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APPLICATION OF WAVELET-TRANSFORMATIONS TO THE TASKS OF MONITORING AND DIAGNOSTICS OF MACHINES AND EQUIPMENT

The paper briefly considers the concepts and principles of vibration diagnostics of machines and equipment. For such problems application of mathematical tools of wavelet transformations is suggested, its advantages are analyzed. The examples of wavelet transformations of some non-stationary signals are presented

Key words: *monitoring, vibration diagnostics, non-stationary signal, amplitude-frequency spectrum, wavelet-transformation, parent wavelet.*

Introduction

In the course of continuous measurement of vibration process in an object the following physical quantities are determined: vibration shift (vibration amplitude), vibration speed (the rate of the coordinate change), vibration acceleration (the rate of vibration speed change), and the rate of vibration acceleration change [1, 2].

At present the methods with full delimitation of monitoring and diagnostics functions are mostly used. Monitoring involves usage of a vibration sensor, installed stationary, and a measuring channel for electrical quantities with microprocessor control. Monitoring provides detection of changes in vibroacoustic state of the object, selection of the changes connected with the possible change of its condition and prediction of the potential development of defects. The monitoring task is solved using hard- and software tools, the task of diagnostics – by software tools that realize one of the known information technologies.

There exist the following information technologies of vibration diagnostics:

- *power* technology based on the measurement of the controlled signal amplitude;
- *frequency* technology that analyzes frequency-amplitude spectrum of the signal;
- *phase-time* technology based on the comparison of signal forms measured at fixed time intervals.

This paper considers the distinguishing features of frequency technology application for monitoring and vibration diagnostics of machines and equipment.

Phase-time technology for the analysis of non-stationary signals

In the overwhelming majority of existing vibration diagnostics methods the controlled signal of rotary machine vibration is assumed to be a *stationary* one, i.e. its amplitude frequency spectrum does not change in time [2].

At the same time, if a signal is being measured for a long time (is being monitored), local deformation might occur in the machine assembly (caused by wear of some of its parts or for another reason), which, in turn, leads to the changes in amplitude-frequency spectrum of the vibration signal. Therefore, it would be expedient to assume that in general case the controlled vibration signal of a certain machine unit is non-stationary, i.e. its amplitude-frequency spectrum changes in time.

Diagram of the amplitude-frequency spectrum of non-stationary signal will not be two-dimensional, but a three-dimensional one. The example of such amplitude-frequency spectrum diagram is presented in fig. 1.

If vibration signal is considered to be stationary, its amplitude-frequency spectra can be determined using ordinary Fourier transformation, which is usually done in diagnostics systems. It should be reminded that Fourier $f(\omega)$ spectrum of one-dimensional signal $f(t)$ is given by the formula

$$f(\omega)=\int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt \tag{1}$$

and does not allow to localize possible frequency changes in time.

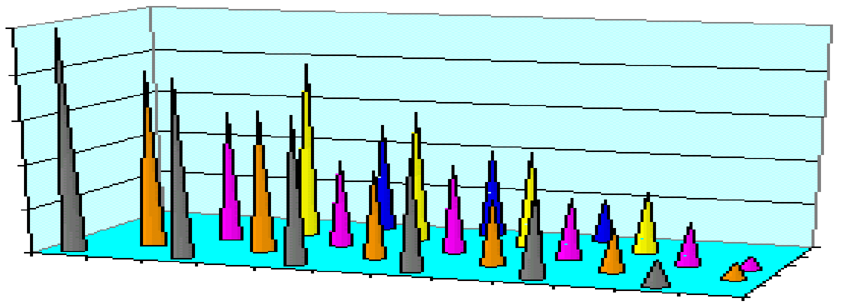

Fig. 1. Three-dimensional diagram of the discrete amplitude-frequency spectrum (along horizontal axes frequency and time while along the vertical axis - amplitude is plotted)

It is evident that if we want to combine monitoring and diagnostics functions in one system, it is necessary to apply such transformation that would provide continuous determination of the amplitude-frequency signal in time.

One of such transformations is the so called w*indowed Fourier transform* (WFT).that is sometimes called *weighted Fourier transform* (WFT). $f(t)$ signal is analyzed only in the limits of a certain window. For this $f(t)$ is multiplied by a function with the compact carrier, e. g., $g(t)=\theta(t-t_1)\theta(t_2-t)$, where θ – ordinary step function that is non-zero only for positive values of the argument, t_1, t_2 – moments of the signal start and finish that are set by the window choice.

In this case

$$f(m,n)=\int_{-\infty}^{\infty} f(t)g(t)e^{-i\omega t}dt \tag{2}$$

or in a discretized form

$$f(m,n)=\int f(t)g(t-nt_0)e^{-im\omega_0 t}dt\ , \tag{3}$$

where $\omega_0, t_0 > 0$ are fixed and m, n – numbers that define scale and location.

