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ELEMENTS OF FUZZY-LOGIC OF "PRODUCT" TYPE

The paper considers the equations of frequency - pulse, band -pulse, and phase - pulse elements operation which realize fuzzy-logic operations of the "product" type – "AND", "OR", "SUM", which are the unique case of logic operations of "minimum", "maximum" and "exclusive OR" correspondingly. There had been described functioning of fuzzy logic elements of "product" type. There had been presented time diagrams of frequency-pulse element "AND", band-pulse element "OR" and phase-pulse element "SUM". There had been developed structural diagram of frequency-pulse element "AND".

Key words: *pulse signal, logic element, fuzzy-logic.*

Introduction

Devices of automated control, functioning on fuzzy-logic base are being applied in complex telecommunication, radioengineering and microelectronic system, input data of which are subject to constant change, and output data require on-line adjusting. Application of specialized devices of automated control, functioning on the base of fuzzy logic, unlike conventional controlling devices, allow to improve accuracy and reliability of control. In separate cases, in devices for automated control, which function on the base of fuzzy-logic, logical output is performed by logic operations of "product" type – "AND", "OR", "SUM", which are the separate case of logic operations "minimum", "maximum" and "exclusive OR" correspondingly [1].

Further designing of devices for automatic control requires the synthesis of elements, which perform operations of fuzzy-logic of "product" type, and the synthesis of the above elements requires the development of their mathematical models. Mathematical models of fuzzy-logic elements are developed in [2 – 4], but without the consideration of type of information signal. References [3 – 5] present the diagrams of fuzzy-logic elements in which input and output information is presented by analogue signals of alternative voltage or alternative current, which stipulates for their low accuracy. At the same time, use of digital signals results in decrease in operating speed of the device.

It is suggested to improve the accuracy of the system, which function on the base of fuzzy-logic, due to use of analogue but not pulse signals, since the coding of values of fuzzy-logic values by pulse parameters, considering possible errors and influences, will allow to achieve high accuracy, in contrast with formation of analogue signals, the form of which corresponds to the form of membership function, since it is almost impossible to form a signal, the form of which is mathematical ideal.

Therefore the objective of this work is to improve efficiency of fuzzy-logic elements design.

The objective requires the solution of the following problems:

– to develop methods for coding fuzzy-logic values by signal impulse parameters;

–to determine the main operations of fuzzy-logic of the type "product";

–to develop the working equation for fuzzy- logic elements of "product" type.

Pulse coding of fuzzy-values

At the initial stage of elements development, which function by the rules of fuzzy-logic, it is necessary to determine the method for presenting fuzzy-values by parameters of signals of such devices. Since in future there will be developed elements, signals in which are pulses, it is necessary to determine the method for coding the fuzzy-values by parameters of pulse- modulated signals. In this case pulse signals are frequency – band – and phase-pulse signals. The authors developed methods for pulse coding of fuzzy values μ. They are given in table 1.

Table 1

Pulse coding of fuzzy values

Coding type	Fuzzy logic zero	Fuzzy logic unit	Auxiliary value	Additional condition	Coding of fuzzy value
Frequency pulse	f_α	f_β	f_γ	$f_\beta > f_\alpha$, $f_\gamma = f_\beta - f_\alpha$	$f_\mu = f_\alpha + \mu \cdot f_\gamma$
Band – width and pulse	t_α	t_β	t_β^n, t_α^n	$t_\beta > t_\alpha$, $t_\beta^n < t_\alpha^n$	$t_\mu = \mu \cdot t_\beta + (1-\mu) \cdot t_\alpha$
Phase and pulse	φ_α	φ_β	f_1^x	$\varphi_\alpha = 0$, $\varphi_\beta = 2\pi$	$\varphi_\mu = \mu \cdot 2\pi$

During frequency and pulse coding, the input and output signal of devices are pulses, frequency of filling of which f_μ is proportional to the value of fuzzy value μ. In this method of coding, the frequency of pulse filling f_α corresponds to zero value of fuzzy – value, f_β - to unity value, f_β; frequency range with values of fuzzy- values from 0 to 1 are determined by frequency of filling pulse f_γ.

