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CONCRETE STRENGTH PREDICTION USING LINGUISTIC VARIABLES OF THE FUZZY LOGIC APPARATUS

Fuzzy logic-based calculation procedure is proposed to determine the influence of physical-mechanical parameters of concrete components, the technological parameters of its processing, mixing water quality on the strength of concrete. Hierarchic system of logic equations used as a knowledge base for mathematical model elaboration makes it possible to estimate the value of the predicted concrete strength index as a function of the above-mentioned factors.

Key words: fuzzy logic, mathematical model, prediction, the strength of concrete.

Introduction

At present, high-rise building construction requires high-strength concrete application to reduce the specific consumption of concrete, which leads to corresponding reduction of the construction pressure on the foundation. It is evident that selection of the concrete composition with pre-defined properties, particularly with the ultimate compression strength, is especially important today. Mathematical modeling is one of the efficient methods for obtaining concrete with pre-defined properties, particularly with predefined compression strength. A number of authors proposed procedures for designing concretes with pre-defined properties [1 – 6]. Each of the procedures has its own advantages but none of them takes fully into account the concrete strength influence factors that are characterized not only by quantitative, but also by qualitative indices. Fuzzy logic-based modeling is one of the effective tools of taking into account qualitative and quantitative influence factors of the concrete mix components and of the technological processing type on the ultimate strength of concrete [7, 8, 9].

The research goal is creation of an expert modeling system for multifactor analysis of the influence of the main concrete mix composition indices on the predicted strength of concrete. Those are physical-mechanical parameters of the fillers and technological parameters of the concrete mix treatment. For solving this problem, mathematical apparatus, based on the fuzzy logic theory, is proposed. The method of nonlinear object identification using fuzzy knowledge bases [7] as an interrelated set of mathematical models makes it possible to use expert linguistic information to determine the optimal composition of concrete fillers and the type of treatment proceeding from the results of virtual experiment.

Research materials and results

In order to reveal hierarchic connections and factors that affect the strength of concrete, a “black box” model was proposed (fig. 1). Factors that affect the strength of concrete are given as input signals, as an output signal the concrete compression strength is indicated. All the factors of influence are divided into two groups: technological factors of the concrete mixture treatment and physical-mechanical characteristics of the concrete mixture components. To determine hierarchical connections of the factors that affect making decision on the choice of parameters for the formation of concrete with a predictable strength, their corresponding classification was conducted according to quantitative and qualitative indices: physical-mechanical and technological ones.

Technological indices include: the influence of pressure, the influence of solidification temperature, the influence of humidity, the influence of the method for concrete mixture compaction. Physical-mechanical factors are: water / cement ratio, additives (plasticizers), the quality of water, the concrete brand, the type of concrete (portland cement, portland cement with additives and portland slag cement, pozzolan cement, composite cement) as well as the type and quality of the surface and the form of the cement coarse filler, gradation factor of sand [6].

All the influence factors that affect the optimal composition of concrete with a given strength are considered to be linguistic variables that are defined on the corresponding universal sets and are evaluated by fuzzy terms. A qualitative fuzzy term is a linguistic variable the meaning of which is expressed by the word [7].

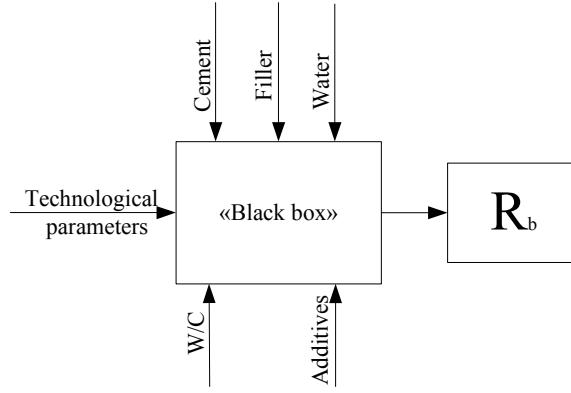


Fig. 1. Model of the influence of factors that determine the strength of concrete

As fuzzy terms for linguistic variables evaluation the following quantitative expressions are adopted: “low” (L), “below average” (bA), “average” (A), “above average” (aA) and “high” (H). Application of fuzzy terms enables building of fuzzy knowledge bases that reflect relationships between input and output variables [7]. Formalization and hierachic classification of the factors affecting the concrete ultimate strength were performed. This makes it possible to build membership functions of fuzzy estimates for the influence of qualitative and quantitative parameters. These functions will be used for modeling concrete strength with a given value.