Function $g(t)$ may have another form, the main thing being its having a compact carrier [3, 4].

By means of WFT the signal is localized in time, but the window must be of a fixed size, which is main disadvantage of this transformation. It should be noted that time-frequency transformations are subjected to the universally known *Heisenberg uncertainty principle* that in our case is formulated as follows: *there does not exist a fixed time moment for which it is possible to determine spectrum components of the signal.*

Proceeding from this principle we can determine only *time intervals* when the signal contains *frequency bands*.

From this it follows that if a window (i.e. time interval) is small and this points to the high time localization of the spectrum, the frequency band will be rather indistinct (smeared) and, vice versa, more accurate determination of spectrum components requires a big window [3, 4].

Real non-stationary signals consist of short-time high-frequency and long low-frequency components and so for the analysis it is expedient to use transformations that would provide different windows for different frequencies (narrow ones for high and wide ones for low frequencies).

*Wavelet transformation*s meet these requirements.

At the verbal level it can be represented as displacement of a certain analytical function (the so called *parent wavelet*) along time axis and its interaction with the signal that is being controlled. Different functions can be used as parent wavelets, such as the wavelets of Haar, Shannon, Daubechies, Meyer, the "Mexican hat" wavelet etc.

If parent wavelet is designated $\psi(t)$, then wavelet transformation of signal $f(t)$ with scale parameter s and time shift τ is defined as

$$Wf(\tau,s) = \frac{1}{\sqrt{s}} \int_{-\infty}^{+\infty} f(t) \psi^* \left(\frac{t-\tau}{s} \right) dt, \tag{4}$$

where ψ^* is a conjugate parent wavelet [3 – 5].

If signal $f(t)$ is given in analytical form, formula (4) reflects *continuous* wavelet transformation (CWT) of signal $f(t)$.

It is evident that the domain of function $Wf(\tau,s)$ is the set of all possible combinations of s and τ.

Scale parameter s is, in fact, the value reverse to frequency. As it is in the denominator, then $s<1$ compresses it.

CWT calculation algorithm is rather simple. First, a researcher chooses a parent wavelet and then $Wf(\tau,s)$ is calculated for all points of the domain.

In this way a matrix of wavelet coefficients values for all combinations of s, τ is obtained.

Let's consider some examples. Let a signal (fig. 2) with a continuously diminishing value is given analytically as

$$f(t) = 0.5 \, sin\left(\frac{2\pi\sqrt{t}}{10} \right). \tag{5}$$

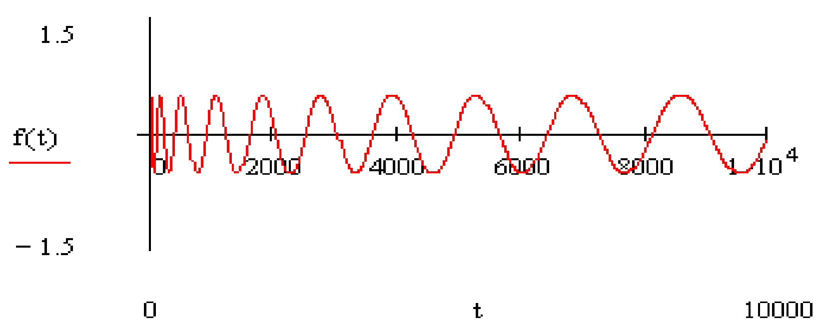

Fig. 2. Diagram of the signal with diminishing frequency

Let's perform wavelet transformation for this signal. As a parent wavelet, function

$$\psi(t) = (1-t^2) e^{\frac{-t^2}{2}} \tag{6}$$

is chosen. It is called the "Mexican hat" (fig.3) and often used for wavelet transformations.

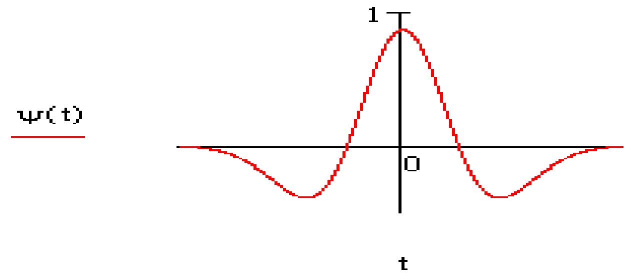

Fig. 3. Diagram of the “Mexican hat” parent wavelet

Let’s determine a matrix of wavelet coefficients for this signal.

It is obvious that the range of this matrix must be finite as scale parameter s and time shift τ are given with a definite step.