During band width coding, the duration of pulse t_μ corresponds to the values of fuzzy value μ. Duration of pulse corresponds to zero value of fuzzy-value, and t_β to unit value.

During phase pulse coding, the difference in phases of input and reference pulse φ_μ corresponds to the fuzzy value μ. Difference in phases of input and reference signals $\varphi_\alpha = 0$ corresponds to the zero value of membership function, and difference in phases of input and reference signal $\varphi_\beta = 2\pi$ - to unit.

Operation of fuzzy logic of "product" type

Mathematically, fuzzy logic operations of "product" type may be presented as follows [3]:

$$\mu_{AND}^y = \mu_1^x \cdot \mu_2^x, \quad (1)$$

$$\mu_{OR}^y = \mu_1^x + \mu_2^x - \mu_1^x \cdot \mu_2^x, \quad (2)$$

$$\mu_{SUM}^y = \mu_1^x + \mu_2^x - 2 \cdot \mu_1^x \cdot \mu_2^x. \quad (3)$$

The suggested equations for work of frequency - pulse, band – width pulse and phase pulse elements, which realize operations "AND", "OR", "SUM" are presented in table 2.

Table 2

Equation of fuzzy – logic elements of "product" type operation

	Frequency and pulse element	Band –width pulse element	Phase and pulse element
Input values	$f_1^x = f_\alpha + \mu_1^x \cdot f_\gamma$, $f_2^x = f_\alpha + \mu_2^x \cdot f_\gamma$	$t_1^x = \mu_1^x \cdot t_\beta + (1-\mu_1^x)\cdot t_\alpha$, f_2^x	$\varphi_1^x = \mu_1^x \cdot 2\pi$, $\varphi_2^x = \mu_2^x \cdot 2\pi$
Result of logic operation AND	$f_{AND}^y = f_\alpha + \mu_{AND}^y \cdot f_\gamma = f_\alpha + (\mu_1^x \cdot \mu_2^x) f_\gamma$	$t_{AND}^y = \mu_{AND}^y \cdot t_\beta + (1-\mu_{AND}^y)\cdot t_\alpha = \mu_1^x \cdot \mu_2^x \cdot t_\beta + (1-\mu_1^x \cdot \mu_2^x)\cdot t_\alpha$	$\varphi_{AND}^y = \mu_{AND}^y \cdot 2\pi = \mu_1^x \cdot \mu_2^x \cdot 2\pi$
Result of logic operation OR	$f_{OR}^y = f_\alpha + \mu_{OR}^y \cdot f_\gamma = f_\alpha + (\mu_1^x + \mu_2^x - \mu_1^x \cdot \mu_2^x) f_\gamma$	$t_{OR}^y = \mu_{OR}^y \cdot t_\beta + (1-\mu_{OR}^y)\cdot t_\alpha = (\mu_1^x + \mu_2^x - \mu_1^x\mu_2^x)t_\beta + [1-(\mu_1^x + \mu_2^x - \mu_1^x\mu_2^x)]t_\alpha$	$\varphi_{OR}^y = \mu_{OR}^y \cdot 2\pi = (\mu_1^x + \mu_2^x - \mu_1^x\mu_2^x)\times 2\pi$
Result of logic operation SUM	$f_{SUM}^y = f_\alpha + \mu_{SUM}^y \cdot f_\gamma = f_\alpha + (\mu_1^x + \mu_2^x - 2\mu_1^x\mu_2^x) f_\gamma$	$t_{SUM}^y = \mu_{SUM}^y \cdot t_\beta + (1-\mu_{SUM}^y)\cdot t_\alpha = (\mu_1^x + \mu_2^x - 2\mu_1^x\mu_2^x)t_\beta + [1-(\mu_1^x + \mu_2^x - 2\mu_1^x\mu_2^x)]t_\alpha$	$\varphi_{SUM}^y = \mu_{SUM}^y \cdot 2\pi = (\mu_1^x + \mu_2^x - 2\mu_1^x\mu_2^x)\times 2\pi$

The suggested equations of work may be used during the development of elements of fuzzy – logic of "product" type for construction of pulse elements which realize operations "AND", "OR", "SUM".

