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NUMERICAL METHODS OF SOLVING BOUNDARY PROBLEMS IN FOUNDATION ENGINEERING OBJECTS DESIGN

The paper studies modern numerical methods of solving boundary problems while designing foundation engineering objects. The comparison with classic design in accordance with current specifications is performed. Economically efficient type of foundation constructions is suggested.

Key-words: *stressed-strained state, soils, numerical modeling, boundary element method, finite element method, finite difference method.*

Introduction

The necessity for creating numerical models of the objects with sub-definite information according to the known procedures (mathematical models) with purposeful parameter change makes no doubt today.

In order to find an acceptable variant of engineering solution as to defining the stressed-strained state (SS) of building foundation, comparison of the alternative methods of foundation construction design was performed using the finite difference method (FDM) and elasto-plastic model by the boundary element method (BEM).

BEM uses the superposition principle and, therefore, it can be applied either to linear systems, or to the systems that are linear in terms of the argument increment, or could be approximated as such. So the last category expands the field of BEM application on many engineering problems. In this work BEM was applied to the problems of soil mechanics and foundation engineering problems.

All potentially variable data about the situation, on which the calculation results depended, were adopted as the model parameters. In the first model two parameters of physical-mechanical soil characteristics (E, ρ) were used, the second model is an octa-parametric one (it includes 8 input data of physical-mechanical characteristics of the base: E, υ, *c*, φ, ρ, ρ^{min}, ρ^{max}, P_0).

In the construction diagram development the advantages of the existing modifications were taken into account and their disadvantages were eliminated.

Problem statement, defining relationships

Calculation according to the first model includes finding SS of the maximally loaded section of the shallow foundation.

Classical design of strip foundations as a beam on elastic support involves finding the solution of the differential equation of its equilibrium:

$$\frac{\partial^2}{\partial x^2}\left[EI(x)\frac{\partial^2 w(x)}{\partial x^2}\right]+K_{II}(x)\ w(x)=q(x), \tag{1}$$

where $EI(x)$ – bending stiffness of the beam; E – elasticity module of the beam material; $I(x)$ – momentum of inertia of the beam cross-section; $K_{II}(x)$ – linear coefficient of the base stiffness; x – variable coordinate along the beam length; $w(x)$ – beam deflection in cross-section with x coordinate.

For free support of the beam ends limit conditions will have the form of

$$w''(0)=0;\ \ w'''(0)=0;\ \ w''(L)=0;\ \ w'''(L)=0. \tag{2}$$

In terms of physics stiffness coefficient of the base $K(x)$ is the load in (kN) acting on the unit area (in m^2) and causing unit settling (in m). Its determination involves the consideration of joint work of bases and foundations because soil base settling depends not only on the soil type and its condition, but also on the foundation form and dimensions. In this work the relationship of S. N. Klepikov was used in calculations according to the first model for finding

$K_{II}(x)$ [1]:

$$K_{II}(x) = K(x) \cdot b_n; \tag{3}$$

$$K(x) = p^н / s, \tag{4}$$

where $p^н$ – average pressure along the foundation foot ($p^н = q^н / b_n$); $q^н$ – linear load from external forces acting along the foundation length, the foundation weight and soil at its ledges; s – average settling of the strip foundation according to СНиП (building norms and regulations) 2.02.01–83 [2]. This document “Foundations of buildings and structures” recommends performing calculations of a base settling using layer-by-layer summation method, although it is based on quite arbitrary suppositions.assumptions. In spite of the complicated stressed state of the soil, only normal vertical stresses are taken into account, soil deformability is taken into account only by deformation modulus without lateral expansion coefficient. Yet, this method allows to find lower limit of the base active zone – the foot of the last layer (H_c is used in calculations according to the second model) and to determine acceptable value of settlement s (fig. 1).

