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QUASI-LINEAR MATHEMATICAL MODEL OF GENERATORS ON THE BASIS OF TRANSISTOR STRUCTURES WITH NEGATIVE RESISTANCE

The paper suggests quasi-linear model of generator of sinusoidal waves with fixed frequency, active element of which is transistor structure with negative resistance. Method of phase plane allowed to receive analytical correlations between amplitude and frequency of steady-state oscillation as well as dispersive values of fluctuation of amplitude and phase of the generating signal in real-time. The received correlations are simple and obvious and may be used for engineering calculations of such generators on the stage of their designing.

Key words: *generator, transistor structure, negative resistance, quasi-linear model, fluctuation of amplitude and phase.*

1. Introduction

Transistor structures with negative resistance (TSNR) are extensively used for building generators of electric oscillation with the aim of compensation for losses of energy in passive networks of settings and oscillation system of generator [1, 2]. There had been developed approaches to the researches of basic schema of generators of electric oscillations (GEO) on the basis of TSNR, which helped receive equations of main parameters of generators, conditions of self-excitations and steadiness [3, 4]. However, practical application requires the study of problems dealing with steadiness of the developed GEO on the basis of TSNR, one of the tasks of which is to research fluctuations of amplitude and phase of steady-state generated oscillations.

The objective of the work is to develop the quasi-linear model of GEO on the basis TSNR, convenient for receiving analytical correlations of relative fluctuations of amplitude and phase of steady-state generated oscillations.

2. Quasi-linear mathematical model of GEO on the basis of TSNR

Majority of practical GEO schemes on the basis of TSNR during operation on the fixed frequency of generating quasi – harmonic oscillations may be presented as the tank circuit of the first genus [5].

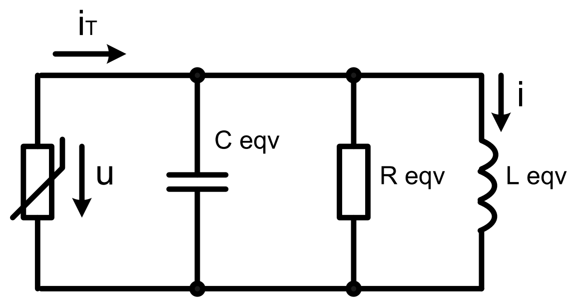

Fig. 1. Equivalent scheme of GEO on the basis of TSNR with quasi – harmonic generated signal

Fig. 1 assumes the following symbols: $i_T(u)$ – controllable voltage source, presenting dependence of current through TSNR on voltage, which is determined by the mode of generator power supply; C, L, and R – equivalent capacity , inductivity and resistance of active losses of selective system of generator. Equivalent capacity of oscillatory circuit:

$$C_{\text{eqv}} = C(u) + C_L + C_{cap}, \qquad (1)$$

where $C(u)$ – equivalent capacity , value of which is determined by reactive component of full resistance of TSNR; C_L – capacity of setting elements and loading of generator, calculated as for oscillatory circuit; C_{cap} – capacitance of circuit.

Equation, describing the dependence of equivalent capacity of reactive component of full resistance of TSNR may be presented as power series [6]

$$C(u) = C_0 + \sum_{k=1}^{\infty} C_k u^k, \tag{2}$$

where C_0 – equivalent capacity, determined by the mode of TSNR power supply.

The first two series members (2) are sufficient to make qualitative analysis of physical processes which take place in GEO on the basis of TSNR;

$$C(u) = C_0 + C_1 u + C_2 u^2. \tag{3}$$

During the operation of GEO on the basis of TSNR on the fixed frequency with quasi-harmonic generated signal, the factors C1 and C2 of the degree series (3) are relatively small [6], therefore, the small amplitude of generated oscillation is

$$C(u) \approx C_0. \tag{4}$$

Equivalent inductivity of oscillatory circuit:

$$L_{eqv} = L_c + L_{str}, \tag{5}$$

where L_c – coil inductivity, L_{str} – calculated as for the oscillatory circuit stray inductance of transistors outputs in active element of generator.