Calculations in this example and in the subsequent ones are performed in MathCAD environment. The listing is given below.

$$\mathrm{MHAT}(t) := \frac{d^2}{dt^2}\exp\left(\frac{-t^2}{2}\right) \qquad \Psi(a,b,t) := \mathrm{MHAT}\left(\frac{t-b}{a}\right)$$

$$W(a,b) := \int_{-25}^{25} \Psi(a,b,t)\cdot\frac{1}{\sqrt{a}}\cdot f(t)\,dt$$

$$i := 0..10 \qquad b := 0..25 \qquad a_i := \frac{(i+15)^2}{10^2}$$

$$N_{i,b} := W\left[\left(a_i\right), 2\cdot b - 25\right]$$

Three-dimensional diagram of wavelet coefficients N is shown in fig. 4.

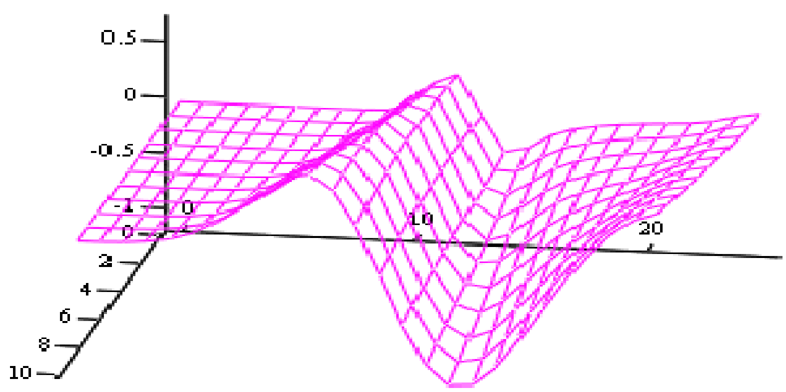

Fig. 4. Diagram of the continuous wavelet transformation of a signal with diminishing frequency

The diagrams of wavelet transformations have the following distinguishing feature: the time and scale parameter are plotted on the horizontal axes (instead of frequency).

And now we will consider the signal (fig.5) with continuously increasing frequency given in the form of

$$f(t) = sin\left(\frac{2\pi t^{1.5}}{50}\right). \tag{7}$$

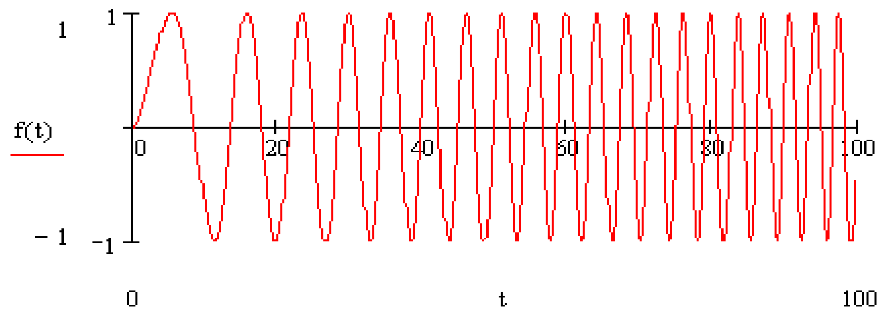

Fig. 5. Diagram of a signal with increasing frequency

Using the same parent wavelet and the same algorithm, we conduct CWT and build the diagram (fig. 6).

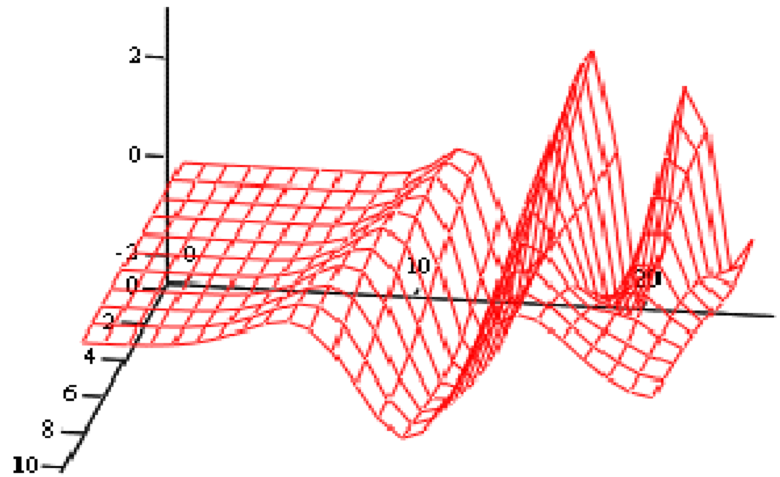

Fig. 6. Diagram of the continuous wavelet transformation of a signal with increasing frequency

Unfortunately, real signals (vibration signals in particular) cannot be represented in analytical form. They come from sensors in the form of sequences of numbers at definite time intervals and are *discrete* by their nature. In such cases *discrete wavelet transformations* (DWT) by means of numerical algorithms are used [3 – 6].

