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MATHEMATICAL MODEL OF TWO-AXEL VEHICLE IN THE TASK OF CONTROLLING OVER ITS MOVEMENT WITH ABSENCE OF DETOUR AND OVERTAKING

There had been built mathematical model of two-axel vehicle, suitable for solving the problem of vehicle movement control on the main lane with the absence of detours and overtaking. The model is set in a Cartesian coordinate system with georeferencing to the map, and describes the movement of the vehicle center mass along the main lane. The paper is first in a series of articles dedicated to the development of mathematical models of different types of vehicles and car trains moving on the road with dry, wet, snow or iced asphalt coverage.

Key words: *mathematical model, vehicle movement, two-axel, road turn, center of mass.*

Initial conditions and problem statement

In the papers [1, 2, 3] we had built the mathematical models of movement of the electric train, assembled from certain number of coupled carriages (one-two – tram, three-four – fast train, five-eight – subway train, nine-fourteen – passenger electric train, fifteen and more - freight train with electric traction).

Since the movement on the railways from stop to stop following the schedule, which makes this movement the determined one both in space and time, is peculiar characteristics of electric trains, it follows that mathematical models built to describe their movement cannot be used without substantial transformation for mathematical simulation of other types of wheeled vehicles such as automobiles.

Cars do not have the determined reference to the highway neither on time or space, because during their movement from the starting point they have to accelerate or slow down suddenly, make detours or overtaking the other vehicles moving or standing on the same road all the way down the target point.

Therefore we dedicate a series of scientific papers with building mathematic models fit for describing movement of the car which will consider different types of automobiles and automobiles trains running on the road with dry, wet, snow or ice asphalt coverage.

The first paper will suggest mathematical model of two-axel vehicle, which is the simplest in terms of mechanics, in the task of controlling over its movement with no detour, overtaking or slowing down.

Problem solution

Let the two-axle vehicle move at the speed of $\vec{v}(t)$ on the road segment AB (Fig. 1), being in a given time *t* at the point with coordinates (x, y).

Newton's second law runs for this vehicle as well as for any other moving physical body.

$$m\frac{d\vec{v}(x,y,t)}{dt} = \sum_i \vec{F}_i(x,y,t), \quad (1)$$

where m — vehicle mass, $\sum\limits_i \vec{F}_i(x,y,t)$ — vector sum of all forces applied to the center of mass Ц. Let us find out what kind of forces they are.