Equivalent active resistance of oscillatory circuit of generator

$$R_{eqv} = \frac{R_l \cdot R_{los}}{R_l + R_{los}}, \tag{6}$$

where R_l – lead resistance, R_{los} – resistance of ohmic losses in the oscillatory circuit and generator adjustment networks.

On the basis of the known approximation of static VAC of device with negative resistance [7], considering the peculiarities of statistic VAC of TSNR of the Λ-type, the work [8] suggests the approximation

$$i_T(u) = I_S - g(u - U_S) + h(u - U_S)^3, \tag{7}$$

where U_S, I_S –coordinates of the center of descending section of VAC of TSNR (section of negative resistance); g, h – factors of approximation, determined from the experimental data.

Removing brackets and re-uniting similar in the correlation (7), we receive the approximations of static VAC of TSNR polynomial of the third degree [8]

$$i_T(u) = \left(I_S + gU_S - hU_S^3\right) - \left(g - 3hU_S^2\right)u - 3hU_S u^2 + hu^3. \tag{8}$$

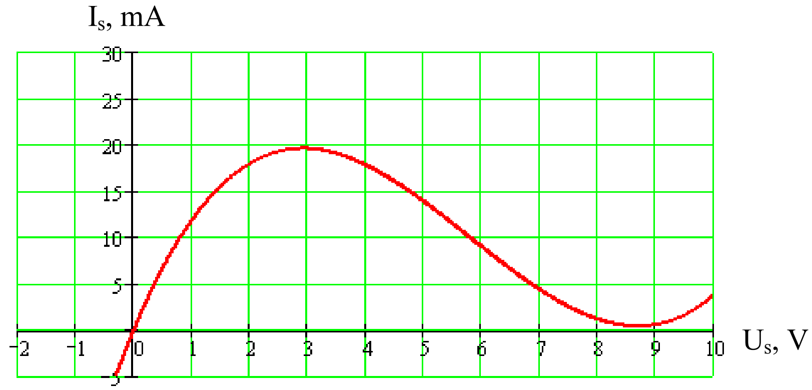

Fig. 2. Graph of approximated statistic VAC of TSNR by equation (8)

Fig. 2 presents graph of approximated statistic VAC of TSNR, which was built with help of equation (8) in MathCad 11.0. Lengh of descending section of statistic VAC may be determined upon researching the function (8) on extremums. Coordinates of the beginning and the end of the section with negative resistance is determined from condition

$$\frac{di_T(u)}{du} = -g + 3h(u - U_S)^2 = 0, \tag{9}$$

from which

$$U_{\max} = U_S - \sqrt{\frac{g}{3h}}, \tag{10}$$

$$U_{\min} = U_S + \sqrt{\frac{g}{3h}}, \tag{11}$$

and the value of maximum and minimum currents in the section with negative resistance.

$$I_{\max} = I_S + g\sqrt{\frac{g}{3h}} - h\left(\sqrt{\frac{g}{3h}}\right)^3, \tag{12}$$

$$I_{\min} = I_S - g\sqrt{\frac{g}{3h}} + h\left(\sqrt{\frac{g}{3h}}\right)^3. \tag{13}$$

Values I_S, U_S, g and h in the equation of approximation (8) in descending section of VAC of TSNR may be determined from the system linear algebraic equations (10) – (13) according to experimentally received points of the beginning ($U_{\max}$, $I_{\max}$) and the end ($U_{\min}$, $I_{\min}$) on the section with negative resistance.

