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THE VACUUM PERMITTIVITY AND PERMEABILITY TENSORS IN THE PRESENCE OF THE ELECTROMAGNETIC FIELD

In this paper we deduce the expressions of the vacuum permittivity and permeability tensors components induced by the electromagnetic background radiation. To achieve this goal we start with the expressions of these tensors modified by an electromagnetic field obtained by Euler and Kockel. These relations are particularized for: plane polarized electromagnetic wave, electromagnetic wave with a certain direction and a monochromatic background wave. In the presence of electromagnetic waves the vacuum is inhomogeneous and anisotropic. In the presence of background of monochromatic electromagnetic waves the vacuum is homogeneous and isotropic.

Key words: *vacuum, permittivity tensor, permeability tensor, electromagnetic field*

Introduction

In the presence of an electromagnetic field, the vacuum becomes inhomogeneous and anisotropic. In the quantum electrodynamics, this phenomenon is known as an effect of nonlinearity of the electromagnetic fields composition which results from the effect of the vacuum polarization (the interaction photon-photon).

The first work in which the expressions of permittivity and permeability were obtained belongs to Euler, H. and Kockel, B [1] in the case of the fields of relatively low strength [2]. The production of the laser radiation, the detection of the cosmic background of equilibrium, thermal radiation with $T = 2{,}7\text{K}$ [3] and of cosmic electromagnetic radiations of high energy determined the necessity of a study of this process, for high intensities too, concerning the phenomena of autofocalization [4, 5, 6] and radiation beam collapse [7, 8].

This paper proposes to deduce the expressions of the permittivity and permeability tensors inducted by an electromagnetic wave and a background monochromatic electromagnetic radiation.

In the first part of the paper there are deduced the expressions of the components of the tensors permittivity and permeability of the vacuum in the presence of an electromagnetic wave. In the second and the third part we will find out the expressions of the components of the same tensors and the refraction coefficient for a plane wave polarized on a certain direction. In the fourth part, the purpose is to obtain the relations necessary for calculating the vacuum parameters in the presence of a background of monochromatic electromagnetic waves.

The tensors of the vacuum permittivity and permeability modified by an electromagnetic wave

In the presence of electromagnetic field, the vacuum becomes inhomogeneous and an-isotropic [3], which results in the following relations between the quantities which characterize the field:

$$D_i = \varepsilon_{ik} E_k , \tag{1}$$

$$B_i = \mu_{ik} H_k , \tag{2}$$

With the permittivity and permeability tensors having the form of:

$$\varepsilon_{ik} = \delta_{ik} + a\left[2\left(\varepsilon_0 E^2 - \mu_0 H^2\right)\delta_{ik} + 7\left(\mu_0 H_i H_k\right)\right] + ... , \tag{3}$$

$$\mu_{ik} = \delta_{ik} + a\left[2\left(-\varepsilon_0 E^2 + \mu_0 H^2\right)\delta_{ik} + 7\left(\varepsilon_0 E_i E_k\right)\right] + ... \tag{4}$$

The constant a has the expression

$$a = 4e^4\hbar \big/ \left(45 m^4 c^7\right) , \tag{5}$$

in which: $e^2 = q_e^2/(4\pi\varepsilon_0)$, q_e is the electron charge and m is the electron mass.

The constant inverse

$$\frac{1}{a} = \left(\frac{45}{4}\right)\left(\frac{e^2}{hc}\right)\frac{mc^2}{\left[e^2/(mc^2)\right]^3} \cong \frac{mc^2}{\left[e^2/(mc^2)\right]^3} \cong 10^{29}\ Jm^{-3} \tag{6}$$

represents the density of the electron energy (in the classic model) or the density of an electric field energy generated by the electron at an equal distance with a classical radius $r_e \cong e^2/(mc^2)$.

We notice that for an electromagnetic field with sources, the two tensors are different, $\varepsilon_{ij} \neq \mu_{ij}$, because $\varepsilon_0 E^2 \neq \mu_0 H^2$.

