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CAPACITY OF DEVICES OF BITWISE ADDITION IN AM-CALCULATION SYSTEMS

This paper considers the class of calculation systems with additive and multiplicative correlations between the digit weights. There had been described the bitwise addition in such systems of calculation and the boundaries, within which the capacity of bitwise summators are, had been determined.

***Key words**: bitwise addition, calculation system, additive transformation.*

Urgency of the problem

Increase in productivity of efficiency of technical equipment may appear possible due to distributed processing of data streams. Efficient organization of distributed processing requires to solve the problem of information connection between the devices. The problem is that the increase in number of distributed devices causes an enormous increase in total number of information connections with them.

One of solutions to this problem is the conveyor bitwise processing of consecutive codes [1-5]. Execution of all bitwise conveyor operations in the unique stream is done according to the known algorithms beginning with the highest orders. The redundant calculation systems, since they have the restricted lengths of transfer to higher digits in bitwise addition and subtracting, are used for this.

In bitwise conveyor addition, beginning with higher digits, the transfer has a specific length, which determines the capacity of bitwise summators. Capacity of bitwise summators, in turn, influences significantly the hardware costs during building means of conveyor bitwise codes processing. The redundant calculation systems with binary digits may exist in big numbers, but not each of them allows for restriction of transfer length when executing arithmetic operations. The authors determined the class of redundant positional system, called *AM*-calculation systems, which allow to execute bitwise all the arithmetic operations, beginning with higher digits [6]. During the bitwise addition in *AM*-calculation systems the transfer is executed on the base of additive transformation. The length of the transfer determines the capacity of bitwise summator. However, there are no famous theoretical developments, which allow to determine the length of transfer depending on parameters of calculation systems and thus to compare the capacities of bitwise summers for any *AM*-calculation systems.

Objective

The objective of the paper is to improve the efficiency of designing the devices for bitwise processing in any *AM*-calculation system.

Tasks

The objective requires the following tasks to be solved:

1. Research of *AM*-calculation systems, which generalize the known and allow to create the new calculation systems with the ability of conveying bitwise execution of all the arithmetic operations on digit codes beginning with higher digits;
2. Research of addition transformation – conditional digit operations in *AM*-calculation systems and determination of their connection with the known operations of transfer and borrowing;
3. Research of bitwise addition and determination of dependence of transfer length on parameters of additive correlation.

AM-calculation systems

Let's introduce the notion of the class of the calculation systems with additive and multiplicatory correlations between the digit weights (*AM*-calculation systems). *AM*-calculation systems are positioning redundant calculation systems in which the weight of each digit is the degree of foundation of calculation system and there is the additive correlation of the definite kind with the set limitations between the set weights of digits. Arbitrary *AM*-calculation system may be described the following aggregate of parameters

$$\left\{ \begin{array}{l} C_k = \{0,...,c_{k-1}\}; \\ w; \\ {}^{t}A^{\tau,p} : w^{\tau p+t} = R^{\tau,p} \end{array} \right\}, \qquad (1)$$

where $k \geq 2$ – significance of the calculation system; C_k –set of numbers; w – base of the calculation system; ${}^{t}A^{\tau,p}$ – additive correlation (A-correlation) of the order(t, τ, p); t, τ, p – parameters of additive correlation ($t>0, \tau>0, p \geq 0$ – whole); $R^{\tau,p} = \sum_{i=0}^{p} r_i \cdot w^{\tau i}$ – limiting value $\left(r \in C_k\right)$.

And the following restrictions are applied to the parameters of additive correlations:

$$\begin{cases} r_{\tau i} \geq r_{\tau(i-1)} > 0; \\ \tau_{\bmod t} = 0. \end{cases} \qquad (2)$$

Additive correlation is set by parameters t and τ, as well as by set of digits $r_{\tau p},...,r_0$, which do not depend on the number of digit. Polynomial $R^{\tau,p}$ may be presented as a code

$$r_{\tau p}\underbrace{0\cdots 0 r_{\tau(p-1)}}_{\tau}\cdots\underbrace{0\cdots 0 r_{\tau}}_{\tau}\underbrace{0\cdots 0 r_0}_{\tau}.$$

Considering that

$$R_{i-\tau p}^{\tau,p} = w^{i-\tau p} \cdot R^{\tau,p}.$$

There is a connection between the additive correlation, base of the calculation system and the set of digits: the base of the calculation system is the positive actual rule of additive correlation in which the factors with the unknown are digits. Proceeding from this, *AM*-calculation system may be set by any of parameters pair $\{C_k, {}^{t}A^{\tau,p}\}$ or $\{C_k, w\}$ according to (1). Considering the necessity in observing the restrictions which are applied to the additive correlations in (2), it is very simple to set *AM*-calculation system by parameters C_k *u* ${}^{t}A^{\tau,p}$.