Considering feasibility of the technological and physical-mechanical factors of influence (TPMFI) on the strength of concrete at the system level, linguistic variable (Y), that characterizes the concrete strength, can be presented in the form of the relationship

$$Y=f(Y_1, Y_2, Y_3, X_4), \quad (1)$$

where Y_1 is a linguistic variable (LV), that describes technological parameters; Y_2 – linguistic variable (LV), that describes physical-mechanical parameters of the fillers; Y_3 – linguistic variable (LV), that describes intermediate parameters of the concrete strength; X_4 – linguistic variable (LV) that describes pH of mixing water.

Linguistic variable that describes technological parameters can be presented by the expression

$$Y_1=f_1(X_{11}, X_{21}, X_{31}, X_{41}), \quad (2)$$

where X_{11} – (LV) “the influence of pressure”; X_{21} – (LV) “the influence of temperature”; X_{31} – (LV) “the influence of humidity”; X_{41} – (LV) “the influence of sealing”.

In a general form mathematical model of the concrete strength is a hierachic fuzzy system where intermediate values of the knowledge bases of variables f_1, f_2, f_3 are the input values for the upper knowledge base Y, for the description of intermediate variables fuzzy terms being used with the expressions “low”(L), “below average” (bA), “average” (A), “above average” (aA) and “high” (H). All the input data concerning the parameters that affect the final variable Y are processed in the “MATLAB 7” software environment.

An example of the fuzzy knowledge matrix taking into account the introduced qualitative terms for modeling the relationship (2) is given in table 1. Hierachic fuzzy system of the concrete strength prediction is represented as a tree in fig. 2.

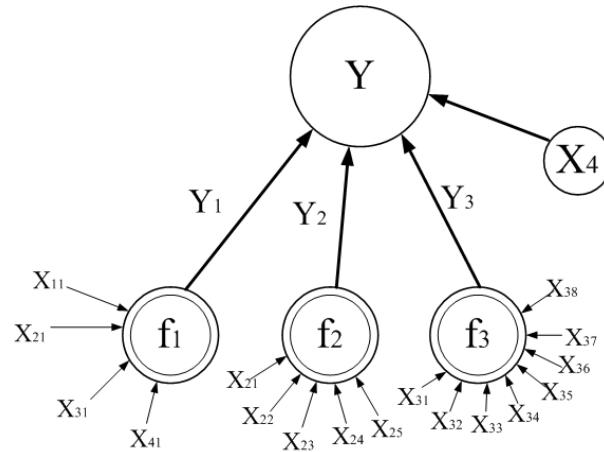


Fig 2. Hierarchical fuzzy system for concrete strength prediction

Analogous knowledge matrices (bases) are built on the basis of expert estimates for the rest of influence factors of the mathematical model for concrete strength evaluation, i.e. Y_2 , Y_3 .

Table 1

Technological parameters									
If				Then	If				Then
X_{11}	X_{21}	X_{31}	X_{41}	Y_1	X_{11}	X_{21}	X_{31}	X_{41}	Y_1
L	L	L	L	Low (L)	A	bA	A	A	Average (A)
L	L	L	A		A	bA	A	H	
L	L	L	H		A	bA	H	L	
L	L	A	L		A	bA	H	A	
L	L	A	A		A	bA	H	H	
L	L	A	H		A	A	L	L	
L	L	H	L		A	A	L	A	
L	L	H	A		A	A	L	H	
L	L	H	H		A	A	A	L	
L	bA	L	L		A	A	A	A	
L	bA	L	A		A	A	A	H	
L	bA	L	H		A	A	H	L	
L	bA	A	L		A	A	H	A	
L	bA	A	A		A	A	H	H	
L	bA	H	L		A	aA	A	A	
L	bA	H	A		A	aA	A	H	
L	A	L	L	Below average (bA)	A	aA	H	A	Above average (aA)
L	A	L	A		A	aA	H	H	
L	A	A	L		A	H	L	H	
L	A	A	A		A	H	A	L	
L	A	H	L		A	H	A	A	
L	A	H	A		A	H	A	H	
L	aA	L	L		A	H	H	L	
L	aA	L	A		A	H	H	A	
L	aA	L	H		A	H	H	H	
L	aA	A	L		H	bA	L	A	
L	aA	A	A		H	bA	L	H	
L	aA	A	H		H	bA	A	L	
L	aA	H	L		H	bA	A	A	
L	aA	H	A		H	bA	A	H	
L	H	L	L		H	bA	H	L	
L	H	L	A		H	bA	H	A	
L	H	L	H		H	bA	H	H	
L	H	A	L		H	A	A	A	
L	H	A	A		H	A	L	H	