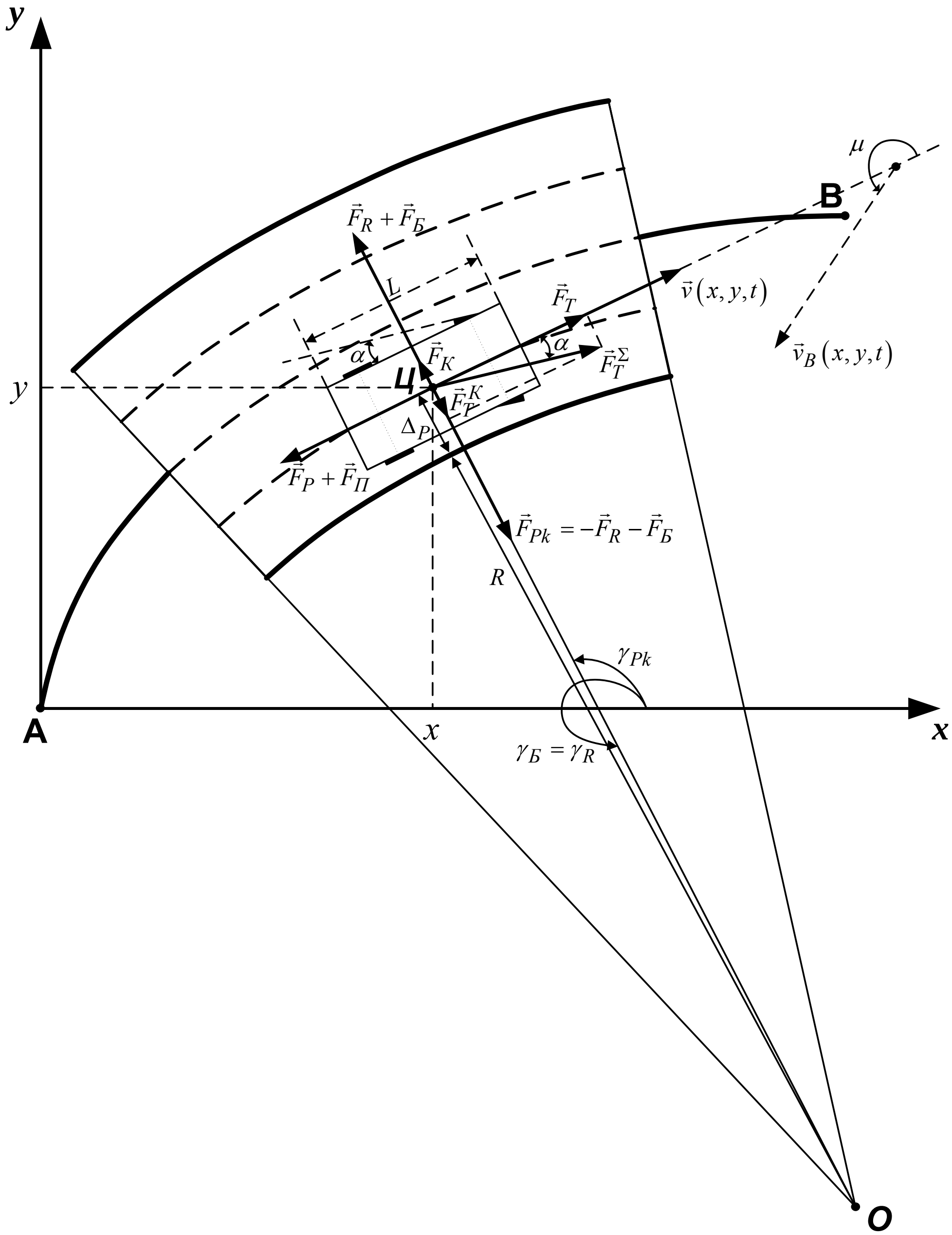

Fig 1. Vector diagram of forces acting on the car, moving with linear velocity $\vec{v}(x,y,t)$ on the rounded section of the road AB with only one line for each direction

It is obvious that the main moving force is vehicle thrust engine $\vec{F}_T$ (fig. 1), acting direction of which coincides with the acting direction of the linear velocity vector $\vec{v}(x,y,t)$ during uniform and the, module is balanced by the sum of oppositely directed force of friction of rolling wheels $\vec{F}_P$ on the road and force of frontal air resistance $\vec{F}_П$, determination which requires to consider the speed of wind $\vec{v}_B(x,y,t)$. It is clear that the vehicle will speed up only when the traction force exceeds on module the sum of friction force of rolling and frontal resistance. But when driving on curvature of the road with radius R, which is determined by segment from the centre of curvature to the nearest edge of the roadbend *O*, as is shown on Fig. 1, the thrust $\vec{F}_T^{\Sigma}$, except for overcoming vector the sum of friction force of rolling and frontal resistance, needs to overcome Coriolis force $\vec{F}_K$, which is the vector sum of centrifugal force $\vec{F}_R$, Coriolis force $\vec{F}_K$, lateral air resistance force $\vec{F}_Б$, which appears on the lateral surface of vehicle during the turn, and lateral sliding friction of the wheels $\vec{F}_{Pk}$, which balances the centrifugal force and the only side-effect of air resistance.