Additive transformations

Availability of additive correlations between the digits in the *AM*-calculation systems allows to execute the operations of additive transformation of digit codes (*A*-transformation), which imply for digit code change when saving its digit equivalent. *A*–transformations are the specific type of conditionary digital operations which are done on part of the digit code or on the whole code. During the execution of *A*-transformations on the code of the digit, higher and lower digits, relating to the given transformations, change their value, hence the value of the whole code is not changed. Change of parts of the codes is done due to operations of addition and subtraction. Operations of increase of one part of the digit code and decrease of the other with stable value of the whole code are known as transfer and borrowing. So, the additive transformations execute the transfer and borrowing during the addition and subtracting of codes in *AM*-calculation systems. Borrowing will also be later called as transfer to the lower digits.

Additive transformations may be classified as for the direction of transfer as well as for the execution condition. Fig. 1. presents the classification of additive transformations in *AM*-calculation systems.

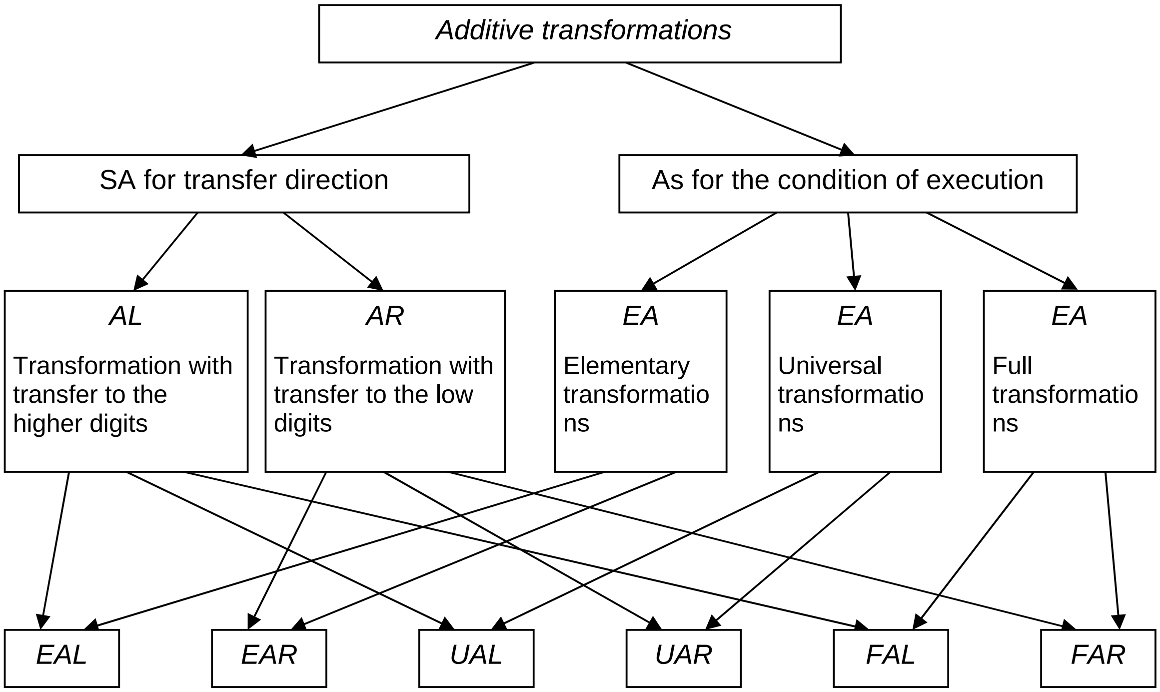

Fig. 1. Classification of additive transformations

Let's consider each of the types of additive transformation separately. As for the direction of transfer, the *A*-transformations are divided into the transformations with transfer to the higher digits (*AL*-transformations) and transformations to the lower digits (*AR*-transformations). During the *AL*-transformation there is the addition to the higher digits and subtracting from the lower ones, and during the *AR*-transformations – vice versa:

$$ {}^{t}AL_{i}^{\tau,p}(X_{0}^{n-1}): X_{0}^{i} - R_{i-\tau p}^{\tau,p} + X_{i+1}^{n-i-2} + w^{i+t}; $$

$$ {}^{t}AR_{i}^{\tau,p}(X_{0}^{n-1}): X_{i+1}^{n-i-2} - w^{i+t} + X_{0}^{i} + R_{i-\tau p}^{\tau,p}. $$

