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DEVELOPMENT OF THE MODELS OF "PRODUCERS – PRODUCTS – CONSUMERS" GENERALIZED SYSTEMS

The system of working models of "producers – products – consumers" distributed system has been constructed. In the model the behaviour of each separate consumer, of a consumer and a retailer is simulated. Investigations were conducted using the models. New results for direct evaluation of the risks are obtained.

Key-words: *distributed system, producer, product, working model, imitational modeling, demand, supply, fuzzy choice, risks, decision making.*

Problem statement. In the modern world of hyper propositions computer models application for planning and forecasting is the necessary condition for a business unit to survive. In the USA economy a bankruptcy of one firm led to 30-70 additional bankruptcies. For satisfactory forecasting it is necessary to model the activity of an individual consumer in a certain field against the background of the activities of all producers in this field and, if possible, of each consumer [1, 2]. Among the sub- and pseudoproblems we distinguish the central problem: construction of imitational models of "producers – products – consumers" distributed systems with the ability to model each component within the period of 1 – 4 months rather than that of 10 – 40 years.

Formulation of the objective. The aim is to develop a working imitational model of the "producers – products – consumers" system ("N×M×K system" for short). We consider the following classes of objects simultaneously:

– *a consumer*, characterized by the income, information awareness – the ability to distinguish the products of different producers and the experience of using the product, as well as by the disposition towards consumption;

– *the system of consumes*, characterized by its scope, distribution of incomes in a certain region, by the market saturation, by the choice probability dependencies on the income level, product rating, "the length and height of the shelf", i.e. the system of the demand functions;

– a *product* (*brand*) at the market is characterized by its cost, price, usefulness, utility, value, supply function;

– *the system of products and producers* at the market;

– *product line* – products of a certain class that are systemized according to the price.

Development of the «N×M×K-system» model structure

This paper considers construction of the model that is a "step" in the sequence of models:

- "one aggregated producer at the market of **M** products**"**;
- "**N** producers at the one-product (aggregated) market;
- "**N** producers at the market of **M** products".

The consumers were considered as a monolithically homogeneous mass having one common function of demand for everybody and for all times. This mass is not taught, generalized and organized. In order to pass to the models with imitation of the behavior of individual consumers and producers, the model of the system was developed and investigated:

- "two producers at the market of two alternatives of the product, having a certain quality, and a consumer who is learning how to choose the best product. This is referred to as a market with asymmetric informational structure or a "lemon market". These models make it possible to perfect methods, technologies and program module for the generalized model construction:
- "the market of **N** producers, **M** products, **K** consumers".

In this work a set of products of one class (purpose) with different prices and of different values is considered. The following order of the model construction has been chosen [3, 5]: a verbal model, graphical model, a working mathematical model, complex testing, comparison with statistical

data and model correction. The two last stages are, in fact, a single complex stage that is similar to the creation of a "flying ship".

Step 1. The verbal model of a real system

A buyer is characterized by the income, information awareness – the ability to distinguish the products of different producers and the experience of using the product, by having personal experience of using the product, by disposition to consumption.

The system of consumers is characterized by the scope of a given region (domain of attraction) distribution of incomes in the given region, by market saturation, by the dependence of the choice probability on the level of income, the product rating, as well as on the "length and height of the shelf", i.e. by the demand functions set.

Product (brand) at the market is characterized by the cost, price, utility, and quality (value), the function of supply and the "consumers' club" function.

The system of products (producers) at the market. In a definite segment of market there are products of different prices and quality. Potentially it is possible to evaluate the usefulness (value) of products and to build the "price – value" dependence. In order to model the selected systems, the following models must be built: the model of sales, the model of choosing and teaching the consumers, the production model, the model of consumption and demand saturation for various types of products – expendable products, multiuse products, "eternal" products etc.

Step 2. Graph model of the "products – consumers" system

Verbal models are mapped into the influence graph (fig.1).

