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DETERMINATION OF SECONDARY CRITERIA AND SIMILARITY INDICATORS IN PROBLEMS OF MODELING SYSTEMS OPTIMAL STATES

Methods of determination of secondary criteria and similarity indicators for establishing of parametric similarity in investigated process are considered. It is shown, that by means of secondary criteria and similarity indicators problems of systems states optimization can be solved.

Key words: *similarity, secondary criteria, indicators, complex systems, optimization.*

Introduction

In applied problems of similarity theory and modeling, secondary similarity criteria, which are the relation of similarity criteria are used [1]. The latter, by definition [1] are dimensionless combinations of parameters, which are numerically the same in all similar processes. By means of secondary criteria of similarity parametric similarity of investigated process is defined. Secondary criteria of similarity are important when they characterize parametric similarity of optimum variants.

Kelvin criterion, for instance, describes optimum relation of expenditures, required for construction of overhead transmission lines [2]. Secondary criteria establish stable relations between separate components of efficiency function, which under certain conditions, can be treated as the laws of optimum control of processes in the system [3]. The given paper considers the methods of determination of secondary criteria and examples of their usage in the problems of systems states optimization.

Problem of determination of secondary criteria and similarity indicators

Let us formulate the problem of optimum control in the following manner:

minimize

$$f(x) = \sum_{i=1}^{m_1} a_i \prod_{j=1}^{n} x_j^{\alpha_{ji}} \tag{1}$$

on condition that

$$g_k(x) = \sum_{i=m_k+1}^{m_{k+1}} a_i \prod_{j=1}^{n} x_j^{\alpha_{ji}} \le G_k, \quad k = \overline{1,h}, \quad x_j > 0, \tag{2}$$

where f(x) – is generalized technical-economic indire of optimized process; g(x) – are limitations, defining possible area of investigation process; a_i , α_{ji} G_k – are constant coefficients values of which are determined by the properties of the system; x_j number of members of efficiency function; m_1 – total number of efficiency function members and limitations; m – member of variables; n – number of variables; h – number of limitations.

For the problem (1) – (2) two types of similarity criteria are possible depending on the selected base. In accordance with the method of integral analogues similarity criteria are defined by means of division of all members of the equation by one of them [1]. If, for instance, the first member is taken as the basic one, then we obtain similarity criteria of the following form

$$\pi_i = \frac{a_i \prod_{j=1}^{n} x_j^{\alpha_{ji}}}{a_l \prod_{j=1}^{n} x_j^{\alpha_{jl}}}. \qquad (3)$$

If f(x) is taken as the base, then similarity criteria have the form

$$\pi_i = \frac{a_i \prod_{j=1}^{n} x_j^{\alpha_{ji}}}{f(x)}. \qquad (4)$$

In latter case, similarity criteria, which belong to efficiency function, have the content of weight coefficients and characterize the contribution of each member into the value of optimality criterion.

Similarity criteria, which belong to limitations, characterize the sensitivity of optimality criterion to the latter. In (4) it is shown that $\lambda_k = \sum_{i=m_k+1}^{m_{k+1}} \pi_i$ – are normalized Lagrange multipliers of k th limitations. It is obvious, that in problems of optimal control the second form of similarity criteria is mainly used, since in this case, they are more informative [4]. Especially, when similarity criteria are defined for extreme value of optimality according to the problem (1) – (2).

In this case each similarity criterion shows is the contribution of corresponding member of efficiency function into optimum value of optimality criterion. The system of equations relatively variables x_j , can be formulated from the expressions (4). Optimum values of x_{jo} are defined from this system on condition of optimality π_{io}. It is so-called reverse problem of criteria programming [4].

Depending on the form of expression of efficiency function and restrictions, as well as relation of the number of members m and number of variables n, the problem (1) – (2) is solved by means of various algorithms. When the problem (1) – (2) exists in analytical from, i. e., coefficients a_i , α_{ji}, then two cases are possible : 1) s=m–n+1=0, 2) s=m–n+1>0. In (1) – (2) exponents of power α_{ji} can be known, or partially known or specified by possible range existence.