Time diagrams of elements operation

Lets show an example of time diagrams of operation of frequency pulse element "AND" band-width pulse element "OR", phase -pulse element "SUM" (fig 1 - 3).

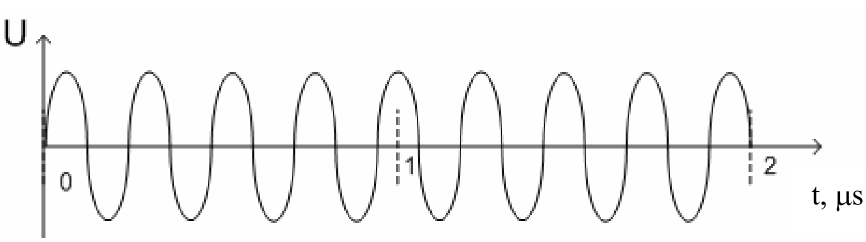

Fig. 1, a. Pulse with frequency f_1^x

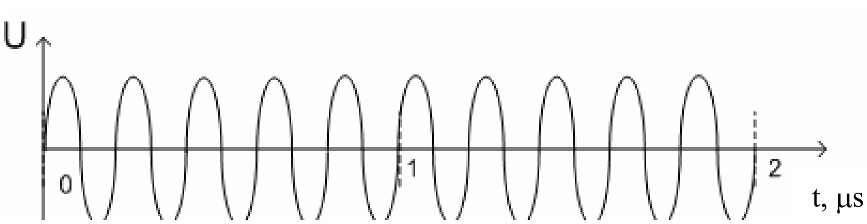

Fig. 1, b. Pulse with frequency f_2^x

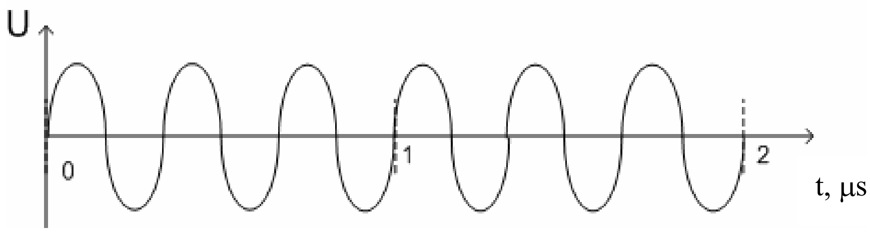

Fig. 1, c. Pulse with frequency f^y_{AND}

During the realization of operation "AND" (1) pulse signals x_1, x_2 enter the elements input, and pulse signal y_{AND} appears on the output. In case of frequency and pulse coding, the signals x_1, x_2 have frequencies of pulse filling f_1^x, f_2^x and the signal y_{AND} – f^y_{AND}. Time diagram for fuzzy-logic element "AND" are presented in Fig. 1. Input values μ_1^x and μ_2^x acquire values 0,3 and 0,4 correspondingly. Then, according to formula (1) - $\mu^y_{AND} = 0{,}12$. According to the developed method of frequency and pulse coding: $f_1^x = 2{,}2\ MHz$ (fig. 1, a.), $f_2^x = 2{,}6\ MHz$ (fig. 1, b.), and $f^y_{AND} = 1{,}48\ MHz$ (fig. 1, c.).