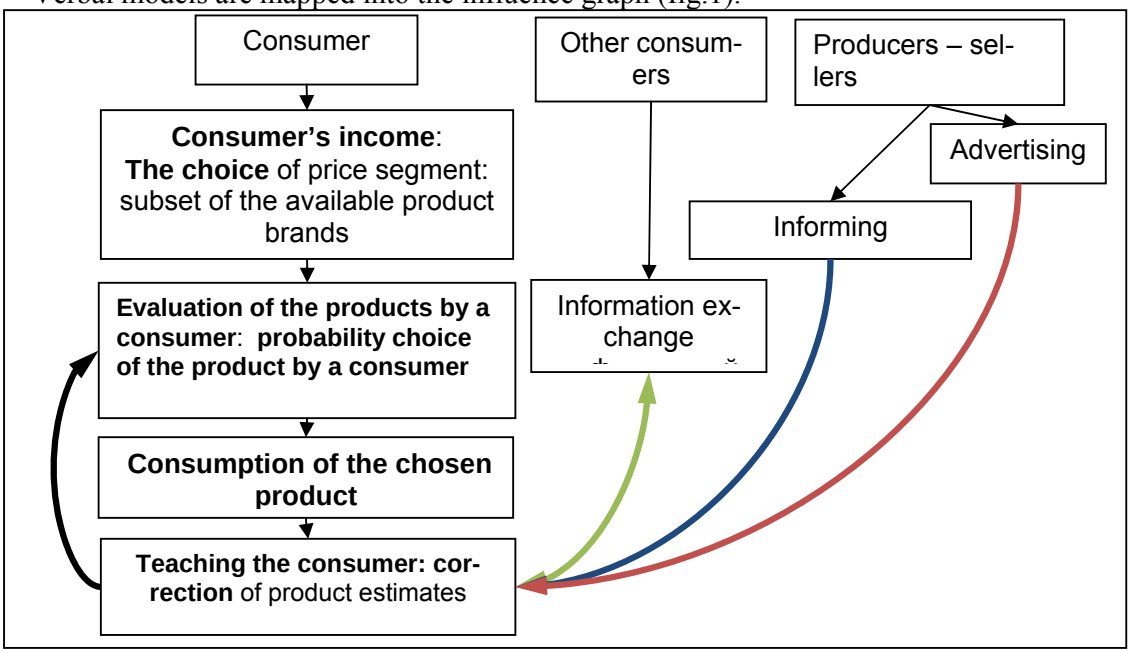
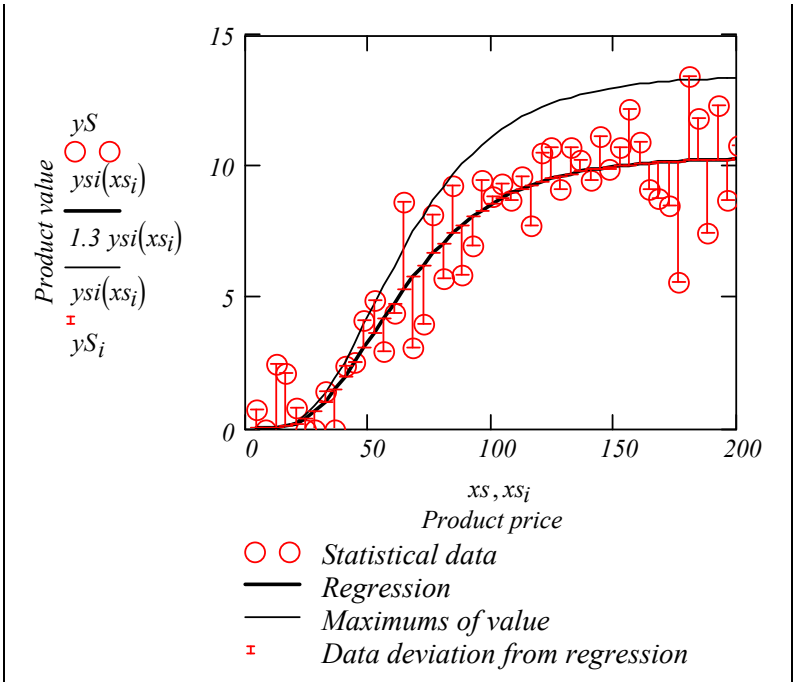