Thus, for the simulation of the general case of vehicle movement on the centerline of its lane, it is possible to write detailed equation instead of the general equation in such a form (1):

$$m\frac{d\vec{v}(x,y,t)}{dt} = \vec{F}_T + \vec{F}_T^K + \vec{F}_P + \vec{F}_П + \vec{F}_R + \vec{F}_K + \vec{F}_Б + \vec{F}_{Pk}, \qquad (2)$$

Or projected on the axes x, y

$$\begin{cases} m\dfrac{dv_x(x,y,t)}{dt} = F_T\cos\gamma_T + F_T^K\cos\gamma_T^K + F_P\cos\gamma_P + \\ F_П\cos\gamma_П + F_R\cos\gamma_R + F_K\cos\gamma_K + F_Б\cos\gamma_Б + F_{Pk}\cos\gamma_{Pk}, \\ m\dfrac{dv_y(x,y,t)}{dt} = F_T\sin\gamma_T + F_T^K\sin\gamma_T^K + F_P\sin\gamma_P + \\ +F_П\sin\gamma_П + F_R\sin\gamma_R + F_K\sin\gamma_K + F_Б\sin\gamma_Б + F_{Pk}\sin\gamma_{Pk}, \end{cases} \qquad (3)$$

where γ denotes the angle, which forms the force vector F at time t in point with coordinates(x,y). The vector is denoted by the same index, with the axis x, a positive value of which is formed by counter-clockwise countdown. This is shown in Fig. 1 for the angles$\gamma_{Pk}, \gamma_R, \gamma_Б$. A similar condition is set to the angle μ between the vector of wind velocity and vector of the linear vehicle velocity. For clarity of record, to facilitate its perception in the equations (2), (3), keeping in mind that F, γ are functions of x, y, t, we don't give these independent variables here and won't give them further using μ angle.

The direction of the vector of each force in the right side of the equation (2) can be seen from the Fig.1, and in the system of equations (3) the directions is given by sines and cosines of the corresponding angles.

Therefore, the identification of the system of equations (3) requires to figure out the way the module of vectors, included in the right part of this system of equations, are determined.

For the analysis of linear movement of the vehicle it is required to know the module of vectors of three forces only $F_P, F_П, F_T$ - since others equal zero.

From the section "Mechanics" of physics [4, 5] it is known that the absolute value of the force of rolling friction F_P is determined from the expression

$$F_P = k_* P, \qquad (4)$$

where P — body weight or its constituents, directed perpendicular to the plane of motion, and the coefficient of rolling friction k_* can take one of the values - $k_c, k_м, k_з, k_o$ (depending on whether wheels are rolling on the dry, wet, snow or ice asphalt). Force of air resistance $F_П$ can be found using the notation in Fig.1 from the following relation

$$F_П = k_{ПЛ} S_Л \left(v - v_B\cos(2\pi - \mu)\right)^2 + k_{ПБ} S_Б \left(v - v_B\cos(2\pi - \mu)\right), \qquad (5)$$

where $S_Л$ — area of the frontal airflow-vehicle contact while driving, $S_Б$ — area of the lateral airflow-vehicle contact while driving, and $k_{ПЛ}, k_{ПБ}$ — coefficients of agreement between corresponding force, areas and velocities squared or just velocities. Correlation (5) follows from the laws of physics, according to which the free stream resistance of the body is proportional to the squared linear velocity of flow on the frontal surface of the physical body and the sliding velocity is proportional to flow over the lateral surface of the body.

A traction force F_T can be easily determined from the relation if we know the torque M_O on the axis of the driving wheel with radius r, which is a function of engine power, working condition of gearbox and accelerator pedal position, which is throttle-related:

$$F_T = 2\frac{M_O}{r}, \tag{6}$$

if the vehicle has two driving wheels or

$$F_T = 4\frac{M_O}{r}, \tag{7}$$

if the vehicle has four driving wheels.