Quasi – linear model of GEO on the basic of TSNR shall be built on the basic of physicalparameters of equivalent scheme on fig.1. Taking into account the selected directions of the currents on fig. 1 allows to write down the system of differential equations.

$$\begin{cases} \dfrac{di}{dt} = \dfrac{u}{L_{eqv}}, \\ \dfrac{du}{dt} = \dfrac{1}{C_{eqv}}\left(i_T - \dfrac{u}{R_{eqv}} - i\right). \end{cases} \tag{14}$$

Let’s use the common correlation of impedance and Q factor of oscillatory circuit.

$$\rho = \omega_0 L_{eqv} = \frac{1}{\omega_0 C_{eqv}} = \sqrt{\frac{L_{eqv}}{C_{eqv}}}, \tag{15}$$

$$Q = \frac{\rho}{R_{eqv}} = \frac{1}{\omega_0 C_{eqv} R_{eqv}}. \tag{16}$$

The system of equation in the normalized time

$$t_n = \omega_0 t, \tag{17}$$

where $\omega_0 = \frac{1}{\sqrt{L_{eqv} C_{eqv}}}$ – resonance frequency of oscillatory circuit will look like.

$$\begin{cases} \frac{d\rho i}{dt_n} = u, \\ \frac{du}{dt_n} = -\rho i + \left[-\frac{u}{Q} + \rho i_T \right]. \end{cases} \tag{18}$$

Considering that equivalent quality of oscillatory system of GEO on TSNR is bigger than the unity [1].

$$Q >> 1 \tag{19}$$

and current, flowing through TSNR is Q times lower than the current of inductivity, the system of differential equations (18) will look as (19).

$$\begin{cases} \frac{d\rho i}{dt_n} = u, \\ \frac{du}{dt_n} = -\rho i + \mu F, \end{cases} \tag{20}$$

where

$$\mu F = -\frac{1}{Q} u + \rho i_T. \tag{21}$$

Constituent (21) is a small parameter of external action on oscillatory circuit, which takes into consideration the non-linear peculiarities of generator on the basic of TSNR. Having accepted the small parameter $\mu = 0$, which corresponds to harmonic generated signal, we solve the system of differential equations using the method of phase plane, which is close to the linear conservative.

$$\begin{cases} \frac{d\rho i}{dt_n} = u, \\ \frac{du}{dt_n} = -\rho i. \end{cases} \tag{22}$$

Phase picture, corresponding to the system (22), in coordinates ρi and u represents the family concentric circumferences with radius U_m, determined by the energy, stored in the oscillatory system [9]. Solution to the system (22) is equations [9]

$$u = U_m \cos(t_n + \varphi) = U_m \cos\psi, \tag{23}$$

$$\rho i = -\frac{du}{dt_n} = U_m \sin(t_n + \varphi) = U_m \sin\psi . \tag{24}$$

Let's determine the law of amplitude establishment U_m and phase φ of equations (23) – (24) under influence of small forces μF . From the second equation of the system (20) it follows

$$du = -\rho i dt_n + \mu F dt_n . \tag{25}$$

Elementary increase in amplitude dU_m and phase $d\varphi$ shall be described by correlations

$$dU_m = d_\mu u \cos\psi , \tag{26}$$

$$d\varphi = -\frac{d_\mu u}{U_m} \sin\psi . \tag{27}$$

Differential equations for establishing instantaneous values of amplitude and phase of equations (23) and (24) look like [9]

$$\frac{dU_m}{dt_n} = \left(-\frac{u}{Q} + \rho i_T \right) \cos\psi , \tag{28}$$

$$\frac{d\varphi}{dt_n} = -\frac{1}{U_m} \left(-\frac{u}{Q} + \rho i_T \right) \sin\psi . \tag{29}$$

Subsequent analysis of the processes of establishing oscillations in GEO on TSNR shall be conducted, presenting current in TSNR as complex action which contains the determinated random components [9]

$$i_T = i_D + i_R , \tag{30}$$

where i_D – determinated component of TSRN current, i_R – random component of TSNR current.