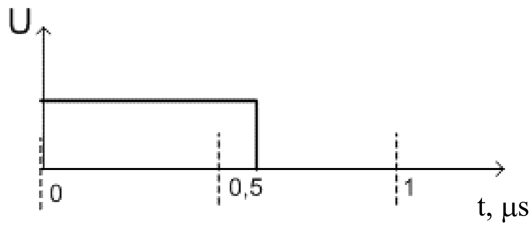

Fig. 2, a. Pulse with duration $t^x{}_1$

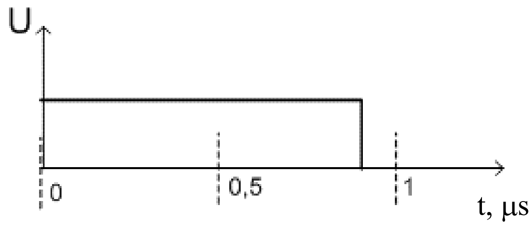

Fig . 2, b. Pulse with duration tx2

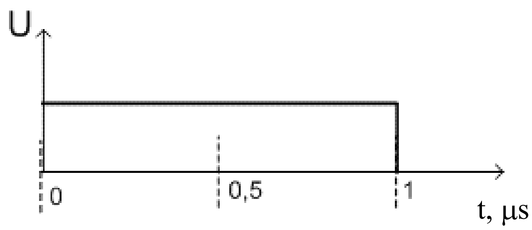

Fig. 2, c. Pulse with duration $t^y{}_{OR}$

During the realization of the operation "OR" (2), pulse signals x_1, x_2 enter the elements input, pulse signal y_{OR} appears on the output. In case of band width pulse coding, signals x_1, x_2 have the durations of pulses t_1^x, t_2^x and signal y_{OR} – t^y_{OR}. Time diagrams for fuzzy-logic elements "OR" are presented in fig 2. Input values μ_1^x and μ_2^x acquire values 0,5 and 0,8 correspondingly. Then, according to the formula (2) – $\mu^y_{OR} = 0{,}9$. According to the developed method of band width impulse coding $t_1^x = 0{,}6\mu s$ (fig. 2, a.), $t_2^x = 0{,}9\mu s$ (fig. 2, b.) and $t^y_{OR} = 1\mu s$ (fig. 2, c.).

During the realization of the operation “SUM” (3) impulse signals x_1, x_2 are supplied on the input of the element, and the impulse signal y_{SUM} appears on the output. In case of phase and impulse coding, signals x_1, x_2 have differences in phases of reference pulse and informational pulses φ_1^x, φ_2^x and signal y_{SUM} – φ_{SUM}^y. Time diagrams for fuzzy-logic element “SUM” are presented in fig. 3. Input values μ_1^x and μ_2^x acquire values 0,7 and 0,1 correspondingly. Then according to formula (3) $\mu_{SUM}^y = 0,66$. According to the developed method of phase and impulse coding $\varphi_1^x = \frac{7}{5}\pi$ (fig. 3, a.) and $\varphi_2^x = \frac{\pi}{5}$ (fig. 3, b.) and $\varphi_{SUM}^y = 1,22\pi$ (fig. 3, c.).

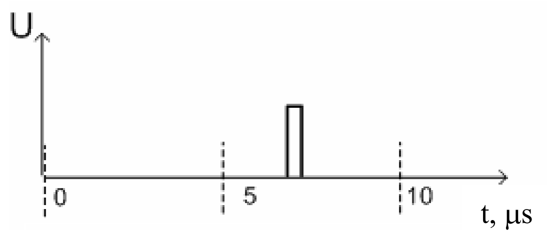

Fig. 3, a. Pulse with phase φ_1^x

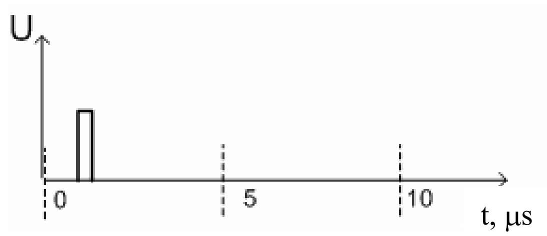

Fig. 3, b. Pulse with phase φ_2^x

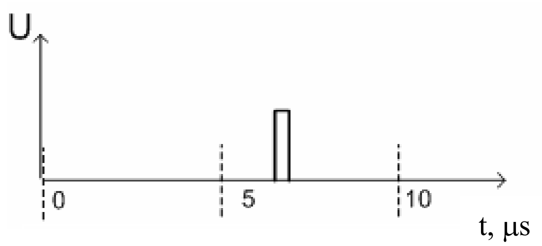

Fig. 3, c. Pulse with phase φ_{SUM}^y

Frequency and pulse element “AND”

According to the above equations, there had been developed the frequency and pulse element executing fuzzy-logic operation “AND” (fig. 4).

