let's analyze the process of mastering the curriculum by promising student on the phase plane within time half-interval $[t_2^{(k)})$, within which a student gains additional knowledge learning independently. In this case the phase plane points $O_1(0,0)$ and $O_2(\frac{\alpha_{22}-\beta_{22}}{\alpha_{21}},\frac{\alpha_{11}}{\alpha_{12}})$, lying within the allowed region (5), will be the singular points of the process as it is presented in the work [2].

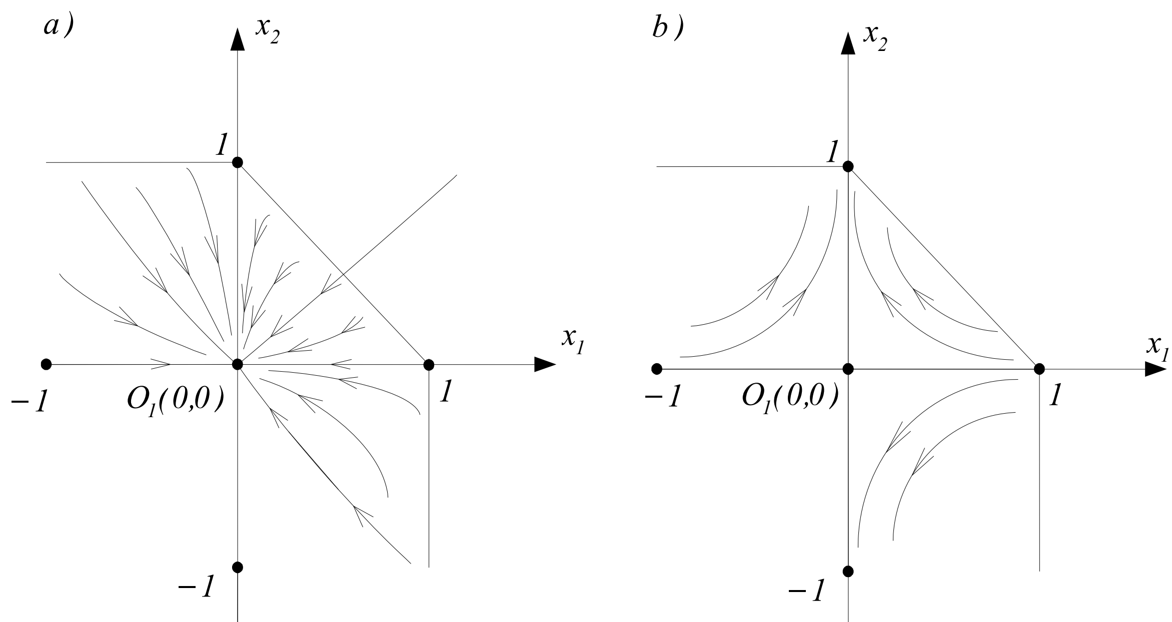

Fig. 5. Phase trajectories of the process of mastering the curriculum by promising student in the singular point vicinity in the period of time, when a student works over the subject independently and a student is «smarter» than a text book (a), as well as when a text book is «smarter» than a student (b)

The characteristics of the singular point $O_1(0,0)$ as well as the character of phase trajectories in its vicinity are determined in [3]: they look as it is shown in fig. 5a for the case, when a student is «smarter» than a text book ($\beta_{22} < \alpha_{22}$), and in fig. 5b for the case, when a text book is «smarter» than a student ($\beta_{22} > \alpha_{22}$).

Let's define the characteristic of the singular point $O_2(\frac{\alpha_{22}-\beta_{22}}{\alpha_{21}}, \frac{\alpha_{11}}{\alpha_{12}})$ and phase trajectories' character in its vicinity. To do this, just like in the previous cases we need to linearize the right part of the equations (3) in the vicinity of this singular point. Therefore, expanding right part of the equation system (3) in the vicinity of the singular point in formal power series and accepting only linear components we will obtain:

$$\begin{aligned}
&-\alpha_{11}x_1+\alpha_{12}x_2x_1=-\alpha_{11}x_{1o}+\alpha_{12}x_{2o}x_{1o}+(-\alpha_{11}+\alpha_{12}x_{2o})(x_1-x_{1o})+\\
&+\alpha_{12}x_{1o}(x_2-x_{2o}),\\
&(-\alpha_{22}+\beta_{22})x_2+\alpha_{21}x_1x_2=(-\alpha_{22}+\beta_{22})x_{2o}+\alpha_{21}x_{1o}x_{2o}+\alpha_{21}x_{2o}(x_1-x_{1o})+\\
&+(-\alpha_{22}+\beta_{22}+\alpha_{21}x_{1o})(x_2-x_{2o}).
\end{aligned} \tag{37}$$

