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## METHOD FOR DETERMINATION OF MAXIMUM LENGTH OF THE SHORT ELECTRIC POWER TRANSMISSION LINE


#### Abstract

There had been suggested mathematical models and method for the determination of maximum length of short electric power transmission line. This method may be used at initial stages of designing when real length of lines, their loads and cross-sections are unknown. That allows justifying the use of optimal design methods suitable for short electric power transmission lines which will ensure optimum and tolerances of electric power supply system as a whole.


Key words: short electrical network, maximum length of short line, lower boundary of maximum length, optimal power system.

Designing of optimum electric-power supply system (PSS) is a complex task which requires the consideration of mutual influences of one design stage on others. Selection of separate subsystems of the uniform PSS by the results of independent optimization calculations as a rule does not ensure the systems optimality as a whole and does not provide its tolerances at all. Formally this results from the fact that the design decisions made at the previous design stages can change the area of feasible solutions of problems of sampling of optimum parameters, elements and subsystems of PSS on the following design stages [1]. Therefore the efficiency of optimum design solutions, taken at the following design stages can be decreased and lead to decrease in efficiency of the design in general. In particular it concerns problems of selection of optimum cross-section of distributive networks which follow from the power supply (PS) to transformer substations of electric power consumers, and co-ordinates of the PS itself.

Optimum co-ordinates of PS location depend on technical and economic performances of power lines, in particular on cross-sections and lengths of these lines. Lengths of lines in turn directly depend on co-ordinates of PS location and may affect an admissibility of use of these or those cross-sections of these lines. There is a dilemma which could have been avoided, solving at once both problems, as one optimization problem with common controlled variables. However at present there are no universal methods of solution of this problem. The analysis of difficulties which appear and some ways of their overcoming are shown in paper [1]. This paper proves, that the above dilemma disappears, if distributive nets are short.

Concept of a short line and maximum length of such line is considered in [1-3]. The line is short if with any allowed load the voltage loss in this line is admissible. The short line has the maximum possible length if with any greater length and maximum allowed load the voltage loss in this line becomes inadmissible.

Paper [1] shows, that optimum cross-section of a short line which is chosen by such parameters of efficiency as cost, active power losses, the non-ferrous metal constituency, the annual expenditures and others, do not depend on its length. Therefore the optimum cross-section of short lines by criteria of a minimum efficiency parameters listed above can be determined, using specific values of these parameters per unit of line length. After that the optimum co-ordinates of the location of the power supply may be determined using all technical and economic parameters necessary for chosen in advance lines. It will allow to avoid complicated calculations upon agreement of results for solving the problem of choosing optimal line cross-sections and optimal coordinates of PS location. That is why, at the initial stage of PSS design it is important to be able to estimate, whether all the lines of distributive network will be short. In that case their optimum cross-sections do not depend on the length of lines and may be chosen prior to the determination for PS location.

Any line of the set brand and cross-section will be short if its length $L_{\mathrm{S}}$ does not exceed maximum length of correspondingly short line $\mathrm{L}_{\text {Smax }}$ [1], i.e.

$$
\begin{equation*}
L_{\mathrm{S}} \leq L_{\mathrm{Smax}} \tag{1}
\end{equation*}
$$

The maximum possible values for $L_{\mathrm{S}}$ may always be evaluated proceeding from conditions of designing PSS. Therefore it is important to be able to determine the maximum length of short line $L_{\text {Smax }}$ of the set brand and cross-section. It will enable, using a relation (1) to estimate, whether the line with the length $L_{\mathrm{S}}$ will be short .

Let's make a substantiation of a method for definition of maximum length of short cable (CL) and air (AL) power transmission lines (PTL) $0,38-35 \mathrm{kV}$.

Since for the local area networks with voltage below 110 kV , voltage loss $\Delta U$ is approximately equal to longitudinal constituent voltage drops [4] it may be determined by formula:

$$
\Delta U=\frac{R_{0}(x) \cdot P+X_{0}(x) \cdot Q}{U} \cdot L=\frac{\sqrt{3} \cdot U \cdot I \cdot\left(R_{0}(x) \cdot \cos (\varphi)+X_{0}(x) \cdot \sin (\varphi)\right)}{U} \cdot L
$$

where $R_{0}(x)$, - specific active resistance of a line with cross-section $x ; P$ - active load of line; $X_{0}$ $(x)$ - specific reactive resistance of line with cross-section $x ; Q$ - reactive loading of line; $L$ - length of line; $U$ - line voltage; $\varphi$ - phase angle between voltage and current.