The given paper considers algorithms of solution of the problem, analytically formulated with the level of complexity s=0 and s>0.

Definition of secondary criteria and similarity indicators

Optimum values of similarity criteria for the problem (1)–(2) are defined from the conditions of orthogonality and normalization [4]:

$$\boldsymbol{\alpha\pi} = \mathbf{b}, \qquad (5)$$

where $\boldsymbol{\alpha} = \begin{vmatrix} \alpha_{11}\alpha_{12}\ \alpha_{13}\dots\ \alpha_{1m} \\ \alpha_{21}\alpha_{22}\alpha_{23}\dots\alpha_{2m} \\ \dots\dots\dots\dots\dots\dots \\ \alpha_{n1}\alpha_{n2}\alpha_{n3}\dots\alpha_{nm} \\ 1\quad 1\dots\ 0\ \ \dots\ 0 \end{vmatrix}$; $\boldsymbol{\pi} = \begin{vmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \\ \dots \\ \pi_m \end{vmatrix}$; $\mathbf{b} = \begin{vmatrix} 0 \\ 0 \\ 0 \\ \dots \\ 1 \end{vmatrix}$.

In the last raw of the matrix $\boldsymbol{\alpha}$ the number of 1s equals the number of members in efficiency function (1), namely m_1. If the system of equations (5) is definite (s=0), then similarity criteria can be found applying Kramer rule[5]:

$$\pi_i = \frac{\|\boldsymbol{\alpha}\|_i}{\|\boldsymbol{\alpha}\|} = \|\boldsymbol{\alpha}\|^{-1} \sum_{t=1}^{n+1} b_t \|\boldsymbol{\alpha}_{ti}\|, \quad i = \overline{1,m}, \tag{6}$$

where $\|\boldsymbol{\alpha}\|$ – determinant of matrix $\boldsymbol{\alpha}$ of equation system (5); $\|\boldsymbol{\alpha}\|_i$ – determinant of matrix $\boldsymbol{\alpha}$, where instead of i th column vector of free members **b** is written; $\|\boldsymbol{\alpha}_{ti}\|$ – are algebraic complements, in i th column of matrix $\boldsymbol{\alpha}$, where instead of i th column the vector of free members **b** is written.

Taking into account values of vector **b** components, the latter expression is rewritten as:

$$\pi_i = \|\boldsymbol{\alpha}\|^{-1} \|\boldsymbol{\alpha}_{n+1,i}\|, \quad i = \overline{1,m}. \tag{7}$$

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Now let us make use of the expression (7) and write secondary similarity criterion as the relation of p-th and q-th similarity criterion:

$$\pi_{pq} = \pi_p / \pi_q = \|\boldsymbol{\alpha}_{n+1,p}\| \, \|\boldsymbol{\alpha}_{n+1,q}\|^{-1}. \tag{8}$$

From (7) and (8) it is seen, that if m=n+1both similarity criteria and secondary similarity criteria are defined only by exponents of power of x_j parameters.

Particular case is when certain parameters x_j enters only in two members of equation – p-th and q-th. In this case the condition of orthogonality for x_j parameter will be written as

$$\alpha_{jp}\pi_p + \alpha_{jq}\pi_q = 0.$$

From the last equation it follows that secondary similarity criterion is

$$\pi_{pq} = \pi_p / \pi_q = -\alpha_{jq} \, \alpha_{jp}^{-1}. \tag{9}$$

It should be noted that the expression (9) unlike (8) can be used when the system of equations (5) is uncertain (in criteria programming this corresponds to the problem with the degree of complexity s=m-n-1>0 [4]). It means that secondary criteria can be defined while investigation both of canonical and non-canonical models.

In problems of optimum control of dynamic systems states instead of similarity criteria indicators of similarity can be used, which are defined by the scales of values, belonging to (1) – (2).