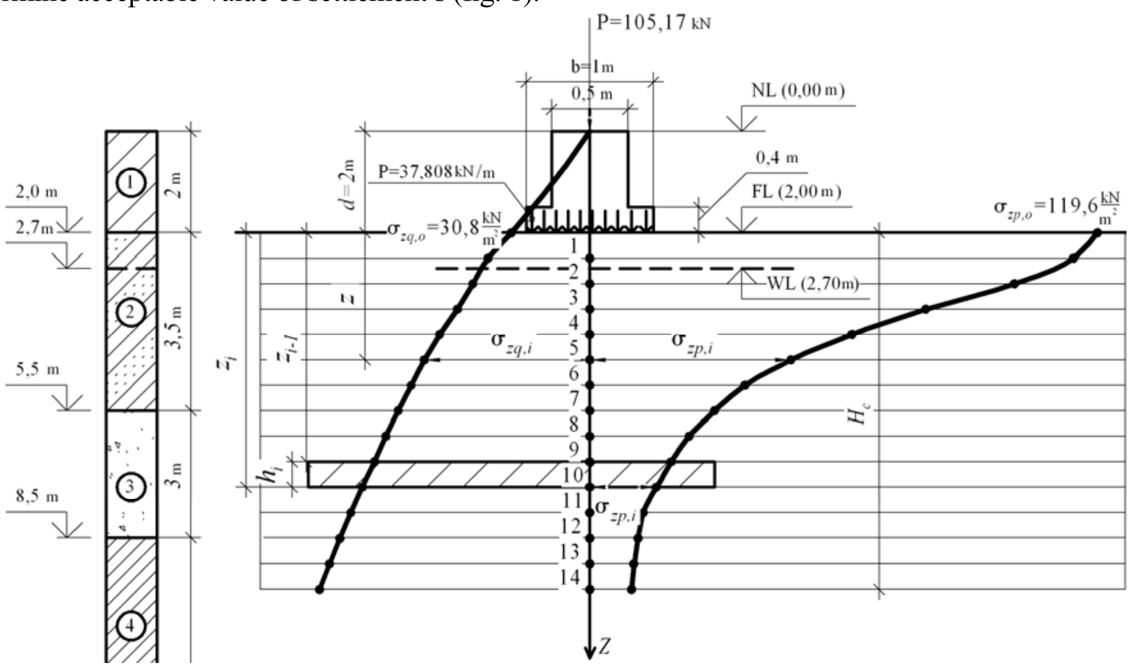

Fig.1. Foundation settling calculation using the method of layer-by-layer summation for finding the base stiffness coefficient. 1 – fill-up soil; 2 – clay sand; 3 – dust-like sand; 4 – clay; b – foundation width; d – foundation laying depth (FL) from natural relief surface (NL); $\sigma_{zq,o}$, $\sigma_{zq,i}$ – vertical stresses from the soil dead weight under the foundation base at the depth z from foundation base (natural pressure of the soil); $\sigma_{zp,o}$, $\sigma_{zp,i}$ – additional vertical weight from external load; H_c – thickness of the compressed zone of the base soil (active zone).

The value of the linear load from external forces $q^н$ is determined on the results of the building design calculations using software complex "ЛІРА 9.4". Finite-element model is presented in fig.2. A beam on elastic support is statistically indefinite problem because the calculations include finding internal forces and displacements of the beam that depend on the geometry of the beam intersections. Therefore, the values of the foundation beam cross-section were previously determined on the basis of the linear profile of the soil reaction pressure on the foot. Preliminary dimensions of he strip are shown in fig.1.

For shallow foundation width b = 1,5 m the expected settling value according to the layer-by-layer

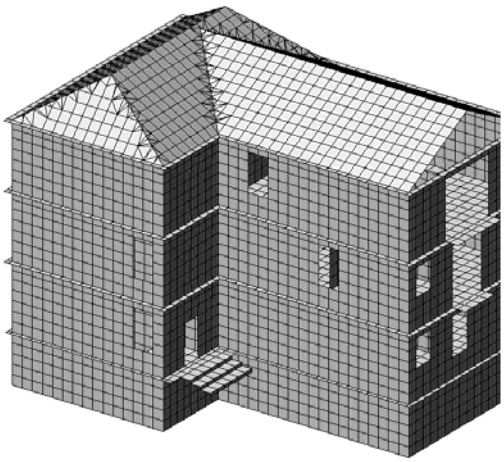

Fig2. Finite - element model

summation method has made s = 1,14 cm. But in order to construct this variant of foundation, 103,6 m^3 are required. For foundation width b = 1 m settling value will be s = 2,018 cm and concrete consumption - 88,8 $м^3$, which is more economical and is adopted in the design.