Additive transformations are executed with necessary and sufficient conditions. Necessary condition of the *i*-th *AL*-transformation means that the value of digits from the 0-th to *i* -th must be not less then the corresponding limiting value, and the value of the (*i*+*t*)-th digit be lower than the higher digit. Necessary condition of the *i*-th *AR*-transformations means that the value of digits from the 0-th to the *i*-th must be not higher than the difference between the maximum code in these digits and corresponding limiting value, and the value of the (*i*+*t*)-th digit must be higher than zero.

As for the sufficient conditions, the execution of *A*-transformation is divided into elementary (*E*), universal (*U*) and full (*F*). In the elementary *A*-transformations (*EA*-transformations) the sufficient conditions in each separate digit of the changeable part of the code are checked:

$$ {}^{t}EAL_{i}^{\tau,p}(X_{0}^{n-1}) = \begin{cases} X_{0}^{n-1}\ when\,(x_{i+t} = c_{k-1}) \vee \underset{0\le j\le p}{\exists}(x_{i-\tau(p-j)} < r_{\tau j}); \\ (X_{i+1}^{n-i-2} + w^{i+t}) + (X_{0}^{i} - R_{i-\tau p}^{\tau,p})\ when \\ (x_{i+t} < c_{k-1}) \wedge \underset{0\le j\le p}{\forall}(x_{i-\tau(p-j)} \ge r_{\tau j}); \end{cases} $$

$$ {}^{t}EAR_{i}^{\tau,p}(X_{0}^{n-1}) = \begin{cases} X_{0}^{n-1} \; when\,(x_{i+t}=0) \vee \underset{0\le j\le p}{\exists}(x_{i-\tau(p-j)}+r_{\tau j}>c_{k-1}); \\ (X_{i+1}^{n-i-2}-w^{i+t})+(X_{0}^{i}+R_{i-\tau p}^{\tau,p}) \; when \\ (x_{i+t}>0) \wedge \underset{0\le j\le p}{\forall}(x_{i-\tau(p-j)}+r_{\tau j}\le c_{k-1}). \end{cases} $$

Universal additive transformations (*UA*-transformations) are analogical to the elementary in the sense that they mean the equivalent change of code in a way that the value of the whole code remains unchanged though the value for separate higher and lower digits changes. Unlike *EA*-, in *UA*-transformations the execution of sufficient conditions shall be checked not for the values of each separate digit, but for the general values of parts of the codes. This allows to execute *UA*-transformations for all values of the obverted part of the code which satisfy the necessary conditions of transformation:

$$ {}^{t}UAL_{i}^{\tau,p}(X_{i-\tau b}^{\tau b+t}) = \begin{cases} X_{i-\tau b}^{\tau b+t} \; when\,(x_{i+t}=c_{k-1}) \vee X_{i-\tau b}^{\tau b} < R_{i-\tau p}^{\tau,p}; \\ X_{i-\tau b}^{\tau b+t}+w^{i+t}-R_{i-\tau p}^{\tau,p} \; when \\ (x_{i+t}<c_{k-1}) \wedge X_{i-\tau b}^{\tau b} \ge R_{i-\tau p}^{\tau,p}; \end{cases} $$

$$ {}^{t}UAR_{i}^{\tau,p}(X_{i-\tau b}^{\tau b+t}) = \begin{cases} X_{i-\tau b}^{\tau b+t} \; when \\ (x_{i+t}=0) \vee X_{i-\tau b}^{\tau b}+R_{i-\tau p}^{\tau,p} > c_{k-1}\sum\limits_{j=i-\tau b}^{i} w^{j}); \\ X_{i-\tau b}^{\tau b+t}-w^{i+t}+R_{i-\tau p}^{\tau,p} \; when \\ (x_{i+t}>0) \wedge (X_{i-\tau b}^{\tau b}+R_{i-\tau p}^{\tau,p} \le c_{k-1}\sum\limits_{j=i-\tau b}^{i} w^{j}). \end{cases} $$