Fig. 1 Flow diagram of the consumer's choice process

Step 3. Selection and development of the functional submodels

"System of the product brands"module. Current requirements of the production and market determine the necessity for both a producer and retailer to operate with the sets of products of a definite class such as automobile model range, product lines of coffee and mayonnaise etc. Product lines are ranged according to prices or value. The "value" index is rather situational and fuzzy indicator. Fig. 2 shows a stand for the analysis of product line models. The "product line" module takes the "cloud of dots" on the "price – value" plane and returns the dependencies of "average price – value", "maximal value – price", "maximal value per cost unit".



Press ≡ 1, to see new reazation of input data
Specifu:
Number of dots Msf ≡ 50
Range xms ≡ 200
Noise level ash ≡ 4.
Review 5 - 10 identifications:
Parametrs
Test Identified
A = 1 ; Ao = 1
w = 0.04 ; wo = 0.034
s = 8 ; so = 5.63
Evaluate the relationship "accuracy - the number of dots"

Fig. 2. Product system – product line

"Consumer's choice in the system of product brands" module. A consumer with a definite level of income chooses a price and a producer of the product he is going to buy at the current stage of the process. In general case he does not possess full information about the value of alternative products. Among several alternatives of building the consumer's choice model the two-level choice model has been chosen: a consumer chooses a price range first and then he chooses a definite product in this range. We use fuzzy logic: we build target membership functions – the dependence of "value/price" index on price and fuzzy budget constraint – dependence on price. Fig. 3 presents parameterized target membership function and income constraint membership function.

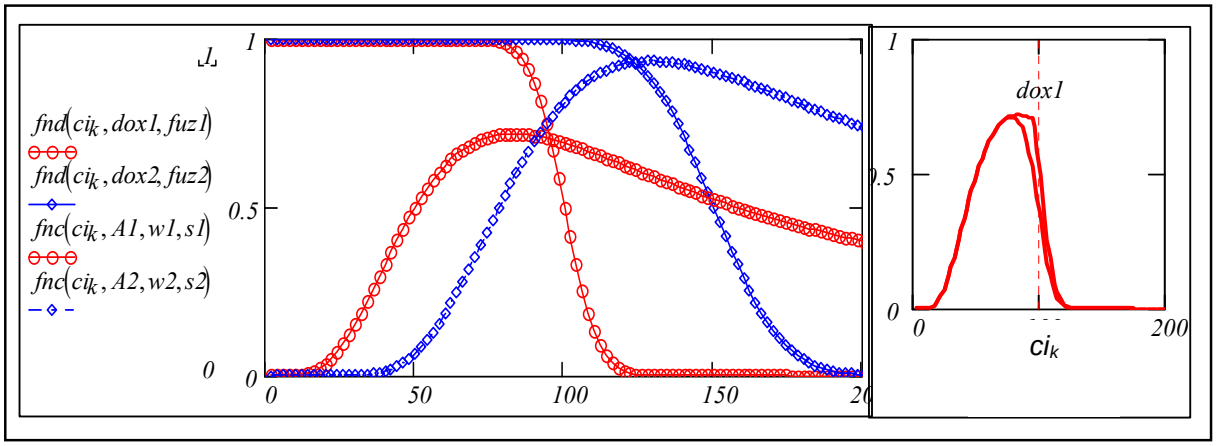


Fig. 3. Examples of the target membership and constraint membership functions

Convolution of target and constraint membership functions. Convolution of the target and constraint membership functions into a single membership function can be achieved in two ways: by multiplying the functions or by taking their minimal value for a definite price. These functions are written for two sets of parameters:

$$ZFNmak1(cin, dox) := \min(fnd(cin, dox, fuz1), fnc(cin, A1, w1, s1));$$

$$ZFNmul2(cin, dox) := fnd(cin, dox, fuz2) \cdot fnc(cin, A2, w2, s2).$$

Then we build graphs for these two cases. Fig. 4 presents the results of the module operation for consumers with low and high incomes. This module is distinguished by a fuzzy subset of "desired-possible" products

