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REALIZATION OF TERNATY LOGIC OPERATIONS WITH THE HELP OF TWO-THRESHOLD NEURONS

The paper suggests the mathematical models for two-threshold neurons for ternary logic. There had been shown five neuron nets with linear and two-threshold neurons. The first one functions as an elements of ternary, disjunction, the second – as element of ternary conjunction, the third – ternary "exclusive or", the fourth – ternary inversion, the fifth – ternary cycle. For the developed neuron nets there had been determined mathematical models.

Key words: *neuron nets, ternary logic, two-threshold neurons, disjunction, conjunction, inversion.*

Introduction

Methods for calculating intellect unite the different constituents intellectual technologies – fuzzy logic, neuron nets, genetic algorithms – into the hybrid systems. Hybrid systems , such as fuzzy neuron nets with genetic tuning of parameters, demonstrate mutual strengthening of advantages and reduction of number of drawbacks of some methods. At present there prevail the neuro-fuzzy hybrid systems. The number of fuzzy-genetic , neuro-genetic and neuro-fuzzy genetic systems increases.

The main task of combining the perception system with logic processing lies in building schemes, operating digits (perception) and discrete signals of truth (logic). One of the peculiarities of logic - oriented hybrid neuron nets is their ability to perform complicated operations having pretty simple structure. This, in turn, allows to decrease the number of correlations between the elements, which all allow to increase the fast-activity in performing logic operations [1]. Schemes of some of them contain classical as well as AND and OR neurons [2].

Ternary logic became of interest prior to appearance of computers due to peculiarities of symmetrical code of digits. This interest is rising again due to the new possibilities of semi-conducting technology. As a result, there will appear cheap integral elements with three states. There have also been conducted theoretical researches of three level equipment. The results are expected to find practical application.

In [3] it is mentioned that use of ternary coding of information in combination with schemes, which signals are quantified on three levels, in digital devices for controlling, managing and prognosing of failures may influence the structural organization of digital systems. Advantages may appear due to the use of more comfortable devices for integrating digital systems for information transfer and performing devices, signals of which, as a rule, are of ternary character.

It is expedient to develop ternary elements of disjunction, conjunction, "exclusive OR", inversion and cycle as the neuron net with linear and two threshold neurons.

Thus, the objective of the work is to improve designing efficiency of neuron nets. This objective requires to solve the following tasks:

– to determine mathematical models of neurons for ternary logic;
– compose a scheme for neuron net;
– determine mathematical models for the developed neuron nets.

Two-threshold neurons for ternary elements

Principles, operations and functions of ternary logic are described in [4]. Let's assume that we have ternary logic of type [0, 1, 2].

Operation of ternary disjunction is performed as:

$$y = x_a \vee x_b = \max(x_a, x_b).$$

Operation of ternary conjunction is performed as:

$$y = x_a \wedge x_b = \min(x_a, x_b).$$

Operation of ternary "exclusive OR" is performed as:

$$y = \min\lfloor \max(x_a, x_b) - \min(x_a, x_b) \rfloor$$

Operation of ternary inversion is performed as:

$$y = 2 - x.$$

Operation of ternary cycle is performed as:

$$y = x \oplus 1(\mathrm{mod}\,3).$$

Works [6 – 8] present schemes of ternary logic elements of minimum, maximum and inversion. Let's change the constituents blocks of the above elements by two-threshold and linear neuron.

For linear neuron we got [2]:

$$net = \sum_i \omega_i x_i,$$

$$f(net) = net,$$

where x_i – input signal; ω_i – weigh of synopsis; net – weighed neuron signal; $f(net)$ – function of activation

For threshold neuron we get [1]:

$$f(net) = \begin{cases} 0 : net < 0 \\ 1 : net \geq 0. \end{cases}$$

Realization of binary operations AND, OR, NO with the help of threshold neurons is considered in [4].

The advantages of using two-threshold neurons in fuzzy and ternary logic are described in [1].

For two-threshold neurons we have:

$$f(net) = \begin{cases} -1 : net < \theta \\ 0 : -\theta \leq net \leq \theta, \\ 1 : net > 0 \end{cases}$$

where θ – threshold.