For numerical realization of the problem according to the first model the region of continuous argument variations (the beam length, Fig.3) was replaced by the discrete multiple number of points – nodes, concentrated forces being located at the boundary of the sections division.

Approximation of the initial differential equation (1) by the finite-difference analogs led to the system of algebraic equations [3] relative to the values of the functions to be found in the nodes of the grid area (linear reactive pressures of the soil).

Calculation of the SS state of the strip on elastic support was performed using the program developed in the algorithmic language Delphi. By the comparison of the calculation results with different numbers of division sections accuracy and stability of the algorithm was evaluated. Results of the strip SS calculations are presented in fig.3.

Non-linear analysis of the soil base was performed according to the elasto-plastic model using numerical method of boundary elements. In the second mathematical model calculation of the foundation plate h=30 cm is performed. Multilayer environment of the soil was considered as quasi-homogeneous isotropic environment with 8 input parameters describing the soil base deformability in the active zone with the depth H_c = 10 m, width of 2 m (fig. 5). Physical parameters of the model state: E = 15,27 MPa, ν = 0,3112, c = 14,406 kPa, φ = 23,47°, ρ = 1,59 t/m^3, ρ_{min} = 1,378 t/m^3, ρ_{max} = 1,978 t/m^3, p_0 = 1700 kPa.