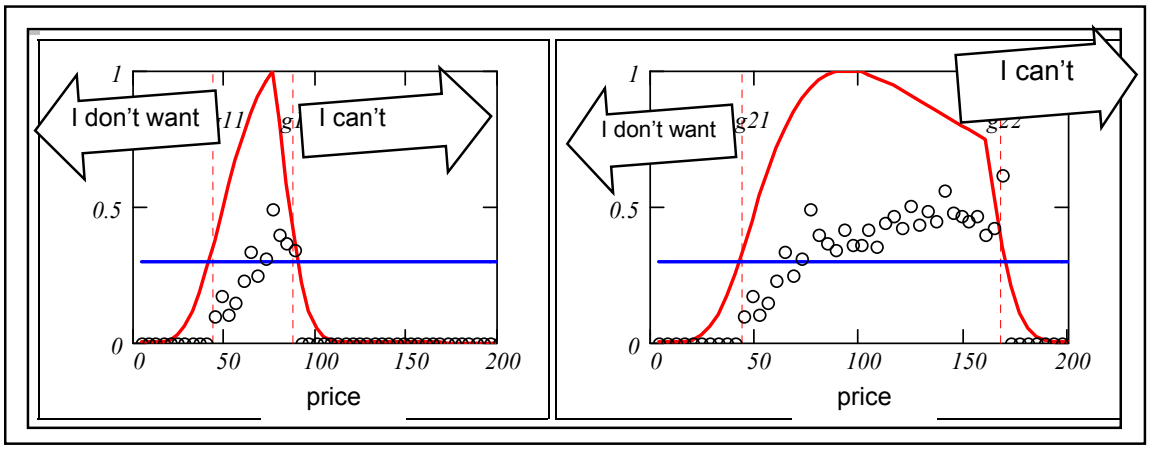


Рис. 4. Control of the function of the available products selection

"Choice of the individual with training" module. We build three alternative models, each of them being acceptable for certain situations of the parameters of products and consumers [4]:

– model 1 based on statistical data used for a simpler training model identification (after a certain period product estimates approach the "true" ones");

– model 2 based on the assumption that the rate of training is proportional to the rate of consumption and information exchange between consumers;

– model 3 based on "crossing out" of the "discredited" products.

We have M alternative products from which a consumer selects a definite product by a random choice with the probability proportional to the products "weight". We believe that there exists the possibility of objective estimation of each product value. Into the imitational simulation program "true value estimates" are introduced. Purchase of a definite product by a definite consumer causes a corresponding change in the individual estimation of value. According to the consumption results value estimation is corrected towards achieving the "true estimate".

"Value re-estimation" module is critical for the system modeling program. We create a module designed for vectorization of calculations. First, a user function for scalar parameters processing is formulated:

$$dMU := \overrightarrow{dmu(Vyb, Bolv, Nvz)},$$

where *Vyb* is a consumer's current choice matrix, *Bolv* – matrix with each column being "true product values", *Nvz* – choice uncertainty matrix. The arrow above the expression is a symbol of vectorization intended for simultaneous realization of this function for all corresponding elements of the matrix.

"Correction of the value estimates" module. We again formulate a vectorization function of a user for processing scalar parameters $\overrightarrow{z \min(Mu, dMU)}$. The function takes the evaluation matrices and returns a new evaluation matrix.