Such two-threshold neuron is suggested in [5]:

$$f(net) = \begin{cases} 0 : net < 0 \\ 1 : 0 \leq net < 2. \\ 2 : net \geq 2 \end{cases}$$

We suggest the following mathematical model of functioning two-threshold neuron for ternary logic.

$$f(net) = \begin{cases} 0 : net < \theta_1 \\ 1 : \theta_1 \leq net < \theta_2. \\ 2 : net \geq \theta_2 \end{cases}$$

Neuron nets for ternary logic elements

Neuron net, performing operation of ternary disjunction is presented on fig. 1. and it operates as follows:

1. The first layer

,

$$f(net_1) = y_1 = \begin{cases} 0 : -2 \le net_1 < 1 \\ 1 : 1 \le net_1 < 2 \\ 2 : 2 \le net_1 \end{cases},$$

$$f(net_2) = y_2 = \begin{cases} -2 : -2 \le net_2 < -1 \\ -1 : -1 \le net_2 < 0 \\ 0 : 0 \le net_2 \end{cases},$$

2. The second layer

$$net_3 = x_a \cdot (+0{,}5) + x_b \cdot (+0{,}5) + y_1(+0{,}5) + y_2(-0{,}5),$$

$$f(net_3) = z_{dis} = net_3 .$$

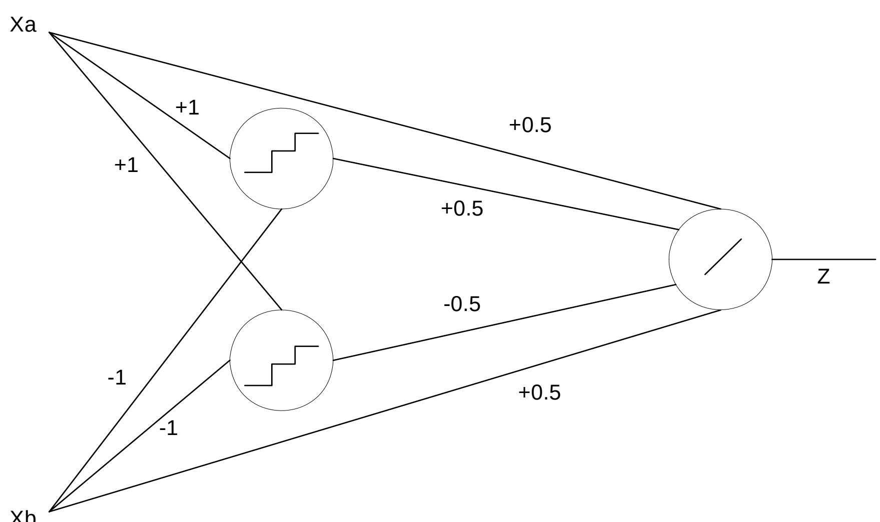

Fig. 1. Neuron net of ternary disjunction

Here x_a, x_b – input net signals; y_1, y_2 – output signals of the first and the second neurons; z_{dis} – output net signal.

Neuron net, performing operation of ternary conjunction is presented on fig. 2 and operates as follows:

1. The first layer

$$net_1 = net_2 = x_a \cdot (+1) + x_b \cdot (-1),$$

$$f(net_1) = y_1 = \begin{cases} 0 : -2 \le net_1 < 1 \\ 1 : 1 \le net_1 < 2 \\ 2 : 2 \le net_1 \end{cases},$$

$$f(net_2) = y_2 = \begin{cases} -2 : -2 \le net_2 < -1 \\ -1 : -1 \le net_2 < 0 \\ 0 : 0 \le net_2 \end{cases},$$

2. The second layer

$$net_3 = x_a \cdot (+0,5) + x_b \cdot (+0,5) + y_1(-0,5) + y_2(+0,5),$$

$$f(net_3) = z_{con} = net_3.$$

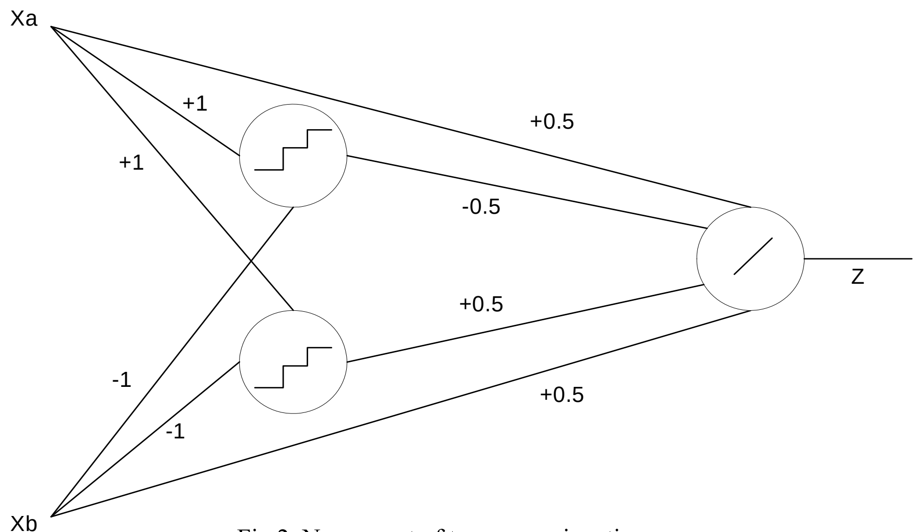

Fig.2. Neuron net of ternary conjunction

Here x_a, x_b – input net signals; y_1, y_2 – output signals of the first and the second neurons; z_{con} – output net signals.