During the analysis of vehicle movement on the curvature of the road, besides the three determined absolute values of forces - $F_P, F_П, F_T$, we need to know how to determine modules of 5 more forces included in the equation (2), (3), - $F_R, F_K, F_Б, F_T^K, F_{Pk}$. We find them as follows:

From the laws of mechanics [4, 5] and designations from Fig. 1 it follows that the model for the centrifugal F_R and Coriolis F_K forces can be written as:

$$F_R = m\frac{v^2 + \omega^2(R+\Delta_P)^2}{R+\Delta_P}, \tag{8}$$

$$F_K = 2mv\omega, \tag{9}$$

where ω - rotational angular velocity of the center of mass Ц of the vehicle around the center of rotation . Applying the same considerations which were expressed regarding the correlation (5), and notation in Fig. 1 for the force $F_Б$ appearing during the turn due to clash of air flow on the lateral surface of the vehicle and slip of the flow on the frontal surface of the vehicle, let's find

$$F_Б = \frac{1}{2}k_{ПЛ}S_Б(\omega L + v_B\sin(2\pi-\mu))^2 + k_{ПБ}S_Л(\omega L + v_B\sin(2\pi-\mu)). \tag{10}$$

Fig. 1 shows that the constituent of traction force F_T^K to be created to implement the rotation of the vehicle by balancing the Coriolis force F_K, will look as

$$F_T^K = -F_K = -2mv\omega. \tag{11}$$

Regarding force F_{Pk}, which balances the sum of forces $F_R + F_Б$, it is formed, as the force F_P by the friction of the wheels on the road, not rolling friction, but sliding friction, i.e., the mathematical model for its calculation will look like

$$F_{Pk} = k_{**}\left(\omega, P\right)P, \tag{12}$$

in which the coefficient of sliding friction k_{**}, which depends on the size of the wheel contact spot with the road, which under normal air pressure in the tire is a function of vehicle weight and its rotational angular velocity around its center of gravity, can be one of the values of $k_c^к, k_м^к, k_з^к, k_o^к$ - (depending on whether the wheel slip dry asphalt, wet, snowy or iced).

Substituting relations (4) - (12) in the system of equations (3), we obtain completely determined model for the analysis of controlled vehicle movement on the straight segment of road as well as on rounded with no of overtaking and detours. Control variables will be the thrust F_T, which can be changed using the accelerator pedal, and compensation power of the Coriolis force F_K, which can be changed using the steering wheel: turning the front wheels at α angle. The value of the angle can be evaluated from the expression

$$\alpha = arctg \frac{-2mv\omega}{F_T}. \quad (13)$$

Conclusions

1. There had been built mathematical model of two-axel vehicle suitable for solving the problem of vehicle movement control on the main lane in the absence of detours and overtaking.

2. The model is given in a Cartesian coordinate system with reference to a map, and describes the movement of the vehicle center of mass along the centerline of the main lanes.

REFERENCES

1. Мокін Борис Іванович. Математичні моделі багатомасових розподілених динамічних систем для задач оптимізації (частина 1) / Б. І. Мокін, О. Б. Мокін // Вісник Вінницького політехнічного інституту. – Вінниця: «УНІВЕРСУМ-Вінниця», 2008. – № 6. – С. 55 – 58.
2. Мокін Борис Іванович. Математичні моделі багатомасових розподілених динамічних систем для задач оптимізації (частина 2) / Б. І. Мокін, О. Б. Мокін // Вісник Вінницького політехнічного інституту. – Вінниця: «УНІВЕРСУМ-Вінниця», 2009. – № 1. – С. 28 – 33.
3. Мокін Борис Іванович. Модель обмеження на лінійну швидкість вагона електропотяга під час його руху по закругленню колії / Б. І. Мокін, О. Б. Мокін // Вісник Вінницького політехнічного інституту. – Вінниця: «УНІВЕРСУМ-Вінниця», 2009. – № 3. – С. 27 – 29.
4. Фейнман Ричард. Фейнмановские лекции по физике / Р. Фейнман, Р. Лейтон, М. Сэндс. – Т. 1 – 2. – М.: «Мир», 1976. – 439 с.
5. Павловський Михайло Антонович. Теоретична механіка / М. А. Павловський. – Київ: Техніка, 2002. – 512 с.

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