Considering that

$$x_{1o}=\frac{\alpha_{22}-\beta_{22}}{\alpha_{21}}, x_{2o}=\frac{\alpha_{11}}{\alpha_{12}}, \tag{38}$$

correlation (37) may be written as follows:

$$\begin{aligned}
&-\alpha_{11}x_1+\alpha_{12}x_2x_1=\alpha_{12}\frac{\alpha_{22}-\beta_{22}}{\alpha_{21}}(x_2-x_{2o}),\\
&-\alpha_{22}x_2+\alpha_{21}x_1x_2=\alpha_{21}\frac{\alpha_{11}}{\alpha_{12}}(x_1-x_{1o}).
\end{aligned} \tag{39}$$

Proceeding to the co-ordinates z_1, z_2, following the equations (14), that is having transferred the beginning of the co-ordinates to the singular point $O_2(\frac{\alpha_{22}-\beta_{22}}{\alpha_{21}}, \frac{\alpha_{11}}{\alpha_{12}})$ and considering correlation (39), the linearized system of equation for the mathematical model (3) in the vicinity of the singular point we will get the same as in the previous cases, i.e. it looks like (15) with that slight difference, that in the first equation there is a coefficient $\alpha_{22}-\beta_{22}$ instead of α_{22}. And this means that for this case all calculations ranging from the matrix (16) to correlations for phase coordinates (32), (33) with the coefficient $\alpha_{22}-\beta_{22}$ instead of α_{22} are also correct.

But due to this change only those results considering the characteristics of the singular point $O_2(\frac{\alpha_{22}-\beta_{22}}{\alpha_{21}}, \frac{\alpha_{11}}{\alpha_{12}})$ and phase trajectories' character in its vicinity, that correspond to a student, who is "smarter" then a text book will have the equivalent to previous values of indicated coefficients for this case. In this very case the phase trajectories' character in the vicinity of indicated singular point will have the same character as it is presented in fig 1b, which for convenience we will show again in fig. 6a with new coordinates.

As for the case, when a text book is "smarter" then a student, instead of correlation (19) we will get

$$\lambda_1=\sqrt{\alpha_{11}(\alpha_{22}-\beta_{22})}, \qquad \lambda_2=-\sqrt{\alpha_{11}(\alpha_{22}-\beta_{22})}, \tag{40}$$

which when $\beta_{22} > \alpha_{22}$ will define complex conjugates. And instead of the equation (22) we will receive

$$z_2^2 = \frac{\alpha_{11}}{\alpha_{22} - \beta_{22}} z_1^2 + 2C, \qquad (41)$$

Which when $\beta_{22} > \alpha_{22}$ will not define a hyperbola, but ellipse with long axis on the axis z_1 and c direction of point moving along phase trajectory «from left to right» in the lower part and «from right to left» in the upper one (fig. 6b). And in this case the singular point $O_2(\frac{\alpha_{22}-\beta_{22}}{\alpha_{21}}, \frac{\alpha_{11}}{\alpha_{12}})$ transforms from «saddle» to «centre».