To determine the established mode of oscillations, during absence of fluctuations, we analyze the reduced equation for establishing amplitude and phase, which we receive in the way of averaging equations (23) and (24) as for the period. It is being assumed, that the values of amplitude and phase of the established oscillations during the period are stable [9].

$$\frac{dU_m}{dt_n} = \frac{1}{2\pi} \int_0^{2\pi} \left[-\frac{U_m}{Q} \cos\psi + \rho i_T \right] \cos\psi d\psi = -\frac{U_m}{2Q} + \frac{\rho}{2\pi} \int_0^{2\pi} i_T \cos\psi d\psi , \tag{31}$$

$$\frac{d\varphi}{dt_n} = -\frac{1}{2\pi U_m} \int_0^{2\pi} \left[-\frac{U_m}{Q} \cos\psi + \rho i_T \right] \sin\psi d\psi = -\frac{\rho}{2\pi U_m} \int_0^{2\pi} i_T \sin\psi d\psi . \tag{32}$$

Introducing cosine (I_{1C}) and sinusoidal (I_{1S}) constituents of the first harmonica of the current decomposition TSNR $i_T \left(U_m \cos\psi \right)$ in Fourier series, equations (31) and (32) may be presented as:

$$\frac{dU_m}{dt_n} = -\frac{1}{2} \frac{U_m}{Q} + \frac{1}{2} \rho I_{1C} , \tag{33}$$

$$\frac{d\varphi}{dt_n} = -\frac{1}{2} \frac{\rho I_{1S}}{U_m} . \tag{34}$$

From equations (33) – (34) it follows that losses in the oscillatory system GEO on the basis of TSNR influence the amplitude correlations only. Integrating the equation (33), allows to determine the increase in amplitude of oscillation, conditioned by series resistance R_{eqv} within the period

$$\Delta_R U_m = -\frac{\pi}{Q} U_m. \tag{35}$$

Further researches of the established oscillations of GEO on the basis of TSNR we conduct for soft mode. Soft self-exciting mode appears when the working point is situated on the descending section of VAC on the section with the biggest steep slope. Using the suggested approximation of cubical polynomial (8), we received the equation of dependence of amplitude of the first harmonica of the current TSNR on voltage amplitude on circuit.

$$I_{m1} = -\left(g - 3hU_S{}^2\right)U_m + \frac{3}{4}hU_m^3. \tag{36}$$

Using the dependence of inertial properties of TSNR on the frequency, the first harmonica of the current of TSNR is shifted by the angle φ_β relating to voltage in the circuit.

$$i_Д = I_{m1}\cos\left(\psi - \varphi_\beta\right). \tag{37}$$

Considering (37) we received the equation of cosine and sinusoidal constituents of the current of the first harmonica of TSNR, which look as follows

$$I_{1C} = \left[-\left(g - 3hU_S{}^2\right)U_m + \frac{3}{4}hU_m^3\right]\cos\varphi_\beta, \tag{38}$$

$$I_{1S} = \left[-\left(g - 3hU_S{}^2\right)U_m + \frac{3}{4}hU_m^3\right]\sin\varphi_\beta. \tag{39}$$

Substituting (38) and (39) correspondingly in (33) and (34), we receive the shortened equations of establishing amplitude and phase of generated oscillations in GEO on the basis of TSNR which look

$$2\frac{dU_m}{dt_n} = -U_m\left[\frac{4}{3h\rho Q\cos\varphi_\beta} + \frac{4}{3}\frac{g - 3hU_S^2}{h} - U_m^2\right]\frac{3}{4}h\cos\varphi\beta, \tag{40}$$

$$2\frac{d\varphi}{dt_n} = \rho\left[g - 3hU_S^2 - \frac{3}{4}hU_m^2\right]\sin\varphi\beta. \tag{41}$$

In accordance with equation (40) we received the condition of soft mode of self-excitation of GEO on the basis of TSNR

$$\left(g - 3hU_S^2\right) > \frac{1}{Q\rho\cos\varphi_\beta}. \tag{42}$$

The equation of stationary amplitude of oscillation which we received, looks like

$$U_{ST} = \frac{2}{\sqrt{3h}}\sqrt{g - 3hU_S^2 + \frac{1}{Q\rho\cos\phi_\beta}}. \tag{43}$$