The element functions as follows. The signal x_1 with the frequency of impulse filling $f_{x1} = f_\alpha + \mu_1^x \cdot f_\gamma$ is supplied to the input of band filter СФ1 – СФ11. Filters are adjusted as follows:

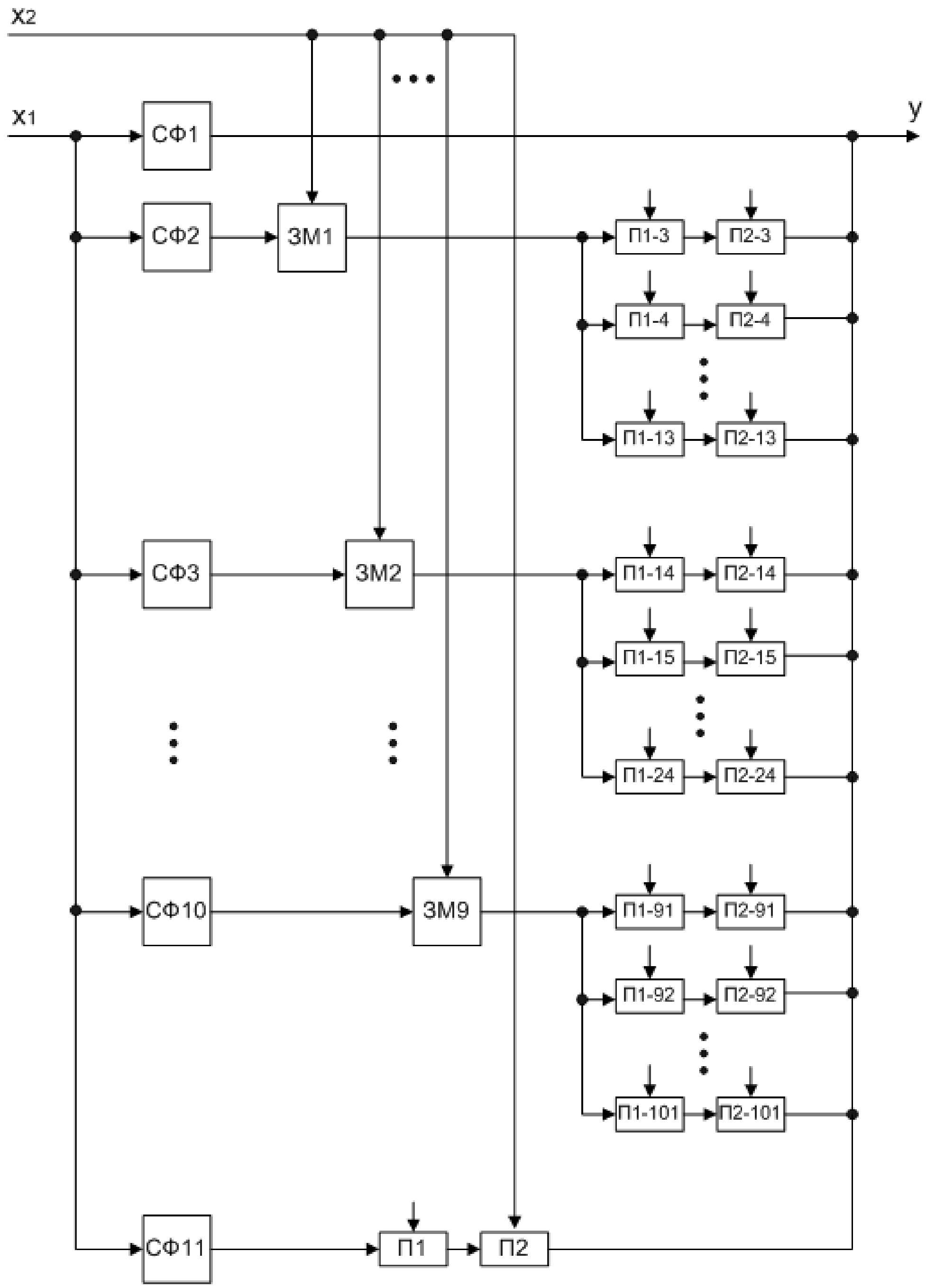

Fig. 4. Structural diagram of device "AND"