L	H	A	H		H	A	A	H
L	H	H	L		H	A	H	L
L	H	H	A		H	A	H	A
A	L	L	L		H	A	H	H
A	L	L	A		H	aA	L	A
A	L	L	H		H	aA	L	H
A	L	A	L		H	aA	A	L
A	L	A	A		H	aA	A	A
A	L	A	H		H	aA	A	H
A	L	H	L		H	aA	H	A
A	L	H	A		H	aA	H	H
L	aA	H	H		H	H	L	L
:L	H	H	H	Average (A)	H	H	L	A
L	aA	H	H		H	H	L	H
A	bA	L	H					

To linguistic variables presented in table 1 the system of fuzzy logic expressions corresponds. They characterize the surface of membership of the variables to the relevant term.

$$\begin{aligned}
\mu_h(X1) = & \mu_h(X11) \wedge \mu_h(X21) \wedge \mu_h(X31) \wedge \mu_h(X41) \vee \mu_h(X11) \wedge \mu_h(X21) \wedge \mu_h(X31) \wedge \mu_c(X41) \\
& \vee \mu_h(X11) \wedge \mu_h(X21) \wedge \mu_h(X31) \wedge \mu_b(X41) \vee \mu_h(X11) \wedge \mu_h(X21) \wedge \mu_c(X31) \wedge \mu_h(X41) \\
& \vee \mu_h(X11) \wedge \mu_h(X21) \wedge \mu_c(X31) \wedge \mu_c(X41) \vee \mu_h(X11) \wedge \mu_h(X21) \wedge \mu_c(X31) \wedge \mu_b(X41) \\
& \vee \mu_h(X11) \wedge \mu_h(X21) \wedge \mu_b(X31) \wedge \mu_h(X41) \vee \mu_h(X11) \wedge \mu_h(X21) \wedge \mu_b(X31) \wedge \mu_c(X41) \\
& \vee \mu_h(X11) \wedge \mu_h(X21) \wedge \mu_b(X31) \wedge \mu_b(X41) \vee \mu_h(X11) \wedge \mu_{HC}(X21) \wedge \mu_h(X31) \wedge \mu_h(X41) \\
& \vee \mu_h(X11) \wedge \mu_{HC}(X21) \wedge \mu_h(X31) \wedge \mu_c(X41) \vee \mu_h(X11) \wedge \mu_{HC}(X21) \wedge \mu_h(X31) \wedge \mu_b(X41) \quad (3) \\
& \vee \mu_h(X11) \wedge \mu_{HC}(X21) \wedge \mu_c(X31) \wedge \mu_h(X41) \vee \mu_h(X11) \wedge \mu_{HC}(X21) \wedge \mu_c(X31) \wedge \mu_c(X41) \\
& \vee \mu_h(X11) \wedge \mu_{HC}(X21) \wedge \mu_b(X31) \wedge \mu_h(X41) \vee \mu_h(X11) \wedge \mu_{HC}(X21) \wedge \mu_b(X31) \wedge \mu_c(X41) \\
& \vee \mu_h(X11) \wedge \mu_c(X21) \wedge \mu_h(X31) \wedge \mu_h(X41) \vee \mu_h(X11) \wedge \mu_c(X21) \wedge \mu_h(X31) \wedge \mu_c(X41) \\
& \vee \mu_h(X11) \wedge \mu_c(X21) \wedge \mu_c(X31) \wedge \mu_h(X41) \vee \mu_h(X11) \wedge \mu_c(X21) \wedge \mu_c(X31) \wedge \mu_c(X41) \\
& \vee \mu_h(X11) \wedge \mu_c(X21) \wedge \mu_b(X31) \wedge \mu_h(X41) \vee \mu_h(X11) \wedge \mu_c(X21) \wedge \mu_b(X31) \wedge \mu_c(X41);
\end{aligned}$$