From this modules we build a function of normalization of consumers product evaluation matrices – the module $nrmlz2(Mu, Vyb, Bolv, Nvz, A1)$, that takes the matrix *Mu* of current product evaluation by the consumers, current *Vyb* matrix of the consumer's choice, *Bolv* matrix of true evaluation of the products, *Nvz* matrix of the uncertainty of products evaluation by the consumers and $A1$ matrix of the consumers' training rate and returns the normalized matrix of the current state of product evaluation by the consumers (fig. 5).

$$
\begin{array}{|l|l|}
\hline
nrmlz2(Mu, Vyb, Bolv, Nvz, Al) := & \overrightarrow{M \leftarrow zmip\left(Mu, \overrightarrow{dmu(Vyb, Bolv, Nvz)}, Al\right)} \\
 & kk \leftarrow cols(Mu) \\
 & qq \leftarrow rows(Mu) \\
 & for \ \ i \in 1..kk \\
 & \quad \left| \begin{array}{l} nn \leftarrow qq \cdot mean\left(M^{\langle i \rangle}\right) \\ M^{\langle i \rangle} \leftarrow M^{\langle i \rangle} \div nn \end{array} \right. \\
 & M \\
\hline
\end{array}
$$

Fig. 5. Module of the product rating matrix correction

This is a purely technological module that provides a satisfactory speed of response for the modeling program.

Step 4. Development of the program of a random choice of the consumers with training

The program takes the vector of parameters that consists of the following components:

$$Vpar = \begin{pmatrix} "Mf - \text{matrix of products estimation by consumers}" \\ "Blv - \text{matrix of the «true value» of products}" \\ "Nevz - \text{matrix of scattering of product estimation by consumers}" \end{pmatrix}$$

Then it returns a vector consisting of the following matrices:

$$Vyxid = \begin{pmatrix} "Vybir - \text{matrix of products current choice by consumers}" \\ "Mf - \text{the corrected matrix of products estimation by consumers}" \end{pmatrix}.$$

"Redistribution of consumers according to their incomes" module. Distribution of incomes is built on the basis of statistical data used for identification of the theoretical model that corresponds to a definite hypothesis concerning the mechanism causing this scattering. Usually it is a non-Gaussian model, even not a unimodal one, if stratification of the society according to age, profession etc. is taken into account.

Income change and redistribution in the society. It is practically important to know how potential market volumes are changing for groups of consumers with different income levels. In the model all possible mechanisms of income redistribution in the society in different situations must be reflected:

– increase and reduction of national (regional) income;
– directive or spontaneous growth of average salary for employees;
– income distribution change due to the change of demographic situation;
– impulsive changes of incomes (elections, holidays, bonuses,"distribution of elephants");
– local changes of salaries at certain firms, corporations, fields.

Then a simpler model is used (for a mathematician rather than for an economist), a socially neutral one: national income change causes **a shift** of income distribution towards higher or lower incomes. Income model is modified on the basis of a simpler hypothesis about distribution shift:

$$doxpoz(x) := d \ln orm(x, mu, roz) \Rightarrow doxpom(x, nd) := d \ln orm(x - nd, mu + 0.1 \cdot nd, roz),$$

where x – income level, nd – parameter of the "national income per capita" type.

Fig. 6 shows 2 series of graphs – distribution of potential consumers and distribution of the market potential profit according to the consumers' income level. This can be represented by mathematical and verbal formulas:

$$doxsum(x, nd) := doxpom(x, nd) \cdot x;$$

"total income of buyers of the category with x income per capita (total income density) = density of probability to meet such a buyer in the "interval"·"income"" sampling.