Filter СФ1 allows the signal with frequency of impulse filling f_α;

СФ2 –signal with the frequency of impulse filling $(f_\alpha + 0{,}1 \cdot f_\gamma)$;

СФ3 – signal with the frequency of impulse filling $(f_\alpha + 0{,}2 \cdot f_\gamma)$;

СФ4 – signal with the frequency of impulse filling $(f_\alpha + 0{,}3 \cdot f_\gamma)$;

СФ5 – signal with the frequency of impulse filling $(f_\alpha + 0{,}4 \cdot f_\gamma)$;

СФ6 – signal with the frequency of impulse filling $(f_\alpha + 0{,}5 \cdot f_\gamma)$;

СФ7 – signal with the frequency of impulse filling $(f_\alpha + 0{,}6 \cdot f_\gamma)$;

СФ8 – signal with the frequency of impulse filling $(f_\alpha + 0,7 \cdot f_\gamma)$;

СФ9 – signal with the frequency of impulse filling $(f_\alpha + 0,8 \cdot f_\gamma)$;

СФ10 – signal with the frequency of impulse filling $(f_\alpha + 0,9 \cdot f_\gamma)$;

СФ11 – signal with the frequency of impulse filling f_β.

If the frequency of the first input signal $f_{x1} = f_\alpha$, which corresponds to fuzzy-logic zero, then from filter output СФ1 the signal is supplied to the output of the devices y without any change.

If the frequency of the first input signal $f_{x1} = f_\alpha$, which corresponds to fuzzy logic zero, then the signal passes filter СФ11 and is supplied on controlling input of switcher П1, on informational input of which the signal frequency f_c is supplied, which is controlling for switcher, П2 on informational input of which the second input signal with frequency f_{x2} is supplied. As the switcher П1 under the influence of frequency f_β closes, the frequency f_c will be absent on its output, then the switcher П2 remains open and on its output appears the frequency of input signal f_{x2}, which is supplied on the input of the device.

If $f_{x1} \neq f_\alpha$ and $f_{x1} \neq f_\beta$, then signals appears on the output of one of the filters СФ2 – СФ10, depending on value of frequency f_{x1}.

The second input signal with frequency f_{x2} is supplied on the first inputs of frequency mixers ЗМ1 – ЗМ9. On the second input of one mixers ЗМ1 – ЗМ9 the first input signal from output of one of the filters СФ2 – СФ10 is supplied correspondingly.

If the frequency of the first input signal $f_{x1} = f_\beta$ which corresponds to fuzzy-logic unit, the signal then passes through the filter СФ11 and is supplied on the controlling input of switcher П1, on the informational input of which the signal with frequency f_c is supplied, which is a controlling one for the switcher П2, on informational input of which the second input signal with frequency f_{x2} is supplied. Since the switcher П1 closes under influence of frequency f_β the frequency f_c will be absent on its output, in the result of which the switcher П2 remains open and the frequency of the second input signal f_{x2} which is supplied to the output of the device, will appear on its output.

If $f_{x1} \neq f_\alpha$ and $f_{x1} \neq f_\beta$, the signal then will appear on the output of one of the filters СФ2 – СФ10, depending on value frequency f_{x1}.

The second output signal with frequency f_{x2} will be supplied on the first inputs of frequencies mixers inputs ЗМ1 – ЗМ9. The first input signal from the output of one of filters СФ2 – СФ10 correspondingly is supplied on the second input of one of the mixers ЗМ1 – ЗМ9.

From the mixers output we receive the signal with intermediate frequency $f_{x1} + f_{x2}$. The signal form the mixers output is supplied on the inputs of corresponding switchers, controlling signals of which are given in table 3, and the informational frequency for switchers П1-3 – П1-101 is frequency f_c, controlling for switchers П2-3 – П2-101, informational frequencies of which are given in table 3. Outputs of switchers П2-3 – П2-101 are connected with corresponding outputs of switchers П1-3 – П1-101. Thus, the signal appears on the output of one of the mixers, is stipulated on the outputs 11 switchers which correspond to it, for one of them it is a controlling one and closes it, then the signals appear on the outputs of other ten switchers and close the next ten switchers, on the output of which the signal will be absent. The switcher with no controlling signal on the input remains open and its information signal will be supplied on the output of the device "AND".