With full additive transformation (*FA*-transformation) the universal additive transformations are executed not only in the *i*-th, but also in all the lower obverted codes. The sufficient condition for *FA*-transformation is execution of the sufficient condition of *UA*-transformation at least in one of the obverted digits. The result of the execution of ${}^{t}FA_{i}^{\tau,p}$-transformations on the code digits from the (i-τb)-th to the (u+t)-th one is code in which for each j from the (j-τb)-th to the i-th value of digits from the (j-τb)-th to the j-th do not satisfy the condition ${}^{t}UA_{i}^{\tau,p}$-transformation of the same direction:

$$ {}^{t}FAL_{i}^{\tau,p}(X_{i-\tau b}^{\tau b}) = {}^{t}UAL_{i-\tau b}^{\tau,p}({}^{t}UAL_{i-\tau+t}^{\tau,p}(\cdots {}^{t}UAL_{i-t}^{\tau,p}({}^{t}UAL_{i}^{\tau,p}(X_{i-\tau b}^{\tau b}))\cdots)). $$

$$ {}^{t}FAR_{i}^{\tau,p}(X_{i-\tau b}^{\tau b}) = {}^{t}UAR_{i-\tau b}^{\tau,p}({}^{t}UAR_{i-\tau+t}^{\tau,p}(\cdots {}^{t}UAR_{i-t}^{\tau,p}({}^{t}UAR_{i}^{\tau,p}(X_{i-\tau b}^{\tau b}))\cdots)). $$

Additive transformations are most known operations of transfer and borrowing during the execution of bitwise adding and subtracting. Transfer and subtracting as well as additive transformations change values of separate digits without changing the value of the whole code. Change of digit values during transfer and borrowing is based upon additive correlation between the digit weights. For instance, transfer under overrun of some digit means the adding of the unit to the higher one and subtracting of an equivalent value from the overrun one, which in fact, is an *AL*-transformation. But unlike transfer and borrowing, *A*-transformations in *AM*-calculation systems may be fulfilled not only in cases when the value of a range becomes greater-than the maximum digit or less than zero, but also in case when the specific group of digits reaches its extreme value, expressed by the set code. Therefore, the additive transformations may be executed during the operations of adding and subtracting as well as separate from them in such calculation systems.

Bitwise totalling

In *AM*-calculation systems the totalling is done the same way as in the known positional numerical systems. First, the figures in separate digits are added and then the transfer between the digits is done if necessary. Let's consider in details the process of bitwise totalling of successive codes, beginning with the higher digits. Let's assume that it is necessary to find the code of the result Z which equals the codes sum X and Y $X+Y=Z$. Under the bitwise totalling in *AM*-numerical systems the successive codes of items X and Y enter the output of bitwise summator, beginning with higher digit and the successive code of the sum Z enters from the output, beginning with the higher digits, as is shown on fig. 2.

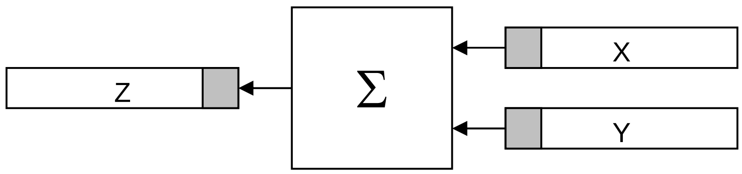

Fig. 2 Total scheme of bitwise totalling

The bitwise totalling of codes in *AM*-numerical systems is executed according to the known methods of non-autonomous processing [3] and represents series of steps of totalling separate digits, beginning with the highest one, on each of which the code of the result Z_i is determined in a way which on the last step equals the code results Z. On the following i-th step, the $(n-i-1)$-th digits of items participate in totalling . During the execution of totalling of the further digits of items there appears the transfer to the other digits of the result. Since in the common case the base of *AM*-numerical system is not an integral, the transfer then may be to higher as well as to the lower digits. Therefore the code of the result Z_i, in general case may have a zero value in the digits which are lower and higher then the $(n-i-1)$-th. Determination of the result Z_i on each step is done by addition of the following digits to the results, received on previous step:

$$Z_i = Z_{i-1} + x_i + y_i \,.$$

It is necessary to note that since *AL*- and *AR*-transformations are done with digits, placed on distance, which is divisible to t from one another, then the transfer to any i-th digit may be only form digits with numbers $(i \pm nt)$.

