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MATHEMATICAL MODELING OF ASYNCHRONOUS MOTORS IN START-UP MODES IN CONDITIONS OF NON-SINUSOIDAL SUPPLY VOLTAGE

The nonlinear mathematical model of the induction motor is developed for the analysis of starting modes in conditions of supply voltage non-sinusoidality.

Keywords: *induction motor, mathematical model, non-sinusoidality supply voltage, starting mode.*

Problem consideration and problem set-up

Non-sinusoidal modes of electric network negatively influence the operation asynchronous motors (AM). Their efficiency factor drops and dynamic characteristics decrease. Inadmissible, according to GOST (State Standard) 13109-97 content of higher harmonics in electric networks creates braking moment that can result in disconnection of AM at certain speed. Application of filter-compensating devices in start-up modes of AM requires technical economic substantiation.

Substantiation of results

At sinusoidal voltage supply the direction and rotation rate of magnetizing forces of stator and rotor coincide. If higher harmonics of direct phase alteration (4^{-th}, 7-*th*, 10^{-th}, 13^{-th}, ...) are available, harmonic components of stator field rotate in the same direction as the main harmonic component with speeds less than the speeds of main harmonic 4, 7, 10, 13, ... times correspondingly [1]. Beginning from the start moment and up to the moment when the speeds of harmonic components of stator field become equal to the speed of rotor, these harmonics create moments, which act in accordance with the moment of main harmonic, and from the period, when their speeds become grater than the speed of rotor, they create moments which are directed towards the moment of main harmonic. Harmonic components of stator field of reverse phase alternation (2^{-th}, 5^{-th}, 8^{-th}, 11^{-th}, ...) rotate in reverse direction with the speed 2, 5, 8, 11,...times less than the speed of main harmonic correspondingly. They create moments, always directed against the moment of main harmonic.

AM equations system we will present in generalized form

$$\mathbf{L}\frac{d\mathbf{i}}{dt} = \mathbf{u} - \mathbf{Z}\mathbf{i}, \qquad (1)$$

where $\mathbf{L}$, $\mathbf{Z}$ – are matrices of inductances and resistances of AM; $\mathbf{u}$, $\mathbf{i}$ – are vectors of voltages and currents of AM; $\frac{d\mathbf{i}}{dt}$ – is the vector of AM derivative currents in time.

The system pf differential equations in orthogonal coordinates is the simplest for AC study, in case of sinusoidal supply voltage [3].

$$\begin{bmatrix} L_s & 0 & L_m & 0 \\ 0 & L_s & 0 & L_m \\ L_m & 0 & L_r & 0 \\ 0 & L_m & 0 & L_r \end{bmatrix} \begin{bmatrix} \frac{di_{s\alpha}}{dt} \\ \frac{di_{s\beta}}{dt} \\ \frac{di_{r\alpha}}{dt} \\ \frac{di_{r\beta}}{dt} \end{bmatrix} = \begin{bmatrix} u_{s\alpha} \\ u_{s\beta} \\ u_{r\alpha} \\ u_{r\beta} \end{bmatrix} - \begin{bmatrix} R_s & 0 & 0 & 0 \\ 0 & R_s & 0 & 0 \\ 0 & \omega_r L_m & R_r & \omega_r L_r \\ -\omega_r L_m & 0 & -\omega_r L_r & R_r \end{bmatrix} \begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \\ i_{r\alpha} \\ i_{r\beta} \end{bmatrix}, \qquad (2)$$

where $u_{s\alpha}, u_{s\beta}$ – are stator voltages by axis α, β correspondingly; $u_{r\alpha}, u_{r\beta}$ – are rotor voltages by axis α, β correspondingly; $i_{s\alpha}, i_{s\beta}$, $i_{r\alpha}, i_{r\beta}$ –are stator and rotor windings by axis α, β correspondingly; R_s, R_r – are active resistances of stator and rotor, correspondingly; L_s, L_r – are complete inductances of stator and rotor; $L_m = \frac{\psi_m}{i_m}$ – is static inductance of magnetizing; ω_r – is frequency of rotor rotation.

Model (2) can be used for investigation of AM starting modes at nonsinusoidal supply voltage. In accordance with [4] we assume, that each of field harmonics is created by the pair of windings, located at the stator and rotor by axes α, β.