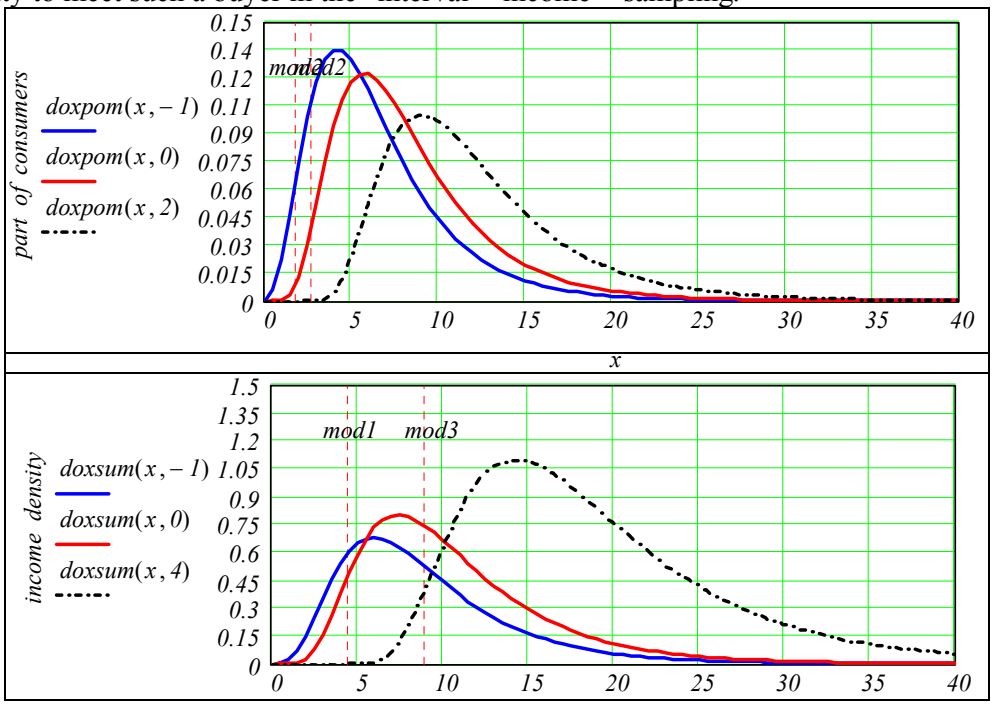


Рис. 6. Distribution of consumers and market profits according to the consumer's income level

Income redistribution model makes it possible to investigate the situations when under moderate growth of the average income (10 – 20%) market profit per product (that is not an essential one) grows and drops considerably. For business units it is important to predict such situations for their business and particularly the following effect: when average income is increased, the sales of the cheap products of the line can be reduced – consumers' demand is shifted towards more expensive, qualitative and prestigious products.

Analysis of the modeling results

Model extension from the "producers – consumers" model to the "producers – products – consumers" model required the creation of new forms for regularizing the modeling results. Fig. 7, 8 present a set of graphs and samples of input and initial data. Fig. 7 shows the state of the system at a definite moment of time – the state of the choice distribution of each consumer and the rate of sales of the products line. On these graphs the "reasons" are superimposed – the function of the products, range and frequency distribution of the consumers according to their income. Fig. 8 presents transient processes function of the line products for the initial "untrained consumers" state. It is a small part of the interface system, but it gives a sufficient visual representation of the distributed system.

It is a common opinion that a working model is designed for giving ever more accurate predictions (by the criterion of minimal integral quadratic error) of the future of a certain system (Microsoft Corporation, French language, the economy of Ukraine).