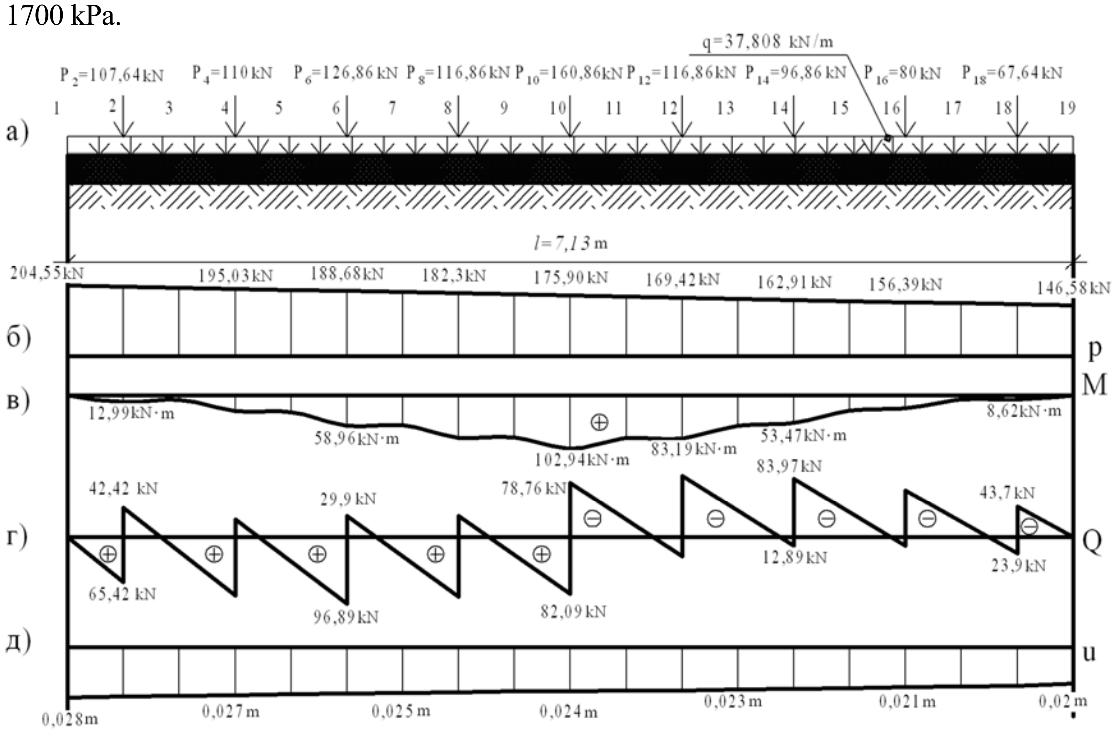

Fig. 3. The example of strip foundation calculation: a) beam discretization; b) profile of the reactive pressures from the load; c) profile of the bending moments; d) profile of the transverse forces; e) settling of the base

While studying the field of stresses and deformations of the “base-foundation” system, the soil was modeled by elasto-plastic body, at the yield point σ – ε relationship was assumed to be linear. Tensor form of representation for the integral limit equilibrium equation, that established the relationship between σ and ε at the boundary of the foundation construction for semi-space, was obtained by K. Brebbia [5]:

$$C_{ij}\dot{U}_j + \int_{\Gamma} p_{ij}^* \dot{U}_{ij}\, d\Gamma = \int_{\Gamma} U_{ij}^* \dot{p}_i\, d\Gamma + \int_{\Omega} \sigma^* \dot{\varepsilon}_{jk}^p\, d\Omega, \tag{5}$$

where the last component contains the integral $d\Omega$ for the soil body area, where plastic deformation is expected to appear; $\dot{\varepsilon}^p$ – vector of plastic deformation; σ^* – derivatives from the fundamental solutions of Mindlin for stresses from P = 1 in the middle of semi-space; p, u – stresses and displacements of the points, basis functions are designated by (*).

Soil behavior at plastic stage was described by incremental theory (non-associated law of plasric flow) where linear relationship between stresses σ and deformation increments $d\dot{\varepsilon}_{ij}$ was adopted

$$d\varepsilon_{ij}^p = d\lambda \frac{dF}{d\sigma_{ij}} \quad , F \neq f, \tag{6}$$

where F – plastic potential (function of the deformation history); $d\lambda$ – scalar coefficient of simple load; σ_{ij} – stress tensor. The criterion of transition to the marginal state f was described by the yield surface of Mizes – Huber – Botkin (fig. 4), that gave relationship between σ_m and σ_i on the octahedral plane, defining the threshold character of plastic deformations:

$$\begin{aligned} f &= \sigma_i + \sigma_m\, tg\psi - \tau_s \qquad for\, \sigma_m \le p_0, \\ f &= \sigma_i + p_0\, tg\psi - \tau_s \qquad for\, \sigma_m > p_0, \end{aligned} \tag{7}$$

where σ_i – stress deviator intensity; σ_m – hydrostatic pressurer; ψ, τ_s – angle of internal friction and cohesion on the octahedral plane.

For numerical realization of the problem lateral surface of the foundation construction and lower contact surface was discretized by linear elements; active zone of the soil base in the foundation vicinity was discretized by triangle cells.

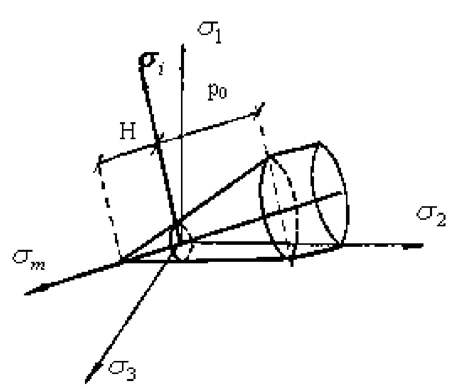

Fig. 4. Yield criterion of Mizes - Huber - Botkin

Discretization scheme and “loading – settling” plots are presented in fig. 5. Building weight, calculated in accordance with current normative documents, has made 5929,025 kN. The expected value of the building settling according to BEM (fig. 5) for plate foundation construction h = 30 cm is 2,7 cm.