That is, for attributing the transfer to the i-th digit, it is necessary to analyze the series which have numbers $(i \pm nt)$, where $n=1,2,3,...$This allows to consider the totalling in any *AM*-numerical system with the parameter of additive correlation $t>1$ as somewhat independent totalling , each of which is executed in the same *AM*-numerical system, but when $t=1$. Therefore for the simplification of the analysis we will consider the case $t=1$

The peculiarity of totalling in *AM*-numerical systems is the possibility of restriction in spreading the transfer to the higher digits due to executing of *FAL*-transformation on group of digits on the previous stage of totalling :

$$Z_i = {}^1FAL^{\tau,p}_{n-1-i}(Z_{i-1} + (x_{n-1-i} + y_{n-1-i}) \cdot w^{n-1-i}) \,.$$

Restriction of transfer length during totalling in *AM*-numerical systems is stipulated for by two peculiarities of *AM*-numerical systems. The first peculiarity is peculiar to all positional numerical systems with the increasing series of digit weights. It is obvious and mans that the result of totalling any group of digits in less than the unit of some higher digit which concerns it:

$$\sum_{i=m}^{b} 2 \cdot c_{k-1} \cdot w^{m-i} \le w^{m+d} \,.$$

The second peculiarity is peculiar to *AM*-numerical systems only. It means that due to execution of universal L-transformation on some group of digits, their value becomes less than the limiting value for the given group.

The evaluation of maximum length of d transfer to the higher digits requires to divide each tact of totalling by two stages. The first stage – totalling separate digits and receiving the code of their sum S_i. The second stage - addition of S_i to the result $T_{i\text{-}1}$, received on previous tact. So, the bitwise totalling may be presented as addition of code S_i to the code $T_{i\text{-}1}$ on each i-th tact.

On the first stage the totalling of separate digits is done by usual way for positional systems. Let on the i-th tact the digits x_i and y_i are added. But for all that there may appear the overrun ($x_i+y_i>c_{k\text{-}1}$). The overrun may overcome by using FAL-transformation. FAL-transformation in general case causes the transfer to both, some higher and to Δlower digits. If the maximum length of transfer to the higher $dSmax$ and lower $\Delta Smax$ digits form totalling two separate series is known, then the code of their sum may be received, using FAL-transformation:

$$(FAL(x_i + y_i)_{i-\Delta(S-1)\max}^{\Delta(S-1)\max+dS\max-1})_{i-\Delta S\max}^{\Delta(S-1)\max+dS\max}. \tag{3}$$

The maximum transfer will be during totalling series with maximum digits $c_{k\text{-}1}$ and will represent the code $Smax$ which consists of $dSmax$+1 of higher and $\Delta Smax$ of lower digits:

$$S\max = 2 \cdot c_{k-1} \cdot w^i = S_{i-\Delta S\max}^{dS\max+\Delta S\max+1}.$$

It is necessary to note that when $c_{k\text{-}1}>r_p$, there may be some codes of maximum value Smax with different length $dSmax$ and different higher digits $S_{i+dS\max}$ depending on digits which underwent the full AL-transformation. With the minimum values of $dS\max_{min}$ the higher digit of the code $Smax$ will have the value no greater than $c_{k\text{-}1}$:

$$(FAL(2 \cdot c_{k-1} \cdot w^i)_{i-b}^{b+dS\max_{\min}-1})_{i+dS\max_{\min}}^{0} \le c_{k-1} \cdot w^{i+dS\max_{\min}}.$$

With the maximum value of $dS\max_{\max}$ the higher digit of code Smax will have value no greater than r_p:

$$(FAL(2 \cdot c_{k-1} \cdot w^i)_{i-b}^{b+dS\max_{\max}-1})_{i+dS\max_{\max}}^{0} \le r_p \cdot w^{i+dS\max_{\max}}.$$

Determination of $dS\max_{\max}$ requires to execute succession of operations similar to EAR-transformation. Unlike EAR-transformation, these operations must be executed even during the overrun in lower digits. The main point is that the succession transformations, beginning with the unit of some digit with transfer to the lower digits is being performed. From the highest significant code digit, received on the previous step, its value must be subtracted on each transformation stage. Equivalent value shall be added in the kind of a code to digits, less then the highest significant. UAL-transformation into digit, lower than the highest significant is performed over the result.