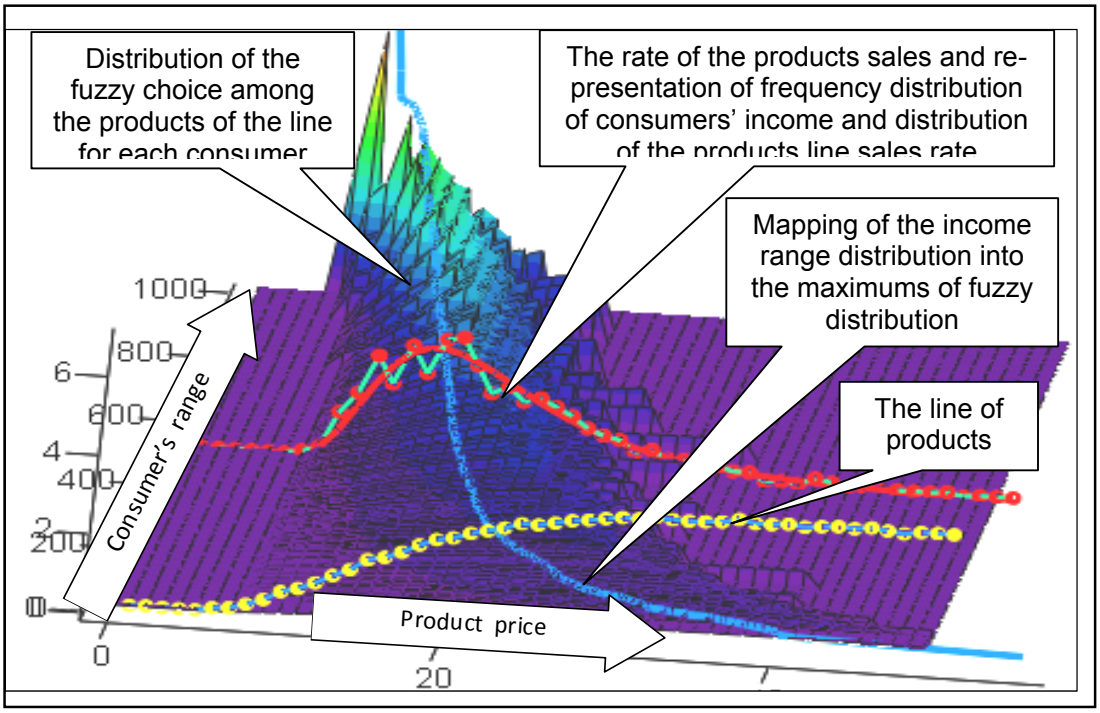


Fig. 7. Analysis of the "4 producers, 50 products, 1000 consumers" system state

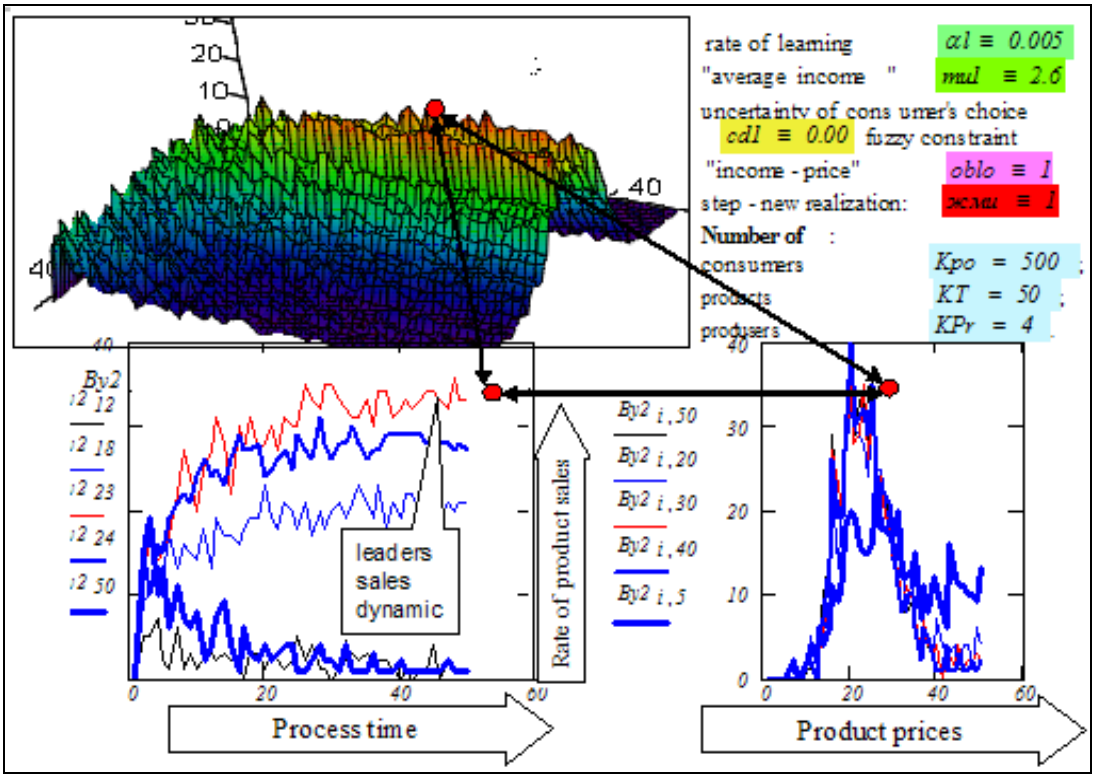


Fig. 8. Analysis of transient processes in the "4 producers, 50 products, 1000 consumers" system

Classics of management and modelling believe that a model is primarily designed for better understanding of the system by a manager and analyst who are working with it. The developed model satisfies this requirement. Already at the stage of the "producers – products – consumers" model refinement and testing a number of new results have been obtained that seemed paradoxical and improbable at first sight. When these results were analyzed and compared with the available statistical

data, logical justification has been found for that. The list of results that could be interesting from theoretical and practical point of view is given below.

☜ **Reaction of "rich" and "poor" markets to small price level variations.** For small growth of prices for a definite class of products under initial globally stabilized conditions the total market profit is growing in the "rich" and decreasing in the "poor" social groups.

☜ **"Erosion" of the sales rate distribution for an uncontrolled line of products.** A line of products is locally unstable to small disturbances of ranging according to prices and value. A simplified explanation is as follows: if in the line of two related products there exists the inverse relationship – a more expensive product has a lower value estimate, then the sales rate of the leader will increase at the expense of the outsider. When the sampling is increased, distribution will not be smoothed out. This is the subject of separate research.

☜ **Simple "reasons – consequences" relationships**. The function of consumers distribution according to their incomes and the "price – value" function are linearly mapped with sufficient accuracy into frequency distribution of the product line sales rate without disturbances and range distribution – into the modes (maximums) of distribution of the consumers fuzzy choice, ranged according to the incomes.