In this paper such algorithms will not be discussed in detail. It should be only noted that the vector of controlled signal and the vector of coefficients corresponding to the definite parent wavelet are used as input signals for them.

Let’s consider the example of a certain discrete signal (fig. 7). Using DWT we determine its matrix of wavelet coefficients.

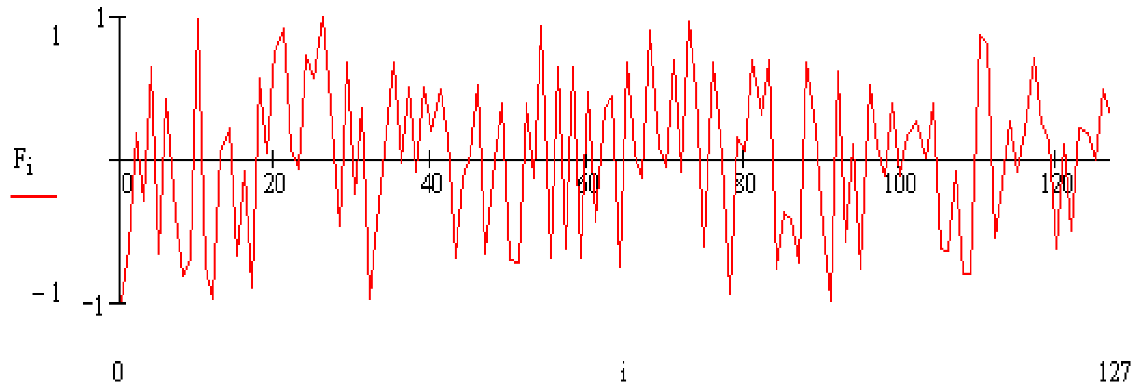

Fig. 7. Diagram of a random signal

Computations are performed in MathCAD environment using built-in function “***Wave***”, that realizes one of the numerical DWT algorithms based on Daubechies parent wavelet.

$$W := \mathrm{wave}(F) \qquad k := 1..6$$

$$\mathrm{coeffs}(\mathrm{level}) := \mathrm{submatrix}\left(W, 2^{\mathrm{level}}, 2^{\mathrm{level}+1} - 1, 0, 0\right)$$

$$C_{i,k} := \mathrm{coeffs}(k)_{\mathrm{floor}\left[\frac{i}{\left(\frac{128}{2^k}\right)}\right]}$$

Diagram of the matrix of C coefficients is shown in fig. 8.

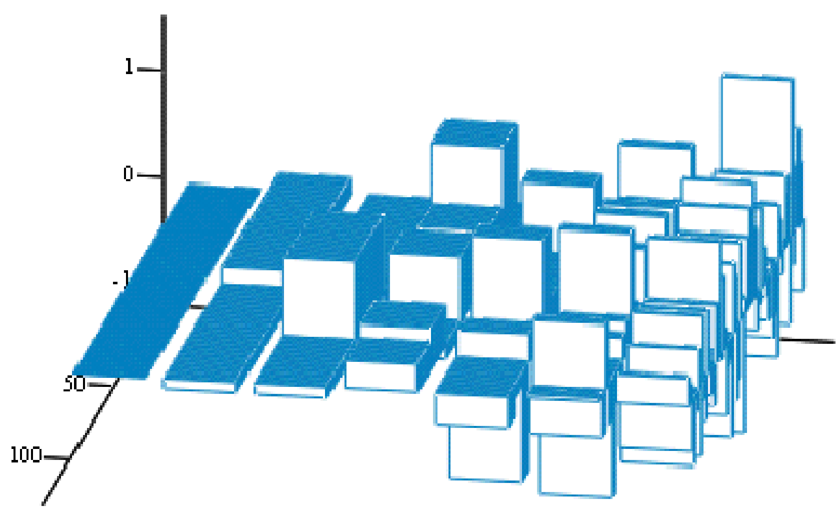

Fig. 8. Diagram of the discrete wavelet transformation of a random signal

From fig. 7 it is evident that we cannot find precise spectrum characteristics of the signal at each moment of time, which is the result of Heisenberg uncertainty principle action. At the same time we know frequency bands in definite time intervals.

Conclusions

1. For the analysis of non-stationary vibration signals it is expedient to use mathematical tools of wavelet transformations;
2. Depending on possible spectrum composition of the vibration signal, the choice of a parent wavelet and numerical algorithm, while using DWT, requires a very responsible attitude.

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