Having expanded the system of equations (2), for instance, if one higher ν^{-th} harmonic is available, to the form:

$$\begin{bmatrix} \mathbf{L}_1 & 0 \\ 0 & \mathbf{L}_\nu \end{bmatrix} \begin{bmatrix} \frac{d\mathbf{i}_1}{dt} & \frac{d\mathbf{i}_\nu}{dt} \end{bmatrix} = \begin{bmatrix} \mathbf{u}_1 \\ \mathbf{u}_\nu \end{bmatrix} - \begin{bmatrix} \mathbf{Z}_1 & 0 \\ 0 & \mathbf{Z}_\nu \end{bmatrix} \begin{bmatrix} \mathbf{i}_1 \\ \mathbf{i}_\nu \end{bmatrix}, \tag{3}$$

Where $\mathbf{L}_1, \mathbf{L}_\nu$; $\mathbf{Z}_1, \mathbf{Z}_\nu$ – are submatrices of inductances and resistance of AM; $\mathbf{u}_1, \mathbf{u}_\nu$; $\mathbf{i}_1, \mathbf{i}_\nu$ – are subvector of voltage and currents of AM; $\frac{d\mathbf{i}_1}{dt}, \frac{d\mathbf{i}_\nu}{dt}$ – are subvectors of derivative currents by time, for basic and ν-th harmonics, and having deduced the equation of mechanical motion

$$\frac{d\omega_r}{dt} = \frac{\frac{3}{2} p_0 L_m \left[\left(i_{s\beta(1)} i_{r\alpha(1)} - i_{s\alpha(1)} i_{r\beta(1)} \right) + \left(i_{s\beta(\nu)} i_{r\alpha(\nu)} - i_{s\alpha(\nu)} i_{r\beta(\nu)} \right) \right] - M}{J}, \tag{4}$$

where M – is mechanical moment; J – is moment of inertia; p_0 – is number of pairs of machine poles, we can analyze starting modes of AM, in case of nonsinusoidal supply voltage.

Models, based on application of nonlinear differential; equations [5] are more complex. Using the inductances, that take into account non-linearity of magnetizing circuit $l_\alpha = (L_\partial - L_m)\left(\frac{i_\alpha}{i_m}\right)^2$; $l_{\alpha\beta} = (L_\partial - L_m)\left(\frac{i_\alpha i_\beta}{i_m^2}\right)$; $l_\beta = (L_\partial - L_m)\left(\frac{i_\beta}{i_m}\right)^2$, where $L_\partial = \frac{d\psi_m}{di_m}$ – is differential magnetizing inductance [6], mathematical model of AM will take the form:

$$\begin{bmatrix} L_s + l_\alpha & l_{\alpha\beta} & L_m + l_\alpha & l_{\alpha\beta} \\ l_{\alpha\beta} & L_s + l_\beta & l_{\alpha\beta} & L_m + l_\beta \\ L_m + l_\alpha & L_{\alpha\beta} & L_r + l_\alpha & l_{\alpha\beta} \\ l_{\alpha\beta} & L_m + l_\beta & l_{\alpha\beta} & L_r + l_\beta \end{bmatrix} \begin{bmatrix} \frac{di_{s\alpha}}{dt} \\ \frac{di_{s\beta}}{dt} \\ \frac{di_{r\alpha}}{dt} \\ \frac{di_{r\beta}}{dt} \end{bmatrix} = \begin{bmatrix} u_{s\alpha} \\ u_{s\beta} \\ u_{r\alpha} \\ u_{r\beta} \end{bmatrix} - \begin{bmatrix} R_s & 0 & 0 & 0 \\ 0 & R_s & 0 & 0 \\ 0 & \omega_r L_m & R_r & \omega_r L_r \\ -\omega_r L_m & 0 & -\omega_r L_r & R_r \end{bmatrix} \begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \\ i_{r\alpha} \\ i_{r\beta} \end{bmatrix}, \tag{5}$$

In order to perform the analysis of energy indices while starting processes current values of active and reactive powers are used; to define these values the integration on sliding interval of time during half period of supply voltage (current) of AM of moment values product is performed [7]:

$$P(t)=\frac{1{,}5}{T/2}\int_{t-T/2}^{t}(u_{s\alpha}(t)i_{s\alpha}(t)+u_{s\beta}(t)i_{s\beta}(t))dt;$$
$$Q(t)=\frac{1{,}5}{T/2}\int_{t-T/2}^{t}(u'_{s\alpha}(t)i_{s\alpha}(t)+u'_{s\beta}(t)i_{s\beta}(t))dt, \qquad (6)$$

where $u'_{s\alpha}(t)$; $u'_{s\beta}(t)$ – are moment values of stator voltage components in α, β-coordinates, that shifted by phase at π/2 angle.