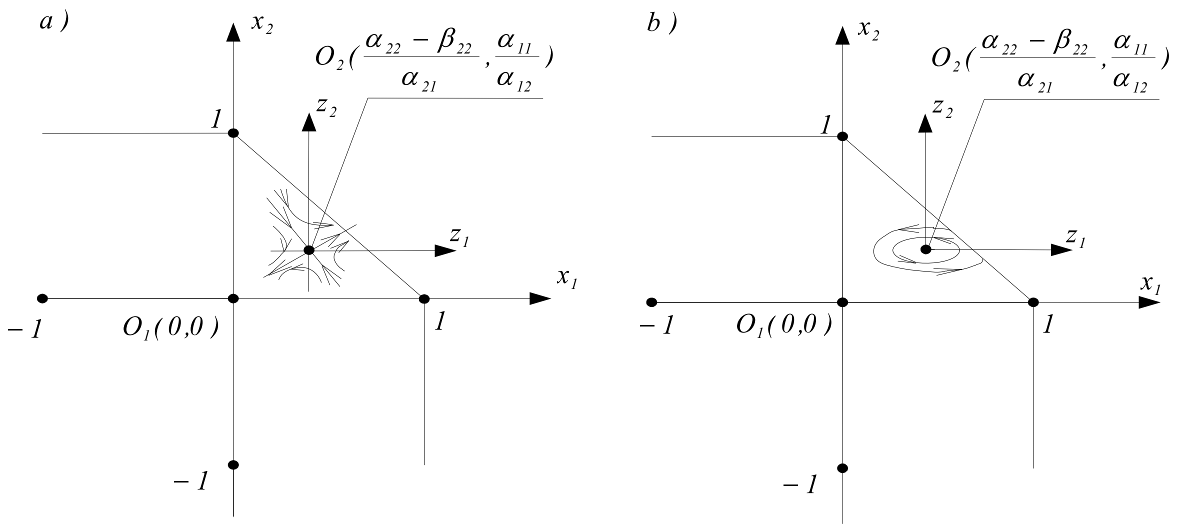

Fig. 6. Phase trajectories of the process of mastering the curriculum by promising student in the vicinity of the singular point $O_2(\frac{\alpha_{22}-\beta_{22}}{\alpha_{21}}, \frac{\alpha_{11}}{\alpha_{12}})$ in the period of time, when a student works over the subject independently and a student is «smarter» than a text book (a), and when a text book is «smarter» than a student (b)

"Compiling" phase trajectories, presented in fig 5a and fig.6a, we will get the phase portrait of the process of mastering the curriculum by promising student in the period of time, when a student works over the subject independently and when a student is «smarter» than the text book (fig. 7a). And "compiling" phase trajectories, presented in fig. 5b and fig. 6b, we will get the phase portrait of the equivalent process when a text book is «smarter» than a student (fig. 7b).

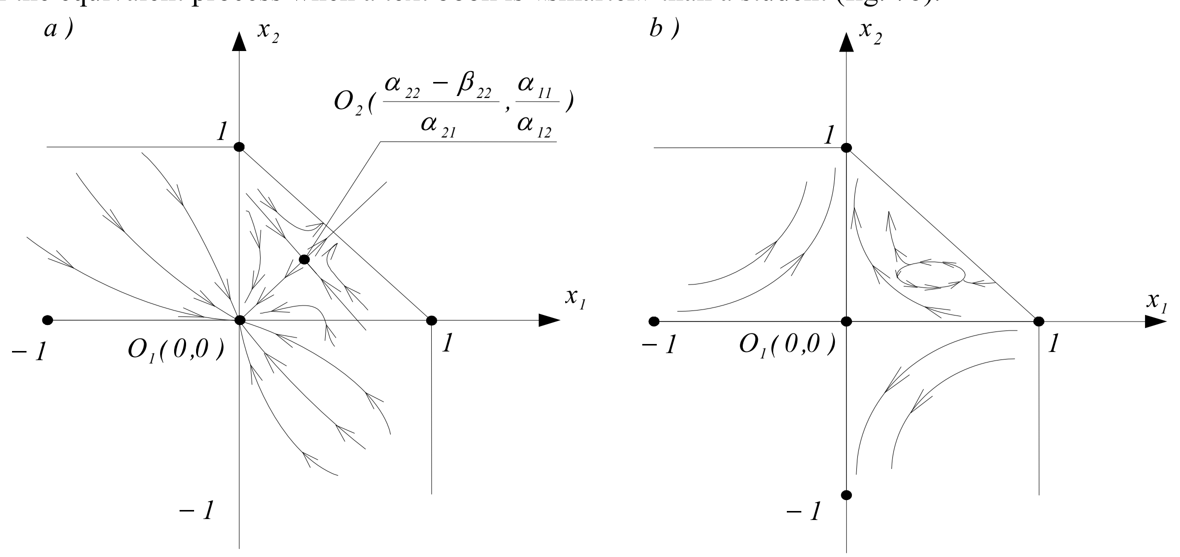

Fig. 7. Phase trajectories of the process of learning the curriculum by the promising student in the period of time, when the student works over the subject independently and the student is «smarter» than the text book(a), and when the text book is «smarter» than the student (b)

Conclusions

The phase portraits of mastering the curriculum by promising students considerably differ from the phase portraits of the equivalent process for the students with average abilities.

On the phase portraits of mastering the curriculum by promising students there had been found the “genius” region, falling into which the phase point reaches the knowledge margin even when the student ceases working over the subject both with lecturer and independently.

There had been shown, that “genius” region is formed due to synergetic effect at subconscious level subsistent only to very promising students and it allows them to create knowledge, that they had not gained with a lecturer or from a study letter, independently.

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