Neuron net, performing operation of "exclusive OR" is presented on fig. 3 and operates as follows:

1. The first layer

$$net_1 = x_a \cdot (+4) + x_b \cdot (-3),$$

$$net_2 = x_a \cdot (-3) + x_b \cdot (+4),$$

$$f(net_1) = y_1 = \begin{cases} 0 : net_1 \le 2 \\ 1 : 2 < net_1 \le 4 \\ 2 : 4 < net_1 \end{cases},$$

$$f(net_2) = y_2 = \begin{cases} 0 : net_2 \le 2 \\ 1 : 2 < net_2 \le 4 \\ 2 : 4 < net_2 \end{cases},$$

2. The second layer

$$net_3 = x_a \cdot (-2) + x_b \cdot (-2) + y_1(+5) + y_2(+5),$$

$$f(net_3) = z_{XOR} = \begin{cases} 0 : net_3 \le 2 \\ 1 : 2 < net_3 < 4 \\ 2 : 4 \le net_3 \end{cases}.$$

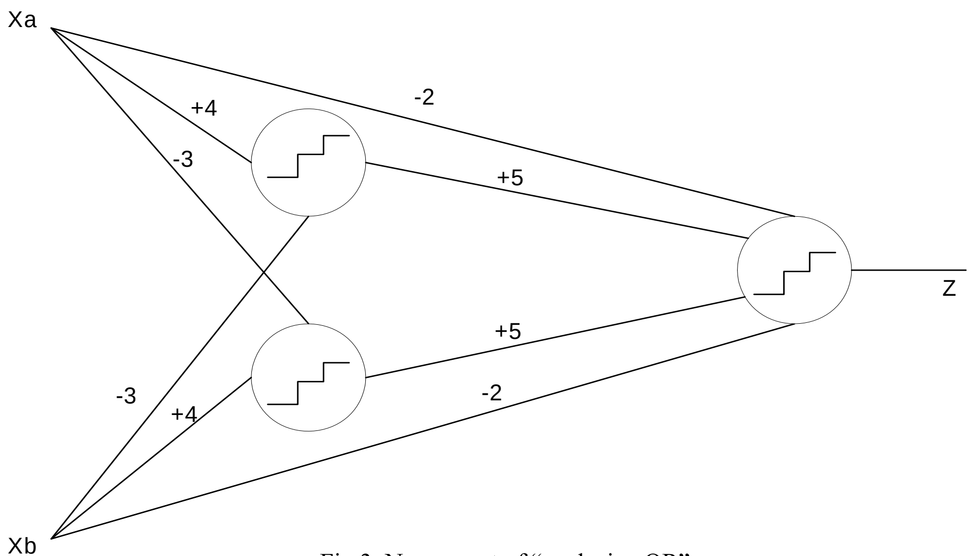

Fig.3. Neuron net of "exclusive OR"

Here x_a, x_b – input nets signals; y_1, y_2 – output signals of the first and the second neurons; z_{XOR} – output net signal.

Neuron net, performing operation of ternary inversions is presented on fig. 4 and operates as follows:

Нейронна мережа, яка виконує операцію трійкової інверсії, представлена на рис. 4 і працює таким чином:

$$net = x_a \cdot (+1) + x_b \cdot (-1),$$

$$f(net) = z_{inv} = net.$$

Here x_a – input net signals; x_b – ancillary net signal; z_{inv} – output net signal.