The application of the latter provides certain advantages. Namely, system of optimum control becomes more rational due to the fact, that similarity criteria are defined not relatively x_j parameters, but relatively control parameters x_j, but relatively control parameters u_j which optimize system states and are functionally with x_j [6].This same is valid for the systems of optimum control with reference model. In this case, it is necessary to establish similarity between original system and its model in control system as well as numeric relations between control parameters of the original u_{jop} and model $u_{jм}$.

In [6] it is shown that similarity indicators of each member of the problem (1) – (2) are defined as

$$\mu_i = \frac{\mu_{a_i} \prod_{j=1}^{n} \mu_{x_j}^{\alpha_{ji}}}{\mu_f} = 1, \tag{10}$$

where μ_{a_i}, μ_{x_j}, μ_f - are scale coefficients of corresponding values.

Similarity criteria are defined by the formula (6), but since they are calculated, in this case, in differential system of relative units, then vector **b** unlike (5) will be equal [3]:

$$\mathbf{b} = |b_1; b_2; \ldots b_n; 1|,$$

where $b_j = \dfrac{\partial f / \partial x_j}{f_o / x_{jo}}, \quad j = \overline{1,n}$.

Thus, we can not obtain, simple expression for definition of similarity indicators, analogous to (9).

As it is known [1] regarding establishment of process similarity, similarity criteria and indicators of similarity are equivalent. However, as it is seen from (10), relations of similarity criteria of separate members of mathematical model of the process and corresponding indicators of similarity are not equivalent.

We can assume that, if parametric similarity is shown by means of relation of similarity criteria, then it can be established by means of relation of similarity indicators. If efficiency function relatively control parameters is approximated by polynomial of the type (1)

$$F(u) = \sum_{i=1}^{m_1} a_i \prod_{j=1}^{n} u_j^{\alpha_{ji}},$$

indicators of similarity for control parameters will be written [6]

$$\mu_{u_j} = \prod_{i=1}^{m} \mu_{a_i}^{\frac{\|\boldsymbol{\alpha}_{ji}\|}{\|\boldsymbol{\alpha}\|}}, \quad j = \overline{1,n}, \tag{11}$$

where $\boldsymbol{\alpha}_{ji}$ – algebraic supplements of α_{ji}, elements, taken with inverse sign.

If parametric similarity exists between p-th and q-th, then their similarity indicator must be the same. Taking into account (11) this condition will be written as

$$\mu_{pq} = \frac{\mu_{u\,p}}{\mu_{u\,q}} = \frac{\prod_{i=1}^{m} \mu_{a_i}^{\|\boldsymbol{\alpha}_{pi}\| \|\boldsymbol{\alpha}\|^{-1}}}{\prod_{i=1}^{m} \mu_{a_i}^{\|\boldsymbol{\alpha}_{qi}\| \|\boldsymbol{\alpha}\|^{-1}}} = 1,$$

which after simplification will have the form

$$\mu_{pq} = \prod_{i=1}^{m} \mu_{a_i}^{(\|\boldsymbol{\alpha}_{pi}\| - \|\boldsymbol{\alpha}_{qi}\|)\|\boldsymbol{\alpha}\|^{-1}} = 1.$$

From the last expression it follows that when in investigated process there is parametric similarity, then to the save it in the model of this process, it is necessary that the condition be satisfied

$$\|\boldsymbol{\alpha}_{pi}\| = \|\boldsymbol{\alpha}_{qi}\|. \tag{12}$$

That is, corresponding algebraic supplements of the same parameters in the original process and its model must be the same

Example

In over-head transmission line 330 – 750 kV losses of active power depend on energy transfer of active and reactive power, as well as on losses, due to corona discharge (corona losses). That is, losses in the line are defined as [7]

$$\Delta P = \Delta P_k + \Delta P_P + \Delta P_Q, \tag{13}$$

where $\Delta P_k = k_k L U^{\alpha_{11}}$ – corona losses; $\Delta P_P = r_o L P^2 U^{-2}$ – are transfer losses of active power P; $\Delta P_Q = r_o L Q^2 U^{-2}$ – are losses as a result of reactive power transfers Q; k_k – is the coefficient, that characterizes level of corona losses in 1 km of the line for the given phase construction and given weather; α_{11} – is an index that characterizes the dependence of corona losses on voltage U at certain weather condition; r_o is specific active resistance of the line.