For a plane electromagnetic wave, the density of the electric field energy is equal to the density of the magnetic field energy and equal to a half of the density of the wave energy.

$$w/2 = \varepsilon_0 E^2/2 = \mu_0 H^2/2\ . \tag{6}$$

Replacing the equality (6), in the expressions (3) and (4), we will obtain:

$$\varepsilon_{ik} = \delta_{ik} + 7a\left(\mu_0 H_i H_k\right), \tag{7}$$

$$\mu_{ik} = \delta_{ik} + 7a\left(\varepsilon_0 E_i E_k\right). \tag{8}$$

Because in an electromagnetic wave, the vectors $\vec{E}$ and $\vec{H}$ are position and time functions

$$\vec{E} = \vec{E}_0 \sin\left(\omega t - \vec{k}\vec{r}\right),\quad \vec{H} = \vec{H}_0 \sin\left(\omega t - \vec{k}\vec{r}\right) \tag{9}$$

and permittivity and permeability tensors are position and time functions as $\sin^2\left(\omega t - \vec{k}\vec{r}\right)$. For a stationary wave, the position and time dependencies are according to the functions $\cos(\vec{k}\vec{r})\sin(\omega t)$ and $\cos^2(\vec{k}\vec{r})\sin^2(\omega t)$, respectively.

The vacuum in the presence of a plane polarized wave

Here is a monochromatic plane polarized wave ($E = E_0 \sin(\omega t - \vec{k}\vec{r})$ $H = H_0 \sin(\omega t - \vec{k}\vec{r})$), which propagates on the direction Ox_j $(x_1 = x, x_2 = y, x_3 = z)$ and the polarized plan is rotated with the angle ψ with respect to the axes Ox_i. Because this wave is divided into two monochromatic waves which propagate through the same direction (Ox_j) at the same speed (speed c) and having the planes for polarization on the direction of the axis perpendicular to Ox_j, i.e. Ox_i (the wave to which components $E_i = E\cos\psi$ and $H_k = H\cos\psi$ correspond) and Ox_k (the wave to which components $E_k = E\sin\psi$ and $H_i = -H\sin\psi$ correspond), the off-diagonal components of the tensors (7) and (8) are equal:

$$\varepsilon_{ik} = 7a\left(\mu_0 H_i H_k\right) = \mu_{ik} = 7a\left(\varepsilon_0 E_i E_k\right) = 7a\left(\varepsilon_0 E^2 \sin\psi\cos\psi\right),\ i,k \neq j\ , \tag{10a}$$

$$\varepsilon_{ij} = \mu_{ij} = 0, i \neq j\ , \tag{10b}$$

where $cB_i = E_k$ and $cB_k = E_i$, $i,k \neq j$.

In this case, the diagonal components are:

$$\varepsilon_{jj} = \mu_{jj} = 1\ , \tag{11a}$$

$$\begin{gathered}\varepsilon_{ii} = 1 + 7a\left(\mu_0 H_i^2\right) = 1 + 7a\left(\mu_0 H^2 \sin^2\psi\right), \mu_{ii} = 1 + 7a\left(\varepsilon_0 E_i^2\right) = \\ 1 + 7a\left(\varepsilon_0 E^2 \cos^2\psi\right), \varepsilon_{ii} = \mu_{kk}, i \neq k \neq j.\end{gathered}\ , \tag{11b}$$

The refraction indexes induced by the wave, on the principal directions are:

$$n_j = \sqrt{\varepsilon_{ii}\mu_{kk}} = \sqrt{\varepsilon_{kk}\mu_{ii}} \neq 1, \tag{12a}$$

$$n_i = \sqrt{\varepsilon_{jj}\mu_{kk}} = \sqrt{\varepsilon_{kk}\mu_{jj}} \neq 1. \tag{12b}$$

Replacing the formulae (11) of the tensors components, we will obtain

$$n_j = \sqrt{\varepsilon_{ii}\mu_{kk}} = 1 + 7a\left(\mu_0 H_i^2\right) = 1 + 7a\left(\varepsilon_0 E_k^2\right), \tag{13a}$$

$$n_i = \sqrt{\mu_{kk}} = \sqrt{\varepsilon_{kk}} = \sqrt{1 + 7a\left(\varepsilon_0 E_k^2\right)} = \sqrt{1 + 7a\left(\mu_0 H_k^2\right)}, \tag{13b}$$

the expressions of the refraction indexes.