Table 3

Controlling signals of switchers П1-3 – П1-101 and information signals of switchers П2-3 – П2-101

MIX1			
switcher	f_{cont}	switcher	f_{inf}
П1-3	$2f_\alpha + 0,1 \cdot f_\gamma$	П2-3	f_α
П1-4	$2f_\alpha + 0,2 \cdot f_\gamma$	П2-4	$f_\alpha + 0,01 \cdot f_\gamma$
П1-5	$2f_\alpha + 0,3 \cdot f_\gamma$	П2-5	$f_\alpha + 0,02 \cdot f_\gamma$
П1-6	$2f_\alpha + 0,4 \cdot f_\gamma$	П2-6	$f_\alpha + 0,03 \cdot f_\gamma$
П1-7	$2f_\alpha + 0,5 \cdot f_\gamma$	П2-7	$f_\alpha + 0,04 \cdot f_\gamma$
П1-8	$2f_\alpha + 0,6 \cdot f_\gamma$	П2-8	$f_\alpha + 0,05 \cdot f_\gamma$
П1-9	$2f_\alpha + 0,7 \cdot f_\gamma$	П2-9	$f_\alpha + 0,06 \cdot f_\gamma$
П1-10	$2f_\alpha + 0,8 \cdot f_\gamma$	П2-10	$f_\alpha + 0,07 \cdot f_\gamma$
П1-11	$2f_\alpha + 0,9 \cdot f_\gamma$	П2-11	$f_\alpha + 0,08 \cdot f_\gamma$
П1-12	$2f_\alpha + f_\gamma$	П2-12	$f_\alpha + 0,09 \cdot f_\gamma$
П1-13	$2f_\alpha + 1,1 \cdot f_\gamma$	П2-13	$f_\alpha + 0,1 \cdot f_\gamma$
MIX2			
switcher	f_{cont}	switcher	f_{inf}
П1-14	$2f_\alpha + 0,2 \cdot f_\gamma$	П2-14	f_α
П1-15	$2f_\alpha + 0,3 \cdot f_\gamma$	П2-15	$f_\alpha + 0,02 \cdot f_\gamma$
П1-16	$2f_\alpha + 0,4 \cdot f_\gamma$	П2-16	$f_\alpha + 0,04 \cdot f_\gamma$
П1-17	$2f_\alpha + 0,5 \cdot f_\gamma$	П2-17	$f_\alpha + 0,06 \cdot f_\gamma$
П1-18	$2f_\alpha + 0,6 \cdot f_\gamma$	П2-18	$f_\alpha + 0,08 \cdot f_\gamma$
П1-19	$2f_\alpha + 0,7 \cdot f_\gamma$	П2-19	$f_\alpha + 0,1 \cdot f_\gamma$
П1-20	$2f_\alpha + 0,8 \cdot f_\gamma$	П2-20	$f_\alpha + 0,12 \cdot f_\gamma$
П1-21	$2f_\alpha + 0,9 \cdot f_\gamma$	П2-21	$f_\alpha + 0,14 \cdot f_\gamma$
П1-22	$2f_\alpha + f_\gamma$	П2-22	$f_\alpha + 0,16 \cdot f_\gamma$
П1-23	$2f_\alpha + 1,1 \cdot f_\gamma$	П2-23	$f_\alpha + 0,18 \cdot f_\gamma$
П1-24	$2f_\alpha + 1,2 \cdot f_\gamma$	П2-24	$f_\alpha + 0,2 \cdot f_\gamma$

Conclusions

Thus, the following operations belong to the fuzzy-logic operations of "product" type. For the elements, which realize the given operations, equations are suggested. The functioning of elements is shown by time diagrams. There had been developed the structural diagram of frequency and pulse element "AND". The given equations are suggested to be used for development of frequency, band width, phase and pulse elements of fuzzy logic, which realize the operations of "product: type "AND", "OR", "SUM".

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