The elaborated model has enabled distinct statement of general management problems and specific problems of optimal management of product lines for a retailer and a separate producer. For a retailer such management is connected with resources spent on changing the product position in the retailer line, while for a producer it consists in the management of resources spent on increasing the value and reducing the product cost.

Practical importance of the model elaboration and research consists in defining the guidelines of data collection in real systems in order to create a system for supporting decision making in the development, production and sales of a certain class of products.

Theoretical importance – defining the guidelines for theoretical substantiation of the revealed properties of the system and namely: dynamic and static relationships between the frequency distribution of incomes and sales rate distribution, between the range distribution of incomes and the parameters of consumers' choice frequency distribution. Within the developed model such relationships are stable to the changes of the model parameters. From the conceptual point of view the role of the "new models construction for solving new problems" conception as a new knowledge generator is demonstrated.

REFERENCES

1. Нэгл Томас Т. Стратегия и тактика ценообразования. Руководство для принятия решений приносящих прибыль / Томас Т. Нэгл. – М.: Питер, 2001. – 375 с.

2. Kelly K. New Rules for the New Economy. 10 radical strategies for a connected world. – Penguin books, 1999. – 180 p.

3. Форрестер Дж. Основы кибернетики предприятия / Дж. Форрестер. – М.: Прогресс, 1971. – 340 с.

4. Боровська Т. М. Основи теорії управління та дослідження операцій. Навчальний посібник / Т. М. Боровська, І. С. Колесник, В. А. Северілов. – Вінниця: УНІВЕРСУМ-Вінниця, 2008. – 242 с. – ISBN 978-966-641-275-4.

5. Моделювання та оптимізація у менеджменті: Навчальний посібник / Т. М. Боровська, В. А. Северілов, С. П. Бадьора, І. С. Колесник. – Вінниця: УНІВЕРСУМ-Вінниця, 2009. – 145 с. – ISBN 978-966-641-287-7.

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