The vacuum in the presence of a monochromatic wave on a certain direction

We will deal with a monochromatic plane wave characterized by the wave vector $\vec{k} = k\vec{\kappa}$, with the components of the vector $\vec{\kappa}$:

$$\kappa_x = \sin\theta\cos\varphi,\ \kappa_y = \sin\theta\sin\varphi,\ \kappa_z = \cos\theta \tag{17}$$

and characterized by the electric field vector $\vec{E} = E\vec{e}$ and the magnetic field vector $\vec{H} = H\vec{h}$ .The wave is plane polarized. The vectors $\vec{e}$ and $\vec{h}$ have the following components:

$$e_x = -\frac{\sqrt{\cos(2\psi) - \cos(2\theta)}\csc\theta\sin\varphi}{\sqrt{2}} - \cos\theta\cos\varphi\sin\psi, \tag{18a}$$

$$e_y = \frac{\sqrt{\cos(2\psi) - \cos(2\theta)}\csc\theta\cos\varphi}{\sqrt{2}} - \cot\theta\sin\varphi\sin\psi, \tag{18b}$$

$$e_z = \sin\psi \tag{18c}$$

$$h_x = \csc\theta\sin\varphi\sin\psi - \frac{\sqrt{\cos(2\psi) - \cos(2\theta)}\cot\theta\cos\varphi}{\sqrt{2}}, \tag{18d}$$

$$h_y = -\frac{\sqrt{\cos(2\psi) - \cos(2\theta)}\cot\theta\sin\varphi}{\sqrt{2}} - \csc\theta\cos\varphi\sin\psi, \tag{18f}$$

$$h_z = \frac{\sqrt{\cos(2\psi) - \cos(2\theta)}}{\sqrt{2}}. \tag{18g}$$

With them, the vectors components $\vec{E} = E\vec{e}$ and $\vec{H} = H\vec{h}$ are:

$$E_x = Ee_x,\ E_y = Ee_y,\ E_z = Ee_z, \tag{20a}$$

$$H_x = Hh_x,\ H_y = Hh_y,\ H_z = Hh_z. \tag{20b}$$

Replacing the components of the two vectors given by the relations (19) and (20) in the formulae of the components of the permittivity and permeability tensors given by (7) and (8), it results:

$$\varepsilon_{xx} = 1 + 7a\left(\mu_0 H_x^2\right) = 1 + 7a\mu_0 H^2 h_x^2, \tag{21a}$$

$$\varepsilon_{yy} = 1 + 7a\left(\mu_0 H_y^2\right) = 1 + 7a\mu_0 H^2 h_y^2, \tag{21b}$$

$$\varepsilon_{zz} = 1 + 7a\left(\mu_0 H_z^2\right) = 1 + \frac{7}{2}a\mu_0 H^2\left[\cos(2\psi) - \cos(2\theta)\right], \tag{21c}$$

$$\varepsilon_{xy} = \varepsilon_{yx} = 7a\left(\mu_0 H_x H_y\right) = 7a\mu_0 H^2\left(h_x h_y\right), \tag{21d}$$

$$\varepsilon_{xz} = 7a\left(\mu_0 H_x H_z\right) = 7a\mu_0 H^2\left\{\frac{\csc\theta\sin\varphi\sin\psi\sqrt{\cos(2\psi)-\cos(2\theta)}}{\sqrt{2}} - \frac{\left[\cos(2\psi)-\cos(2\theta)\right]\cot\theta\cos\varphi}{2}\right\} = \varepsilon_{zx}, \tag{21e}$$

$$\varepsilon_{yz} = 7a\left(\mu_0 H_y H_z\right) = 7a\mu_0 H^2\left\{-\frac{\left[\cos(2\psi)-\cos(2\theta)\right]\cot\theta\sin\varphi}{2} - \frac{\csc\theta\cos\varphi\sin\psi\sqrt{\cos(2\psi)-\cos(2\theta)}}{\sqrt{2}}\right\} = \varepsilon_{zy}, \tag{21f}$$

$$\mu_{xx} = 1 + 7a\left(\varepsilon_0 E_x^2\right) = 1 + 7a\varepsilon_0 E^2\left(e_x^2\right), \tag{21g}$$

$$\mu_{yy} = 1 + 7a\left(\varepsilon_0 E_y^2\right) = 1 + 7a\varepsilon_0 E^2\left(e_y^2\right), \tag{21h}$$

$$\mu_{zz} = 1 + 7a\left(\varepsilon_0 E_z^2\right) = 1 + 7a\varepsilon_0 E^2\sin^2\psi, \tag{21i}$$

$$\mu_{xy} = \mu_{yx} = 7a\left(\varepsilon_0 E_x E_y\right) = 7a\varepsilon_0 E^2\left(e_x e_y\right), \tag{21j}$$

$$\mu_{xz} = 7a\left(\varepsilon_0 E_x E_z\right) = 7a\varepsilon_0 E^2\left\{-\frac{\sqrt{\cos(2\psi)-\cos(2\theta)}\csc\theta\sin\varphi\sin\psi}{\sqrt{2}} - \cos\theta\cos\varphi\sin^2\psi\right\} = \mu_{zx}, \tag{21k}$$

$$\mu_{yz} = 7a\left(\varepsilon_0 E_y E_z\right) = 7a\varepsilon_0 E^2\left\{\frac{\sqrt{\cos(2\psi)-\cos(2\theta)}\csc\theta\cos\varphi\sin\psi}{\sqrt{2}} - \cot\theta\sin\varphi\sin^2\psi\right\} = \mu_{zy}. \tag{21m}$$

It results that in the presence of a wave, the vacuum is inhomogeneous and non-isotropic (anisotropic).