Voltage loss is usually determined in percentage of nominal voltage. In this case the last formula will look like:

$$
\Delta U \%=\frac{\sqrt{3} \cdot I \cdot\left(R_{0}(x) \cdot \cos (\varphi)+X_{0}(x) \cdot \sin (\varphi)\right)}{U} \cdot L \cdot 100
$$

Engineering calculations use measuring units kV , A , the $\mathrm{Ohm} / \mathrm{km}$, km for voltage, current specific resistance and length of the line correspondingly. For the determination of $\Delta U \%$ using these measurement units of corresponding parameters, we will present the as follows:

$$
\Delta U \%=\frac{\sqrt{3} \cdot I \cdot\left(R_{0}(x) \cdot \cos (\varphi)+X_{0}(x) \cdot \sin (\varphi)\right)}{10 \cdot U} \cdot L .
$$

From last expression we will determine length of a line which equal to the voltage loss $\Delta U \%$ and loading $I$ for the accepted measurement units of voltage, current, resistance and length:

$$
\begin{equation*}
L=\frac{10 \cdot \Delta U \% \cdot U}{\sqrt{3} \cdot I \cdot\left(R_{0}(x) \cdot \cos (\varphi)+X_{0}(x) \cdot \sin (\varphi)\right)} \tag{2}
\end{equation*}
$$

To determine the maximum length of short line $L_{\mathrm{Smax}}$ with cross section $x$ which corresponds to maximum allowed loss of voltage $\Delta U \%=\Delta U_{\text {allow }} \%$ and maximum allowed on RBE to [5] loading $I=I_{\text {allow }}(x)$ it is necessary to substitute these maximum allowed values meanings of parameters $\Delta U$ $\%$ and $I$ in the formula (2). As a result we will receive:

$$
\begin{equation*}
L_{\text {Smax }}=\frac{10 \cdot \Delta U_{\text {allow }} \% \cdot U}{\sqrt{3} \cdot I_{\text {allow }}(x) \cdot\left(R_{0}(x) \cdot \cos (\varphi)+X_{0}(x) \cdot \sin (\varphi)\right)} \tag{3}
\end{equation*}
$$

Allowed losses of voltage on final net terminations should be determined depending on power supply voltage levels, proceeding from standardized in State Standard of 13109-97 [6] deviations of terminal voltage of electric receivers in percentage relatively rated voltage. Work [7] substantiates, that for urban distributive networks $6-10 \mathrm{kV}$ it is possible to accept $\Delta U_{\text {allow }} \%=6 \%$.

Formula (3) shows, that the maximum length of a short line depends not only on its brand and cross-section which in aggregate determine numerical values of parameters $I_{\text {allow }}(x), R_{0}(x), X_{0}(x)$, but also on the angle $\varphi$ of phase shift between voltage and current. Therefore it is possible to present maximum length of a short line of the set brand, as function $\mathrm{L}_{\mathrm{Smax}}(x, \varphi)$ of two arguments -cross-section $x$ and phase shifting $\varphi$.

It has been substantiated above, that the necessity in estimation of maximum length of a short
line appears at the initial stage of optimum designing of PSS when values of parameters $x$ and $\varphi$ as a rule are still unknown. Despite this fact in many cases it is possible to prove, that the line, which optimum cross-section will be determine at the following designing stages of PSS, is short.

For this purpose we should define low boundary of maximum length of short line $L B L_{S m a x}$, which, if condition $L_{S} \leq L B L_{\text {Smax }}$ is satisfied, would provide that any supply line of the length $L_{S}$ at any admissible values of its cross-section and angle of phase shift between current and voltage is short.

Such problem can be reduced to the solution of optimization problem of a special aspect. Line cross-section and phase angle between voltage and current must act as controlled variables of this problem, and function $L_{\mathrm{Smax}}(x, \varphi)$ - as solution efficiency parameter to be minimized.

Let us present mathematical model of this problem for a case of the separate cable feeder. Let the biggest possible length CL cannot exceed $L_{\mathrm{S}}$. Generally supply line may consist of one or two cables. The mathematical model of a problem for such line will look like:

$$
\begin{align*}
& L_{S \text { max }}(x, \varphi)=\frac{10 \cdot \Delta U_{a d m} \% \cdot U}{\sqrt{3} \cdot I_{a d m}(x) \cdot\left(R_{0}(x) \cdot \cos (\varphi)+X_{0}(x) \cdot \sin (\varphi)\right)} \rightarrow \min _{x, \varphi}, \\
& 0 \leq \varphi \leq \frac{\pi}{2}, \\
& I_{a d m}(x) \geq I ; \\
& k>1 \Rightarrow k_{a f . m} \cdot I_{a d m}(x) \geq \frac{k \cdot I \cdot k_{a f . f}}{k-1} ;  \tag{4}\\
& x \geq x_{s c}=\frac{I_{s c} \cdot \sqrt{t_{d u r}}}{C} ; \\
& x \in X_{s t},
\end{align*}
$$