For the solution of equations system (3), (4) we use Runger-Kut method of the fourth order. Fig 1 shows time dependences of AM rotor rotation frequency in case of sinusoidal (1) and nonsinusoidal supply voltages if the content of the seventh harmonic is 5 %.. Fig 2 shows dependences of electromagnetic moments, created by total currents (1) and currents of the seventh harmonic (2).

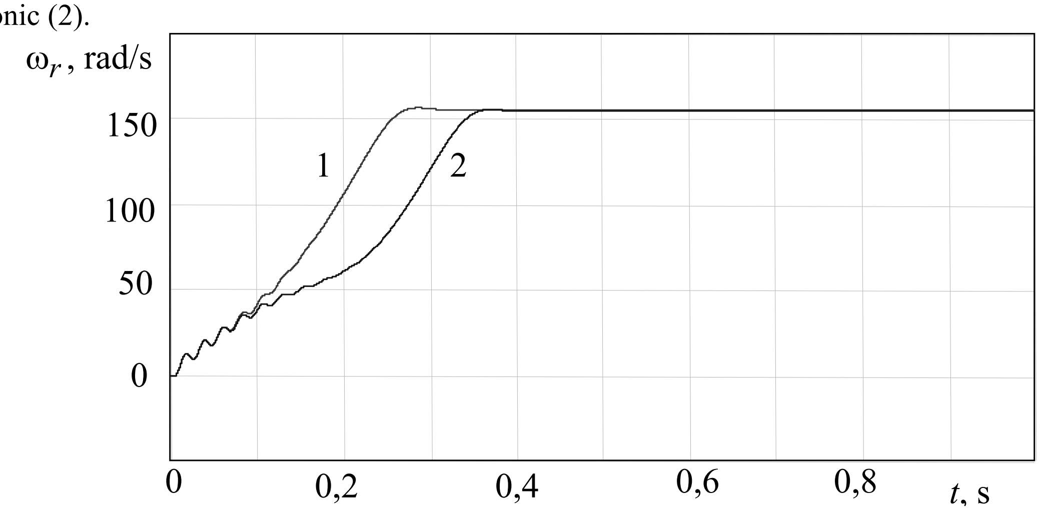

Fig. 1. Dependences of rotation frequency while AM start at sinusoidal and nonsinusoidal supply voltage

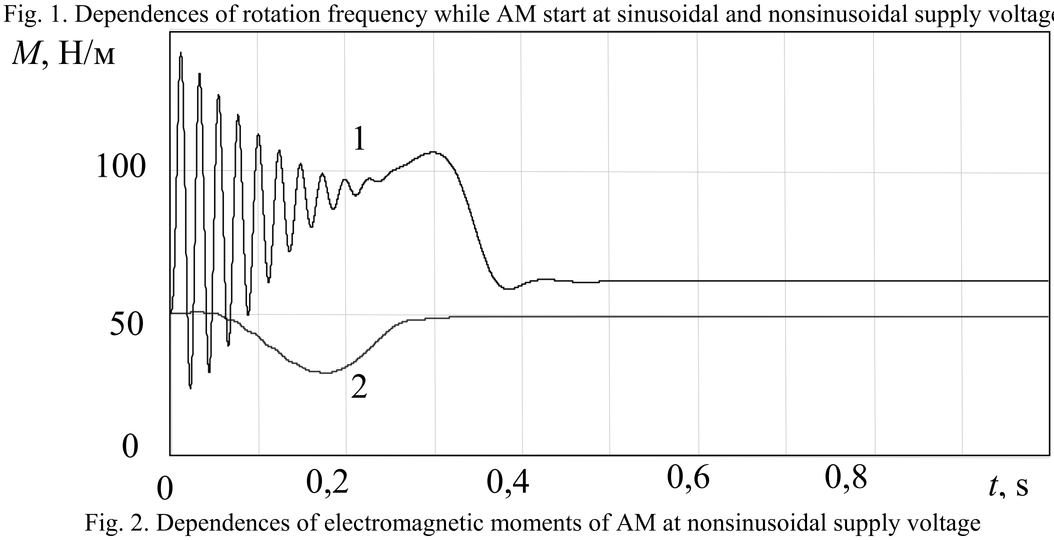

Fig. 2. Dependences of electromagnetic moments of AM at nonsinusoidal supply voltage

Fig 3 and 4 show the dependences of active and reactive powers, obtained while AM modeling at sinusoidal (1) and non sinusoidal (2) supply voltages.