For $\kappa_x = \kappa_y = 0$ and $\kappa_z = 1$, respectively $\varphi = 0$ and $\theta = \pi/2$, there are results for the polarized wave which propagates on a direction Ox.

Homogeneous and isotropic background of monochromatic electromagnetic waves

The homogeneous and isotropic background of monochromatic waves is obtained by the overlapping of the waves with the vectors $\vec{k}$ front towards all the directions with the same probability. In this case, the components of the permittivity and permeability tensors are obtained through the averaging of the formulae (20) and (21) according to the following definitions:

$$\left\langle\varepsilon_{ik}\right\rangle = \frac{1}{8\pi}\left(\int_0^{\pi}\int_0^{\pi}\int_0^{2\pi}\varepsilon_{ik}\sin\psi\sin\theta\,d\psi\,d\theta\,d\varphi\right) = \delta_{ik} + \left(\frac{7a\mu_0}{8\pi}\right)\left(\int_0^{\pi}\int_0^{\pi}\int_0^{2\pi}H_i H_k\sin\psi\sin\theta\,d\psi\,d\theta\,d\varphi\right), \tag{22a}$$

$$\left\langle\mu_{ik}\right\rangle = \frac{1}{8\pi}\left(\int_0^{\pi}\int_0^{\pi}\int_0^{2\pi}\mu_{ik}\sin\psi\sin\theta\,d\psi\,d\theta\,d\varphi\right) = \delta_{ik} + \left(\frac{7a\varepsilon_0}{8\pi}\right)\left(\int_0^{\pi}\int_0^{\pi}\int_0^{2\pi}E_i E_k\sin\psi\sin\theta\,d\psi\,d\theta\,d\varphi\right). \tag{22b}$$

Replacing the expressions (20) and (21) in the definitions of the average (22), the formulae below are obtained:

$$\langle \varepsilon_{xx} \rangle = \langle \varepsilon_{yy} \rangle = \langle \varepsilon_{zz} \rangle = 1 + \frac{7}{3} a \mu_0 H^2 = 1 + \frac{7}{3} a w , \tag{23a}$$

$$\langle \varepsilon_{xy} \rangle = \langle \varepsilon_{xz} \rangle = \langle \varepsilon_{yz} \rangle = 0 , \tag{23b}$$

$$\langle \mu_{xx} \rangle = \langle \mu_{yy} \rangle = \langle \mu_{zz} \rangle = 1 + \frac{7}{3} a \varepsilon_0 E^2 = 1 + \frac{7}{3} a w , \tag{23c}$$

$$\langle \mu_{xy} \rangle = \langle \mu_{xz} \rangle = \langle \mu_{yz} \rangle = 0 , \tag{23d}$$

In the expressions (23) the energy density w is the medium density in time and space. With these values of the tensors components we may calculate the refraction index. Replacing the expressions (23) in the expression (12) of the refraction index we may obtain

$$n = 1 + \frac{7}{3} a w . \tag{24}$$

It results that the vacuum, in the presence of a background of electromagnetic waves with random directions, is homogeneously and isotropically optical. The result is similar to that obtained in the papers [5, 6, 7] for fields with any intensity, within the quantum electrodynamics.

Conclusions

According to the classic and quantum concepts, the vacuum is a physical system which can be modeled. While interacting with the matter (the substance and the physical field) the vacuum modifies its properties. Through this paper we have demonstrated that the optical (electromagnetic) properties of the vacuum are modified by the electromagnetic field of a wave or of a wave system (background of electromagnetic waves). The modification of the optical properties of the vacuum in the presence of electromagnetic field explains the phenomena of autofocalisation and collapse of intense electromagnetic radiation beams. These phenomena are compatible with the current models of Universes, in which the vacuum is modeled as an average manifestation of all the fields generated by the particles from the Universe.

In another paper our goal is to evaluate the parameters of the modified vacuum by the stochastic electromagnetic background at the temperature $T = 0\mathrm{K}$.

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