Thus the code, which equals the unit of some digit, is shifted to the right and increases on each step. Let at the beginning this code equals the unit of some m-th digit $X_0=w^m$. Then, on the i-th step the code value:

$$X_i = UAL_{m-\tau p-i}^{\tau,p}(X_{i-1} - x_{m-i+1} \cdot (w^{m-i+1} - R_{m-\tau p-i}^{\tau,p})),$$

where i=1,2,3,... These steps of transformation must be repeated until $X_i<2\cdot c_{k\text{-}1}\cdot w^{m-i}$. If $X_i=2\cdot c_{k\text{-}1}\cdot w^{m-i}$ after the steps repeats are over, then the maximum length of transfer to the higher digits under adding m-x digits $dS\max_{\max}$ equals number of transformation steps.

If $X_i>2\cdot c_{k\text{-}1}\cdot w^{m-i}$, then $dS\max_{\max}$ is one unit less than the number of transformation steps. $dS\max_{min}$ is determined analogically, only the initial value of the m-th digit is set as equal to the maximum digit $c_{k\text{-}1}$.

On the second stage there appears the transformation from exceedance of limiting value under adding code of separate digits to the intermediate result. It is realized by FAL-transformation of result of addition S_i and intermediate result $T_{i\text{-}1}$, received on previous tact. The maximum length d of this transfer is determined by number of digits, necessary for consumption of this and the following transfers from the addition of separate digits. The total length of transfer to the higher digits d=d+d. The following statement allows to determine the limits within the value d may be positioned in arbitrary *AM*-numerical system.

Statement 1. Let for *AM*-numerical system there had been the multitude of figures $\{0, 1, ..., c_{k-1}\}$ and additive correlation ${}^1A^{\tau,p}$. Let for this system there had been set:

$dS\max_{min}$ – minimum length of transfer to the higher digits under the adding of maximum figures in one digit;

dZ – the biggest number of digits, total maximum value of which is less than the limiting value of additive correlation, that is

$$c_{k-1}\cdot\sum_{i=0}^{dZ-1}w^{\tau p-i}<{}^1R_0^{\tau,p}\le c_{k-1}\cdot\sum_{i=0}^{dZ}w^{\tau p-i}\ . \tag{4}$$

Then on any stage of bitwise totalling the maximum length d of the transfer to the higher digits is within

$$d\ge dZ+dS\max_{\min}+H[p],$$

where $H[p]$ – discrete unit function of Heaviside.

Proof to the statement 1.

To prove the statement it is enough to give examples in which during the bitwise totalling in any *AM*-numerical system with the length of transfer to the higher digits $d<d+dS\max_{min}+H[p]$ there appears the overrun. The proof will come true separately for each of two possible case: p=0 и p>0.

In the first case $H[p]$=0, $dS\max_{min}>0$ and d=0 according to the definition (4). Therefore, for proving the statement it is sufficient to give an example of bitwise totalling which causes overrun in any numerical system, when $d<dS\max_{min}$. Let's consider an example of totalling n-digit codes with maximum digits with the length of transfer

$$c_{k-1}\cdot\sum_{j=0}^{n-1-dS\max_{\min}}w^{j}+c_{k-1}\cdot\sum_{j=0}^{n-1-dS\max_{\min}}w^{j}$$

$d=dS\max_{min}-1$.

During the bitwise totalling of these codes, beginning with the higher digits, the condition for *FAL*-transformation is not executed on the first $dS\max_{min}$ step. On each i-th step of bitwise totalling , beginning with the $dS\max_{min}$-th, there takes place the transfer to the higher digits which are not less than $c_{k-1}\cdot w^{i+dS\max_{\min}}$ upon the condition of determination $dS\max_{min}$. This transfer causes the overrun in the digits from the i-th to (i+ $dS\max_{min}$-1)-th, which cannot be liquidated due to transfer to the lower digits since they also run an overrun on the following tacts. So, for the case p=0 the statement 1 is proved.