$$\begin{aligned}
\mu_{HC}(X1) = & \mu_H(X11) \wedge \mu_{BC}(X21) \wedge \mu_H(X31) \wedge \mu_H(X41) \vee \mu_H(X11) \wedge \mu_{BC}(X21) \wedge \mu_H(X31) \wedge \mu_C(X41) \\
& \vee \mu_H(X11) \wedge \mu_{BC}(X21) \wedge \mu_H(X31) \wedge \mu_B(X41) \vee \mu_H(X11) \wedge \mu_{BC}(X21) \wedge \mu_C(X31) \wedge \mu_H(X41) \\
& \vee \mu_H(X11) \wedge \mu_{BC}(X21) \wedge \mu_C(X31) \wedge \mu_C(X41) \vee \mu_H(X11) \wedge \mu_{BC}(X21) \wedge \mu_C(X31) \wedge \mu_B(X41) \\
& \vee \mu_H(X11) \wedge \mu_{BC}(X21) \wedge \mu_B(X31) \wedge \mu_H(X41) \vee \mu_H(X11) \wedge \mu_{BC}(X21) \wedge \mu_B(X31) \wedge \mu_C(X41) \\
& \vee \mu_H(X11) \wedge \mu_B(X21) \wedge \mu_H(X31) \wedge \mu_H(X41) \vee \mu_H(X11) \wedge \mu_B(X21) \wedge \mu_H(X31) \wedge \mu_C(X41) \\
& \vee \mu_H(X11) \wedge \mu_B(X21) \wedge \mu_H(X31) \wedge \mu_B(X41) \vee \mu_H(X11) \wedge \mu_B(X21) \wedge \mu_C(X31) \wedge \mu_H(X41) \\
& \vee \mu_H(X11) \wedge \mu_B(X21) \wedge \mu_C(X31) \wedge \mu_C(X41) \vee \mu_H(X11) \wedge \mu_B(X21) \wedge \mu_C(X31) \wedge \mu_B(X41) \\
& \vee \mu_H(X11) \wedge \mu_B(X21) \wedge \mu_B(X31) \wedge \mu_H(X41) \vee \mu_H(X11) \wedge \mu_B(X21) \wedge \mu_B(X31) \wedge \mu_C(X41) \\
& \vee \mu_H(X11) \wedge \mu_C(X21) \wedge \mu_H(X31) \wedge \mu_H(X41) \vee \mu_H(X11) \wedge \mu_C(X21) \wedge \mu_H(X31) \wedge \mu_C(X41) \\
& \vee \mu_H(X11) \wedge \mu_C(X21) \wedge \mu_C(X31) \wedge \mu_C(X41) \vee \mu_H(X11) \wedge \mu_C(X21) \wedge \mu_C(X31) \wedge \mu_B(X41) \\
& \vee \mu_H(X11) \wedge \mu_C(X21) \wedge \mu_H(X31) \wedge \mu_H(X41) \vee \mu_H(X11) \wedge \mu_C(X21) \wedge \mu_H(X31) \wedge \mu_C(X41); \quad (4)
\end{aligned}$$