Normalized frequency of stationary oscillation in normalized time t_n is determined by substituting (43) in (41), considering (23) – (24) [9]

$$\omega_{0n} = 1 + \frac{d\varphi}{dt_n} = 1 - \frac{1}{2Q} tg\varphi_\beta . \tag{44}$$

Frequency of generated stationary oscillations in real time in determined from (44) considering (17)

$$\omega_{ST} = \omega_0 \left(1 - \frac{1}{2Q} tg\varphi_\beta \right). \tag{45}$$

3. Determination of fluctuations of amplitude and phase of stationary generated oscillations of GEO on the basis of TSNR.

For the soft mode of self excitation of GEO on the basis of TSNR we present the solution of the system of differential equations (22) considering fluctuation of amplitude and phase as [9]

$$u(t) = U_{ST}\left[1 + \tilde{u}(t)\right] sin\left[t_n + \tilde{\varphi}(t)\right], \tag{46}$$

Where $\tilde{u}(t)$ – relative amplitude fluctuations, $\tilde{\varphi}(t)$ – fluctuations of phase relating to the initial value.

In equation(46) with an aim of simplification, we assume that the phase shift between the voltage on the circuit and on the first harmonica of the current of TSNR equals $\varphi_\beta = 0$, which is acceptable for the majority of schemes of GEO on the devices with Λ-characteristics. If $\varphi_\beta \neq 0$, then transferring to the normalization of system time (17) makes it appropriate to use correlation [9]

$$t_H = \omega_0 \left[1 + \frac{d\varphi}{dt} \right] t. \tag{47}$$

Experimental researches of GEO on the basis of TNSR showed that in stationary mode, fluctuations of amplitude and phase of generated oscillations within a period are relatively small [2]. Therefore for the relative fluctuations of amplitude and phase the correlations come true:

$$\left|\tilde{u}(t)\right| << 1, \tag{48}$$

$$\left|\tilde{\varphi}(t)\right| << 1. \tag{49}$$

Considering correlations (48) – (49) the short and differential equations of relative fluctuations of amplitude and phase

$$\frac{d\tilde{u}}{dt_n} = -b_0 \tilde{u} + \eta_{\tilde{u}}(t), \tag{50}$$

$$\frac{d\tilde{\varphi}}{dt_n} = q_0 - q_1 \tilde{u} + \eta_{\tilde{\varphi}}(t), \tag{51}$$

Where q_0 – constant correction to frequency, which may be neglected ($q_0 = 0$); q_1 – factor, which considers the influence of fluctuation of amplitude of generated oscillations on frequency, considering GEO on TSNR with harmonic generated signal as quasi-isochoric we may accept $q_1 = 0$; b_0 – factor, which considers the degree of stability of boundary cycle of phase picture of GEO on the basis of TSNR; $\eta_{\tilde{u}}(t)$ and $\eta_{\tilde{\varphi}}(t)$ – normal stationary random processes with zero means, equations of which in general kind [9]

$$\eta_{\tilde{u}}(t) = -\frac{1}{2\pi U_{ST}} \int_{t_n}^{t_n+2\pi} i_R \cos t_n dt_n, \tag{52}$$

$$\eta_{\tilde{\varphi}}(t) = -\frac{1}{2\pi U_{ST}} \int_{t_n}^{t_n+2\pi} i_R \sin t_n dt_n, \tag{53}$$

$$\left\langle \eta_{\tilde{u}}(t) \right\rangle = \left\langle \eta_{\tilde{\varphi}}(t) \right\rangle = 0. \tag{54}$$

Functions of autocorrelation of stationary random processes y(t) and n (t)

$$K_{\eta_{\tilde{u}}}(\theta) = K_{\eta_{\tilde{\varphi}}}(\theta) = \frac{1}{2\pi U_{ST}^2} \int_{-\infty}^{+\infty} S_\zeta (1+\Omega) \cos \Omega\theta d\Omega, \tag{55}$$

where S_ζ – energetic spectrum of random constituent of current in TSNR $\Omega = \frac{\Delta\omega}{2}$ – half of bandwidth of generator oscillation system.