In general form (13)as function of voltage losses will be written:

$$\Delta P = a_1 U^{\alpha_{11}} + a_2 U^{-\alpha_{12}}, \tag{14}$$

where $a_1=k_kL$; $a_2=r_oLP^2+r_oLQ^2$; $\alpha_{11}=3\div8$; $\alpha_{12}=2$.

We will define the optimum relation in transmission line between corona losses (the first component in (14)and belong losses the second component in (14)). According to (9)

$$\pi_{12} = \pi_1 / \pi_2 = \alpha_{12} / \alpha_{11} = 2 / \alpha_{11}. \tag{15}$$

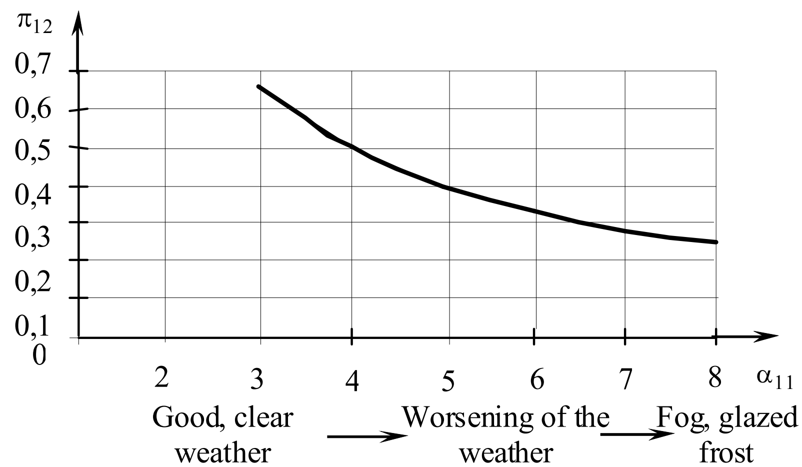

Fig. 1. Optimal relation of losses in overhead transmission line

From the analysis of sensitivity (15) it follows, that optimum relation between corona losses and loading losses depend on weather conditions. If the weather is fine ($\alpha_{11}=3$) relation of corona losses and loading losses in overhead transmission lines mist be 2/3 ($\pi_{12}=0{,}66$). When the weather get worse, share of corona losses in total losses decreases, and loading losses increase. In extreme conditions (fog, glazed frost $\alpha_{11}=8$) $\pi_{12}=0{,}25$. That is, such operation mode of overhead transmission lines will be optimal, when corona losses will be four times less, than loading losses. Fig 1 shows the dependence between optimum ratio of corona losses and loading losses depending on α_{11} coefficient, or on weather conditions while transmission line operation. This dependence can be used for optimum control of transmission line voltage levels and, if it is possible, transmission line power transfer

Conclusions

1. If parametric similarity is available in investigated process this similarity can be established in the form of corresponding similarity criteria – secondary similarity criteria. The value of secondary similarity criteria can be defined as a result of analysis of exponents of power matrix (dimensionalities).
2. Similarity indicators in general case do not allow to reveal parametric similarity in investigated process. They allow to establish and obtain processes adequacy in the original and its model. Information, in the matrix of exponents of power (dimensionalities) of parameters, characterizing the process in sufficient to reach this aim.
3. By means of secondary criteria and similarity indicators some technical problems can be solved. First of all, these are optimization problems, where it is possible and expedient to use stable relations between optimum values of parameters. For instance, problems dealing with the design of transmission lines, optimization of operation modes of transmission lines of superhigh voltages, optimum control of electric networks normal modes are referred to such of problems.

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