In the second case $H[p]$=1. Therefore to prove the statement it is sufficient to give an example of bitwise totalling in which there appears the overrun when $d<d+dS\max_{min}+1$. Let's consider the example of code addition with length of transfer $d=d+dS\max_{min}$.

$$c_{k-1}\cdot\sum_{j=n-1-dZ}^{n-1}w^{i}+c_{k-1}\cdot\sum_{j=0}^{n-1-dZ-dS\max_{\min}}w^{i}$$

and when

$$c_{k-1}\cdot\sum_{j=0}^{n-1-dZ-dS\max_{\min}}w^{i}$$

During the bitwise totalling of these codes, beginning with the higher digits, on the steps from the 1st to ($d+dS\max_{min}$)-th the condition for *FAL*-transformation is not executed. On the ($d+dS\max_{min}$+1)-th step there takes place a transfer to the (n-d-2)-th digit, which is not less than c_{k-1} upon the condition of determination of $dS\max_{min}$. Since the length of transfer to the higher digits

$d=d+dS\max_{min}$, therefore FAL_{n-1}-transformation cannot be executed on this stage, and the condition FAL_{n-2}-transformation is not being executed yet. Therefore the (n-d-2)-th digit of the sum will have the maximum value c_{k-1}. On each following step of bitwise totalling there will be analogically formed the maximum value c_{k-1} of the next digit of the sum. Therefore, considering the transfer to the higher digits, all the received digits of the sum codes will have maximum value c_{k-1}. When $p>0$, the transfer which appears in the result of addition of separate digits, enters both, higher and lower digits. That is, the transfer to the lower digits, which appear on the ($d+dS\max_{min}+1$)-th step it is necessary to add on later steps. As the code of the sum with minimum digits is being formed without the consideration of transfer to lower digits, its registration in some digit will cause the overrun of this digit. This overrun may not be liquidated due to the transfer to the higher digits, since they also have the maximum value. This overrun may not be liquidated due to transfer to the low digits since there will appear the analogical overrun on the following tacts. Consequently, in this case of bitwise totalling of codes there appears the overrun which may not be liquidated under the length of transfer to the higher digits $d<d+dS\max_{min}+H[p]$. So, for case $p>0$ the statement 1 has also been proved. The statement 1 is proved.

It follows from statement 1 that under the bitwise totalling , the maximum length of transfer d to the higher digits not less than $d+dS\max_{min}+H[p]$. Under bitwise totalling , the *FAL*-transformation of the overrun digit of an intermediate result, which may cause the *EAR*-transformation of the given digit, is being executed. Therefore, the length of transfer to the lower digits may not be less than τp. Digit capacity of bitwise summator equals the sum of transfer lengths to the higher and lower digits. That is, for capacity N of bitwise summator *AM*-numerical system the following expression comes true:

$$N \geq dZ + dS\max_{\min} + H[p] + \tau p \,. \qquad (5)$$

The capacity of bitwise summator determines the hardware expenses for its realization with the specific accuracy. Summators are the main part of all arithmetical devices. So, the expression (5) allows to compare different *AM*-numerical systems on hardware expenses to organize the bitwise processing.

Conclusions

The paper researches the *AM*-numerical systems, which generalize the famous and allow to create the new numerical systems with the ability of bitwise conveyor execution of all arithmetic operations, beginning with the higher digits. There had been described the parameters which help set *AM*-numerical systems, and restrictions which are imposed upon these parameters. For *AM*-numerical systems there had been introduced the notion of additive correlation. It is very simply to set *AM*-numerical systems by variety of numbers C_k and additive correlation ${}^{t}A^{\tau,p}$.

There had been researched the additive transformations in *AM*-numerical systems which are conventional arithmetic operations and generalize the known operations of transfer and borrowing. There had been conducted a classification of additive transformations and there had been described the rules for execution of each type of additive transformation.

On the base of additive transformation there had been researched the bitwise totalling in *AM*-numerical systems, beginning with the higher digits. Such totalling has a restricted length of transfer to the higher digits. There had been formulated and proved the statement on the dependence of transfer length to the higher digits under the bitwise totalling on parameters of *AM*-numerical systems.

There had been determined the dependence of length of transfer to the lower digits on parameters of *AM*-numerical systems.

The obtained results enabled to determine the dependence of capacity of bitwise summator on parameters on *AM*-numerical systems. Using the obtained results enables to compare *AM*-numerical systems as for the hardware expences on the organization of bitwise processing.

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