where $I$ - current of separate cable of CL; $k$ - number of lines; $k_{\text {a.f.m }}$ - maximum admissible loading factor of CL in post-emergency conditions of operation; $k_{\text {l.a.f.m. }}$ - part of general loading of electric power consumer which should be transmitted by CL in post-emergency conditions of operation; $x_{\mathrm{sc}}$ - minimum cross-section of a line under condition of thermal impact of short-circuit currents (short circuit); $I_{\mathrm{sc}}$ - short circuit current in the beginning of CL; $t_{\mathrm{n}}$ - time of short circuit; $C$ - thermal factor which depends on rated voltage of the line and conductor material (is set for AL and CL in tables 8, 9 of State Standard 30323-95 [8]); $X_{\text {ст }}$ - set of standard cross-sections of CL.

It is obvious that it is possible to evaluate the lower boundary of maximum length of short line $L B L_{\text {Smax }}$ by the formula $L B L_{\text {Smax }}=L_{S \max }\left(x^{*}, \varphi^{*}\right)$, where $\left(x^{*}, \varphi^{*}\right)-$ model (4) solution, and any supply line the length of which cannot exceed $L_{\mathrm{S}}$ will be short if the condition $L_{\mathrm{S}} \leq L B L_{\mathrm{Smax}}$ is satisfied. Since model (4) helps determine the lower boundary of possible values of maximum length of short line, the reverse statement is incorrect.

In case with lines, RBE requires to check the cross-sections of these lines as well as absence of general coronation and mechanical strength. Therefore model (4) for overhead transmission lines will look like:

$$
\begin{align*}
& L_{S \max }(x, \varphi)=\frac{10 \cdot \Delta U_{a d m} \% \cdot U}{\sqrt{3} \cdot I_{a d m}(x) \cdot\left(r_{0}(x) \cdot \cos (\varphi)+x_{0}(x) \cdot \sin (\varphi)\right)} \rightarrow \min _{x, \varphi}, \\
& 0 \leq \varphi \leq \frac{\pi}{2}, \\
& I_{a d m}(x) \geq I ; \\
& k>1 \Rightarrow k_{a . f l} \cdot I_{a d m}(x) \geq \frac{k \cdot I \cdot k_{a f . l}}{k-1} ;  \tag{5}\\
& x \geq x_{s . c}=\frac{I_{s . c} \cdot \sqrt{t_{d u r}}}{C} ; \\
& x \geq x_{c r} \\
& x \geq x_{\text {mec }} \\
& x \in X_{s t},
\end{align*}
$$

where $x_{\text {cor }}$ - minimum cross-section of the TL without general coronation; $x_{\text {mec }}-$ minimum cross-section of the TL on a condition of mechanical strength.

In case of the cable feeder, the restriction on absence of the general coronation and mechanical strength of cross-section of CL have been omitted, since meeting of these conditions is ensured by cable manufacturing plants.

If the feeder loading is unknown, then in models (4), (5) it is necessary to omit the restriction on admissibility of heating of a line in normal and post-emergency modes of operation. In this case lower boundary of possible values of maximum length of short line defined as a result of models solution will not concern separate power line, but all lines which will depart from the power supply. Therefore all these lines will be short if the largest possible length of each of them does not exceed the found value of $L B L_{\text {Smax }}$.

Controlled variables in mathematical models (4), (5) are the cross-section of a line and a phase angle between current and voltage. The first controlled variable may accept only discrete and standardized values, and the second is a continuous one. This is a principal cause for absence of universal methods for decision theory required for the direct problems solution by the obtained models. Therefore we suggest a special method for solution of the set task which is based on such properties of this problem:

1. All conditions which should satisfy supply line cross-section as it is seen from models (4) and (5), restrict this controlled variable from below. In this case the lower domain boundary of feasible solutions for power line cross-section will be equal $\max _{i}\left(x_{\min _{i}}\right), i=1, \ldots, m$, where $x_{\min _{i}}$ - minimum admissible cross-section of a line which satisfied the $i-{ }^{\text {th }}$ restriction; $m$ - totality of restrictions.


Fig. 1. The example of dependence of CL maximum length on phase shift between current and voltage
2. Dependence of maximum length of short line on phase shift $\varphi$ between voltage and line current at any cross-section of this line on all set of possible values of parameter $\varphi$ is of oneextreme, smooth and convex. The example of such dependence for cable with aluminium conductors and paper impregnated insulation is presented in fig. 1.