Assuming that fluctuation current $i_B(t)$ in bandwidth of circuit has equal energy spectrum [9]

$$S_i(\omega) = S_i(\omega_0) \text{ if } |\omega - \omega_0| < \Omega, \tag{56}$$

Energy spectrums of amplitude and phase fluctuations within the boundaries of brandwidth of the oscillatory circuit [9]

$$S_{\eta_{\tilde{u}}}(\Omega) = S_{\eta_{\tilde{\varphi}}}(\Omega) = \frac{S_i(\omega_0)}{2U_{ST}^2}. \tag{57}$$

We received the stability degree factor of boundary cycle on the basis of correlation

$$\begin{aligned} b_0 &= \frac{1}{U_{CT}} \frac{d}{du} \left[\frac{1}{2\pi} \int_0^{2\pi} \left(-\frac{1}{Q} u \cos t_n + \rho i_T \right) \sin t_n dt_n \right]_u = \\ &= \frac{1}{U_{ST}} \frac{d}{du} \left[\frac{1}{2\pi} \int_0^{2\pi} \left(-\frac{1}{Q} u \cos t_n + \rho \left(I_S - g(u - U_S) + h(u - U_S)^3 \right) \right) \sin t_n dt_n \right]_u \approx \\ &\approx 3hU_S^2 - g. \end{aligned} \tag{58}$$

Solving the shortened equations (52) – (53) considering (57),the author determined dispersion values of fluctuations of amplitude and phase in generated signal in real time

$$\sigma_{\tilde{u}}^2 = \frac{1}{4} \frac{S(\omega_0) \cdot \omega_0^2}{U_{ST}^2 \left(3hU_{ST}^2 - g \right)}, \tag{59}$$

$$\sigma_{\tilde{\varphi}}^2 = \frac{1}{2} \frac{S(\omega_0) \cdot \omega_0^2}{U_{ST}^2} t. \tag{60}$$

Received correlations (59) and (60) for determination of fluctuations of amplitude and phase of stationary oscillations of GEO on the basis of TSNR include parameters of statistic VAC of active element of generator for equation of energy spectrum of fluctuation current within the boundaries of bandwidth of oscillatory circuit of generator.

Idealization of the developed quasi-linear model consists in accepted constant value of capacitive constituent of full resistance of TSNR (correlation 4).Non linear peculiarities of electrically controllable capacity influence the phase correlations in generator only. Equation of fluctuation of phase of generated oscillations of GEO, the equivalent scheme of which is presented in fig.1, considering (2), looks like.

$$\frac{d\varphi}{dt} = -\frac{\omega_0}{C_0}\left[C_1 e_n(t) + C_2\left(\frac{3}{4}u^2(t) + 3e_n^2(t)\right) + C_3 e_n(t)\left(3u^2(t) + 4e_n^2(t)\right) + \ldots\right], \qquad (61)$$

where $e_n(t)$ – equivalent source of noise voltage, which considers internal and external noise of generator. On the basis of equation (61) in work [6] , it was substantiated, that 1) non-linearity of capacity of the first order (first constituent C_1) contributes to transformation of law-frequency noise $e_n(t)$into the noise of sideband close to the carrier ω_0; 2) non-linearity of the second order (constituent C_2) generates phase noise, in the result of transformation amplitude-phase and noise properties of generator active element; 3) non-linearity of the third and higher orders cause morecomplicated behavior of generator noise, which is predetermined by inter- modulation distortions and transformations amplitude-phase.

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