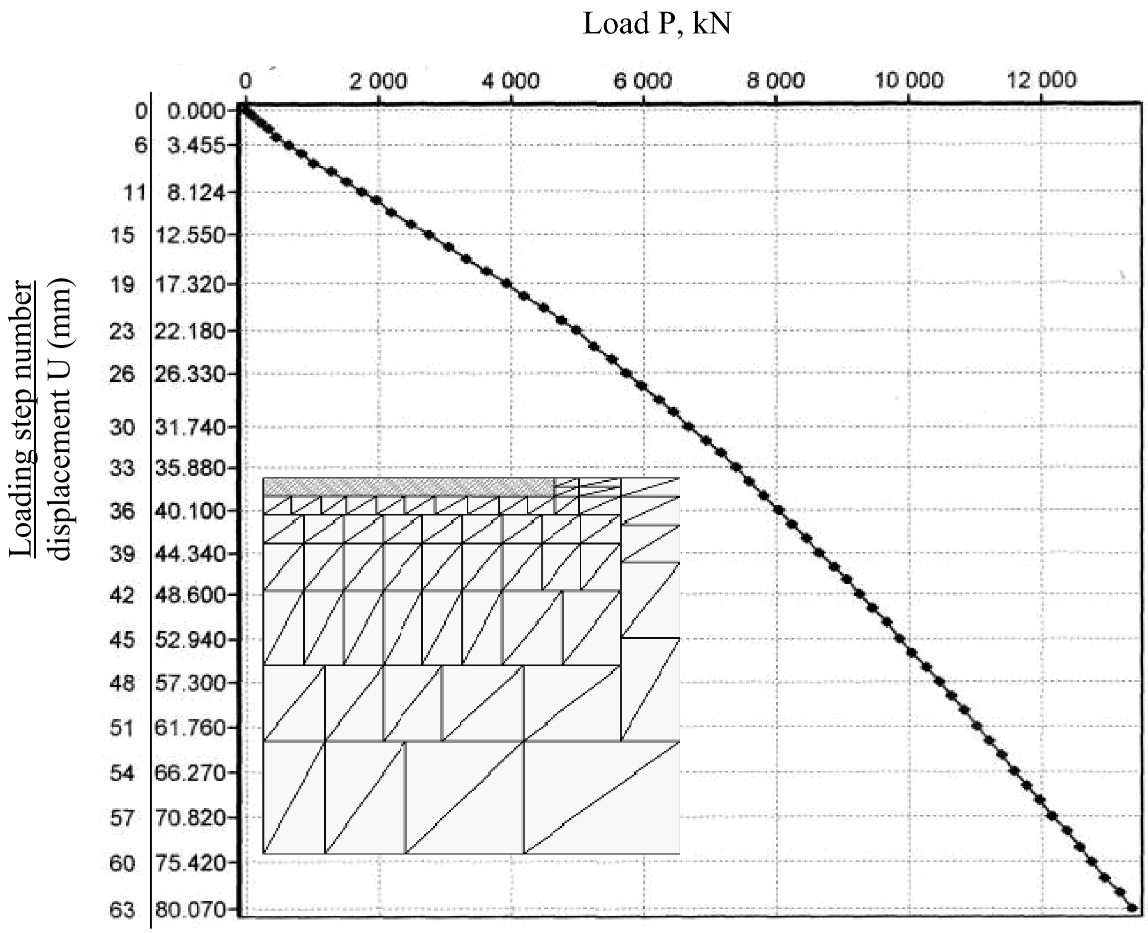

Fig.5. Diagram of "Load – building settlement" dependence on dead weight

Conclusions

1. The building settling values being practically the same (for foundation plate – s=2, 63 cm, for shallow foundations with foot width 1m – s=2,71 cm), concrete consumption for foundation plate erection will be 74% less (88,8 m^3/51,05 m^3 = 1,74). Therefore, foundation plate should be recommended as a foundation construction in terms of economic efficiency.
2. Modern methods of numerical modeling (FDM, FEM and BEM) enable reliable prediction of the stressed-strained state of bases and foundations, which at linear stage corresponds to proven calculation procedures from state normative documents of the construction industry.

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