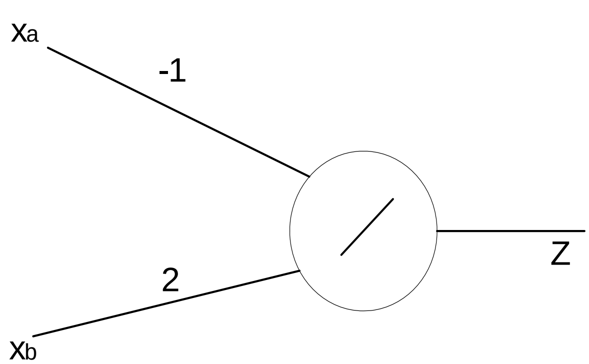

Fig. 4. Neuron net of ternary inversion

Neuron net, performing the operation of ternary cycle, is presented in fig. 5 and operates as follows:

1. The first layer

$$net_1 = x_a \cdot (+2) + x_b \cdot (-3),$$

$$f(net_1) = y_1 = \begin{cases} 0 : net_1 < 1 \\ 1 : 1 \le net_1 < 2 \\ 2 : 2 \le net_1 \end{cases}.$$

2. The second layer

$$net_2 = x_a \cdot (+1) + x_b \cdot (+1) + y_1(-3),$$

$$f(net_2) = z_{sh} = net_2.$$

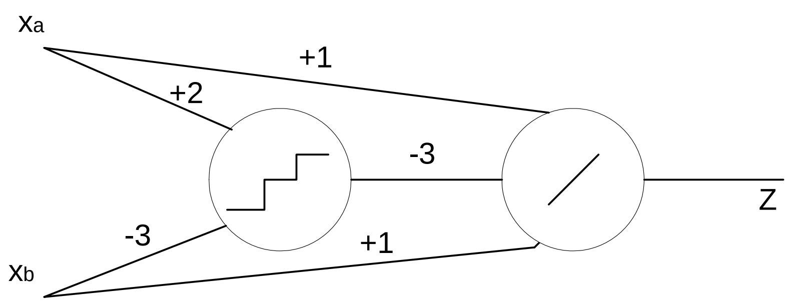

Fig.5. Neuron net of ternary cycle

Here x_a – input net signals; x_b – ancillary net signal; y_1 – output signal of the first neuron; z_{sh} – output net signal.

Conclusions

There had been suggested five neuron nets with linear and two threshold neurons. The first one functions as the element of ternary disjunction, the second as an element of ternary conjunction, the third ternary "exclusive OR", the fourth – ternary inversion, and the fifth – ternary cycle.

Using two – threshold neurons allows to build neuron nets which realize the operations of ternary logic. This will widen the sphere of application of neuron nets and combine the calculating equipment with intellectual technologies.

REFERENCES

1. Masahiro Sakamoto, Mititada Morisue. A study of ternary fuzzy processor using neural networks // Proc. of IEEE International Symposium on Circuits and Systems. – Hong Kong. – 1997. – P. 613 – 616.

2. Ярушкина Н. Г. Нечёткие нейронные сети с генетической настройкой. Научная сессия МИФИ – 2004. VI Всероссийская научно-техническая конференция «Нейроинформатика–2004»: Лекции по нейроинформатике. Часть 1. – М.: МИФИ, 2004. – 199 с.

3. Морозов Д. В., Пилипко М. М., Коротков А. С. Реализация устройств троичной логики на основе стандартной МОП – технологии // Микроэлектроника . – 2009. – Т. 38. – № 3. – С. 224 – 236.

4. Gerald Reif. Neuronale Netze // http://www.iicm.tugraz.at/Teaching/ theses/2000/_idb9e_/greif/node10.html.

5. Ali Haydar, M. Jad Hamdan, M. Backer Rashid, Hassan Hamich, Ahmad Issa, Abdallah Kassem. A novel neural network ternary arithmetic logic unit // Proc. of the 23rd International Technical Conference on Circuits/Systems, Computers and Communications. Kaikyo Messe Shimonoseki Yamaguchi, Japan. – 2008. – P. 593 – 596.

6. Патент №33476 на корисну модель МПК (2006) H03K19/20. Трійковий елемент мінімуму / Семенова О. О., Семенов А. О., Войцеховська О. О. // Номер заявки u2008 01766. Дата подання 11.02.2008. Опубл. 25.06.2008. Бюл. №12. – 4 с.

7. Патент 34465 на корисну модель МПК H03K19/20. Трійковий елемент максимуму / Семенова О. О., Семенов А. О., Войцеховська О. О. // Номер заявки u 2008 30612. Дата подання 21.03.2008. Опубл. 11.08.2008.

Бюл. №15. – 5 с.
8. Патент 35963 на корисну модель МПК H03K19/20. Логічний елемент трійкової інверсії / Семенова О. О., Семенов А. О., Кичак В. М., Войцеховська О. О. // Номер заявки u 2008 05822. Дата подання 05.05.2008. Опубл. 10.10.2008. Бюл. № 19. – 4 с.

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