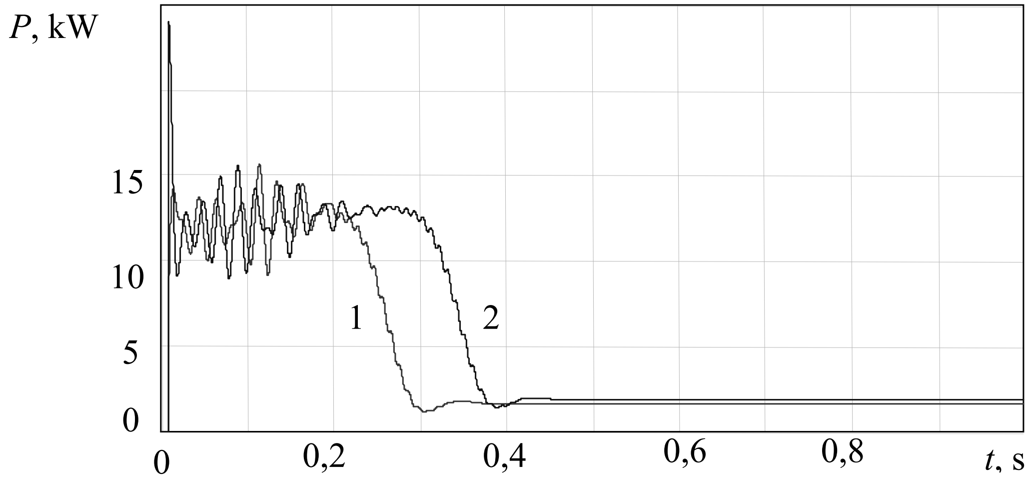

Fig. 3. Dependences of active power at AM start of sinusoidal and nonsinusoidal supply voltage

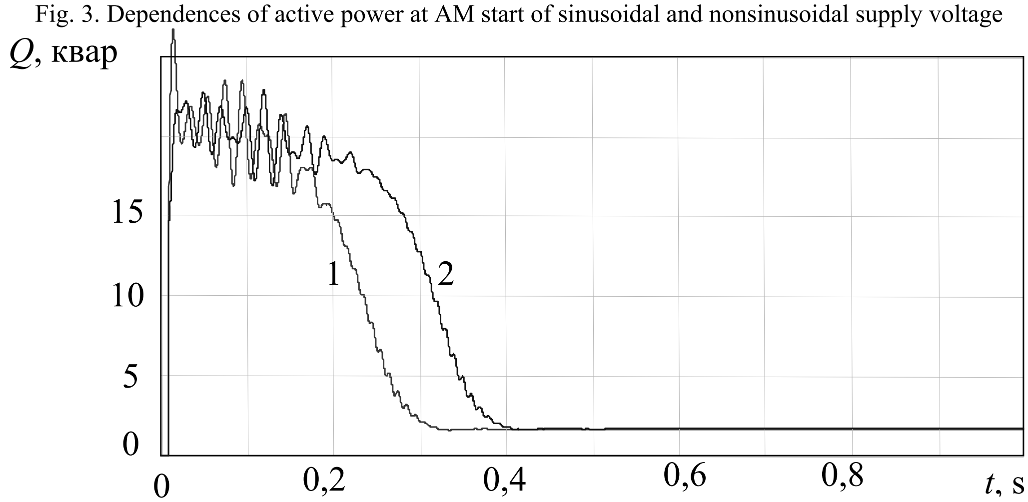

Fig. 4. Dependences of reactive power of AM at sinusoidal and nonsinusoidal supply voltage

Application of time dependences of active and reactive power enables to analyze the efficiency of usage of filter-compensating devices in start modes of asynchronous motors.

Conclusions

Non-linear mathematical model of asynchronous motor for the analysis of starting modes at nonsinusoidality of supply voltage is developed. Dependences of active and reactive powers while motor start are analyzed.

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