$$\begin{aligned} \mu_c(X1) = & \mu_h(X11) \wedge \mu_{BC}(X21) \wedge \mu_b(X31) \wedge \mu_b(X41) \vee \mu_h(X11) \wedge \mu_b(X21) \wedge \mu_b(X31) \wedge \mu_b(X41) \\ & \vee \mu_h(X11) \wedge \mu_{BC}(X21) \wedge \mu_b(X31) \wedge \mu_b(X41) \vee \mu_c(X11) \wedge \mu_{HC}(X21) \wedge \mu_h(X31) \wedge \mu_b(X41) \\ & \vee \mu_c(X11) \wedge \mu_{HC}(X21) \wedge \mu_c(X31) \wedge \mu_c(X41) \vee \mu_c(X11) \wedge \mu_{HC}(X21) \wedge \mu_c(X31) \wedge \mu_b(X41) \\ & \vee \mu_c(X11) \wedge \mu_{HC}(X21) \wedge \mu_b(X31) \wedge \mu_h(X41) \vee \mu_c(X11) \wedge \mu_{HC}(X21) \wedge \mu_b(X31) \wedge \mu_c(X41) \\ & \vee \mu_c(X11) \wedge \mu_{HC}(X21) \wedge \mu_b(X31) \wedge \mu_b(X41) \vee \mu_c(X11) \wedge \mu_c(X21) \wedge \mu_h(X31) \wedge \mu_h(X41) \quad (5) \\ & \vee \mu_c(X11) \wedge \mu_c(X21) \wedge \mu_h(X31) \wedge \mu_c(X41) \vee \mu_c(X11) \wedge \mu_c(X21) \wedge \mu_h(X31) \wedge \mu_b(X41) \\ & \vee \mu_c(X11) \wedge \mu_c(X21) \wedge \mu_c(X31) \wedge \mu_h(X41) \vee \mu_c(X11) \wedge \mu_c(X21) \wedge \mu_c(X31) \wedge \mu_c(X41) \\ & \vee \mu_c(X11) \wedge \mu_c(X21) \wedge \mu_c(X31) \wedge \mu_b(X41) \vee \mu_c(X11) \wedge \mu_c(X21) \wedge \mu_b(X31) \wedge \mu_h(X41); \end{aligned}$$

$$\begin{aligned} \mu_{BC}(X1) = & \mu_c(X11) \wedge \mu_c(X21) \wedge \mu_b(X31) \wedge \mu_c(X41) \vee \mu_c(X11) \wedge \mu_c(X21) \wedge \mu_b(X31) \wedge \mu_b(X41) \\ & \vee \mu_c(X11) \wedge \mu_{BC}(X21) \wedge \mu_c(X31) \wedge \mu_c(X41) \vee \mu_c(X11) \wedge \mu_{BC}(X21) \wedge \mu_c(X31) \wedge \mu_b(X41) \\ & \vee \mu_c(X11) \wedge \mu_{BC}(X21) \wedge \mu_b(X31) \wedge \mu_c(X41) \vee \mu_c(X11) \wedge \mu_{BC}(X21) \wedge \mu_b(X31) \wedge \mu_b(X41) \end{aligned}$$

$$\begin{aligned}
 & \vee \mu_c(X_{11}) \wedge \mu_b(X_{21}) \wedge \mu_h(X_{31}) \wedge \mu_b(X_{41}) \vee \mu_c(X_{11}) \wedge \mu_b(X_{21}) \wedge \mu_c(X_{31}) \wedge \mu_h(X_{41}) \\
 & \vee \mu_c(X_{11}) \wedge \mu_b(X_{21}) \wedge \mu_c(X_{31}) \wedge \mu_c(X_{41}) \vee \mu_c(X_{11}) \wedge \mu_b(X_{21}) \wedge \mu_c(X_{31}) \wedge \mu_b(X_{41}) \\
 & \vee \mu_c(X_{11}) \wedge \mu_b(X_{21}) \wedge \mu_b(X_{31}) \wedge \mu_h(X_{41}) \vee \mu_c(X_{11}) \wedge \mu_b(X_{21}) \wedge \mu_b(X_{31}) \wedge \mu_c(X_{41}) \quad (6) \\
 & \vee \mu_c(X_{11}) \wedge \mu_b(X_{21}) \wedge \mu_b(X_{31}) \wedge \mu_b(X_{41}) \vee \mu_b(X_{11}) \wedge \mu_{HC}(X_{21}) \wedge \mu_h(X_{31}) \wedge \mu_c(X_{41}) \\
 & \vee \mu_b(X_{11}) \wedge \mu_{HC}(X_{21}) \wedge \mu_h(X_{31}) \wedge \mu_b(X_{41}) \vee \mu_b(X_{11}) \wedge \mu_{HC}(X_{21}) \wedge \mu_c(X_{31}) \wedge \mu_h(X_{41}) \\
 & \vee \mu_b(X_{11}) \wedge \mu_{HC}(X_{21}) \wedge \mu_c(X_{31}) \wedge \mu_c(X_{41}) \vee \mu_b(X_{11}) \wedge \mu_{HC}(X_{21}) \wedge \mu_c(X_{31}) \wedge \mu_b(X_{41}) \\
 & \vee \mu_b(X_{11}) \wedge \mu_{HC}(X_{21}) \wedge \mu_b(X_{31}) \wedge \mu_h(X_{41}) \vee \mu_b(X_{11}) \wedge \mu_{HC}(X_{21}) \wedge \mu_b(X_{31}) \wedge \mu_c(X_{41}) \\
 & \vee \mu_b(X_{11}) \wedge \mu_{HC}(X_{21}) \wedge \mu_b(X_{31}) \wedge \mu_b(X_{41}) \vee \mu_b(X_{11}) \wedge \mu_c(X_{21}) \wedge \mu_c(X_{31}) \wedge \mu_c(X_{41}) \\
 & \vee \mu_b(X_{11}) \wedge \mu_c(X_{21}) \wedge \mu_h(X_{31}) \wedge \mu_b(X_{41});
 \end{aligned}$$