We suggest to solve the task in two stages. At the first stage it is necessary to find the lower domain boundary of admissible cross-sections of the feeder $x_{\text {adm }}$, using the first property of the problem:

$$
x_{a d m}=\max _{i}\left(x_{\text {min }_{i}}\right), i=1, \ldots, m
$$

At the second stage for all standard cross-sections of supply line not smaller than $x_{\mathrm{adm}}$ solve the problem:

$$
\left.\begin{array}{l}
L_{S \text { max }}\left(x_{i}, \varphi\right)=\frac{10 \cdot \Delta U_{a d m} \% \cdot U}{\sqrt{3} \cdot I_{a d m}(x) \cdot\left(R_{0}(x) \cdot \cos (\varphi)+X_{0}(x) \cdot \sin (\varphi)\right)} \rightarrow \min _{\varphi},  \tag{6}\\
0 \leq \varphi \leq \frac{\pi}{2}
\end{array}\right\}, i=1, \ldots, n,
$$

Where $n$ - number of standard cross-sections of supply line which satisfy the condition $x_{\mathrm{i}} \geq x_{\text {adm }}$.
The problem of minimization of function $L_{\text {Smax }}\left(x_{\mathrm{i}}, \varphi\right)$ according to model (6) owing to property 2 of this problems may be solved by means of known methods of optimization and, in particular, by means of the gradient methods built- in such mathematical CAD systems as Mathcad, Matlab, Excel and other. As a result we will get $n$ solutions of $\varphi_{i}^{*}, i=1, \ldots, n$. Now the lower boundary of maximum length of a short line is easy to find by the formula:

$$
L B L_{\text {Smax }}=\min _{i}\left(L_{\text {Smax }}\left(x_{i}, \varphi_{i}^{*}\right)\right), i=1, \ldots, n
$$

Having obtained this result, we can be sure, that any line of the set brand with length $L_{\mathrm{S}} \leq$ $L B L_{\text {Smax }}$ will be short.

The research carried out by the author have shown, that the suggest method for calculation of maximum length of a short line is not complicated for realization in the environment of such software products, as Mathcad, Matlab, Excel using the gradient methods of optimization built in them. And in order to obtain the result, it is necessary to substitute the initial data in the corresponding computer models.

Tables 1, 2 contain the examples of the obtained dependences of maximum length of short cable and overhead lines on minimum allowed cross-section of these lines .

Table 1
Maximum length of short $C L$ for 10 kV with aluminium conductor and a paper insulation depending on line cross-section

| Cross-section, <br> $\mathbf{m m}^{2}$ | Length, m |
| :---: | :---: |
| 16 | 2377 |
| 25 | 3094 |
| 35 | 3365 |
| 50 | 3950 |
| 70 | 4652 |
| 95 | 5023 |
| 120 | 5338 |
| 150 | 5709 |
| 185 | 6077 |
| 240 | 6539 |

## Maximum length of the short line 10 kV of AC brand depending on line cross-section.

| Cross-section, $\mathbf{m m}^{2}$ | Length, $\mathbf{~ m}$ |
| :---: | :---: |
| 25 | 1971 |
| 35 | 2266 |
| 50 | 2340 |
| 70 | 2354 |
| 95 | 2271 |
| 120 | 2121 |

Table 1 shows, that the maximum length of short CL of the set brand depending on minimum allowed cross-section of a line can vary from 2,4 to $6,5 \mathrm{~km}$. Following the authors research, the similar picture comes true for power cables of 10 kV of other brands. It allows to state that cable lines of the majority of enterprises and city areas can be referred to short lines of 10 kV .

At the same time Table 2 shows, that the maximum length of a short overhead transmission electric line poorly depends on its minimum allowed cross-section and makes up approximately $2,2 \mathrm{~km}$.

## Conclusions

The suggested method helps to prove that lines of designed system of electric-power supply will be short. The method allows to make this in conditions when the lengths of lines, their loading and crosssections are unknown. Calculations based on this method can be fulfilled easily in the environment of such software products as Mathcad, Matlab, Excel. That allows to substantiate the legitimacy of use of rather simple methods of optimum designing of short networks [1] which will guarantee optimality and admissibility of PSS as a whole. The results of the research carried out by the author using the suggested method show, that the majority of cable networks of the industrial enterprises and city areas may be referred to the short ones. Therefore it is expedient to use such design techniques for their construction.

In case when by means of the suggested method it will not be possible to prove that PSS network will be short, the process of optimum designing of such networks can become considerably complicated, as the existing design techniques in these conditions do not guarantee optimum achievement on PSS as a whole.

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