$$\begin{aligned}
 \mu_b(X_1) = & \mu_b(X_{11}) \wedge \mu_c(X_{21}) \wedge \mu_c(X_{31}) \wedge \mu_b(X_{41}) \vee \mu_b(X_{11}) \wedge \mu_c(X_{21}) \wedge \mu_b(X_{31}) \wedge \mu_h(X_{41}) \\
 & \vee \mu_b(X_{11}) \wedge \mu_c(X_{21}) \wedge \mu_b(X_{31}) \wedge \mu_c(X_{41}) \vee \mu_b(X_{11}) \wedge \mu_c(X_{21}) \wedge \mu_b(X_{31}) \wedge \mu_b(X_{41}) \\
 & \vee \mu_b(X_{11}) \wedge \mu_{BC}(X_{21}) \wedge \mu_h(X_{31}) \wedge \mu_c(X_{41}) \vee \mu_b(X_{11}) \wedge \mu_{BC}(X_{21}) \wedge \mu_h(X_{31}) \wedge \mu_b(X_{41}) \\
 & \vee \mu_b(X_{11}) \wedge \mu_{BC}(X_{21}) \wedge \mu_c(X_{31}) \wedge \mu_h(X_{41}) \vee \mu_b(X_{11}) \wedge \mu_{BC}(X_{21}) \wedge \mu_c(X_{31}) \wedge \mu_c(X_{41}) \\
 & \vee \mu_b(X_{11}) \wedge \mu_{BC}(X_{21}) \wedge \mu_c(X_{31}) \wedge \mu_b(X_{41}) \vee \mu_b(X_{11}) \wedge \mu_{BC}(X_{21}) \wedge \mu_b(X_{31}) \wedge \mu_c(X_{41}) \quad (7) \\
 & \vee \mu_b(X_{11}) \wedge \mu_{BC}(X_{21}) \wedge \mu_b(X_{31}) \wedge \mu_b(X_{41}) \vee \mu_b(X_{11}) \wedge \mu_b(X_{21}) \wedge \mu_h(X_{31}) \wedge \mu_h(X_{41}) \\
 & \vee \mu_b(X_{11}) \wedge \mu_b(X_{21}) \wedge \mu_h(X_{31}) \wedge \mu_c(X_{41}) \vee \mu_b(X_{11}) \wedge \mu_b(X_{21}) \wedge \mu_h(X_{31}) \wedge \mu_b(X_{41}).
 \end{aligned}$$

The obtained system of fuzzy logic equations (3 – 7) is the representation of one of the four knowledge bases that are included into the mathematical model of concrete strength prediction on the basis of fuzzy logic. Further processing of the model parameters in “MATLAB 7” environment [9] will make it possible to teach the model to perform its fine adjustment for adequate representation of the “black box” output signal, i.e. of the strength of concrete as a function of input parameters of the box: concrete components, the type of technological processing, the quality of water.

Conclusions

1. In accordance with the performed classification of the quantitative and qualitative parameters that affect the concrete quality, fuzzy logic equations are obtained on the technological level that relate membership functions of input and output variables, which is determined by the usage of min and max operations during their construction..
2. By means of the mathematical model application, hierachic system of logic equations (3 – 7) enables fuzzy logic-based evaluation of the predicted concrete strength index depending on the technological parameters of influence as well as solving the problem of these factors optimization.

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