V. P. Kozhemiako, Dr. Sc. (Eng), Prof.; V. I. Malinovsky

MATHEMATICAL MODEL OF STERAMS DISTRIBUTION IN GEOINFORMATION-POWER NETWORKS

The paper suggests mathematical model of streams distribution for geoinformation - power networks (GIPN) based on queueing theory. Computer simulation of probabilities changes dynamics of information and power requests in MathCAD environment is carried out. The given model enables to define basic characteristics of GIPN and describes processes of information and power components distribution which is rather promising for the problems of networks topology development.

Key words: *geoinformation-power networks (GIPN), request stream, network loading, information-power, probability distribution function.*

Introduction

Geoinformation-power networks (GIPN) [1,2] – are modern high-tech complex hardware-software solution, providing analysis, processing and decision making both in information and power spheres, optimum control and redistribution of power spatially spaced of resources, supported by intelligence systems, situated in local control centres. Information and power in this case are considered in common information- power field [1].

Problem set-up

While development and investigation of geoinformation-power networks topology problems dealing with determination of principle characteristics of basic components of the network emerge : carrying capacity of nodes and transmission channels, average number of users, necessary processing rate of information traffic and volume of power loading of extreme and intermediate segments. More global problems of efficient management of power resources, optimization of information and power flows (definition of shortest possible lengths of information-power channels), improvement of GIPN operation rate and possibility of their functioning in real-time mode also emerge.

Besides, there appear problems of system energy-saving (determination of routes for power fluxes with least possible power losses and application of efficient power saving technologies, for instance, LED-based illuminators), the solutions of which is impossible without obtaining adequate data concerning basic parameters of geoinformation-power network: carrying capacity, computation rate, users inquiries queueing rate, refusal probabilities. For their determination it is necessary to develop universal mathematical model, which takes into consideration distribution indices both of information and power fluxes.

As any network, geoinformation-power networks are described in terms of queueing theory methods. In [4] mathematical tools for process description were suggested, they enable to determine main indices of information component. But for the solution of problems dealing with the description of geoinformation-power network, apart from information parameters of the network it is necessary to define power parameters: power loading, servicing of energy managements inquiries, downtime while inquiry for power commutation, etc.

That is why it is expedient to elaborate model of the network based on queueing theory, describing processes of information-power fluxes distribution in GIPN, and enabling to perform their complex evaluation.

Principles of formation and distribution of information-power fluxes in GIPN

Basic principles of information processing in geoinformation-power systems were defined in [5].The model of data processing method, suggested by the author in [5] is based on parallel-hierarchial Q-transformation [6] while processing and transmission of information fluxes in GIPS, GIPN. In accordance with this model parallel processing and transmission of information is provided by fiber-optic cable from processing centre (centre of GIPN control) to the end node of geoinformation-power network, and in reverse direction. Parallel-hierarchial model of data processing and transmission provides stable coding and high rate data processing, and makes it suitable for application in geoinformation-power environment [5].

These principles correspond to concept of creation of the greater part of known geoinformation-power networks and can be applied as basic distribution model.

Geoinformation-power networks can be considered as multichannel networks with queues, where processes of information and power exchange are represented by means of queueing theory models. Parameters, characterizing power fluxes and GIPN components are information requests for commutation and redistribution of supplied power. Model of the network with queues cannot be applied directly for the description of physical power parameter (current, optical radiation, etc), since it cannot be delayed in time and put into a queue.

As GIPN provides a great number of information-power interconnections, then such GIPN can be classified as the model of multichannel networks of queueing [7]. In such a model n-number of channels is available, these channels receive the stream of requests with the intensity of λ. The intensity of queueing along one channel equals μ, ratio of general and single levels of intensities equals $\rho = \frac{\lambda}{\mu}$ [7]. In such a case, the condition [7] must be satisfied:

$$\lambda_i = \lambda,\ i=0,1,2\ldots;$$

$$\mu_i = \begin{cases} i\mu, & if\ i \le n; \\ n\mu & if\ i > n. \end{cases}$$

Condition $\rho = \frac{\lambda}{\mu} < n$ defines the probability of single request [7]:

$$\begin{cases} p_i = \frac{\rho^i}{i!} p_0, & i < n; \\ p_i = \frac{\rho^n}{n!} \cdot \frac{\rho^{i-n}}{n^{i-n}} p_0 & i > n, \end{cases} \quad (1)$$

where p_0 - is downtime probability of all the channels of GIPN, defined as [7]

$$p_0 = \frac{p^{n+1}}{(n-1)!(n-\rho)^2} p_0 . \quad (2)$$

Average number of occupied channels k_{Sz} [7]:

$$k_{Sz} = \frac{p^{n+1}}{nn!(1-\rho/n)^2} . \quad (3)$$

Average number of requests is defined as ks $=\rho = \frac{\lambda}{\mu}$ [7]. Time of information requests expectation in a queue [7]:

$$t_z = \frac{k_{Sz} + \rho}{\mu n} . \quad (4)$$

Formulas (1) – (4) define main characteristics of GIPN while their evaluation and comparison applying the model of multichannel queueing network.

Mathematical model of fluxes distribution

Proceeding from the statements, presented in [5], we may underline the following points regarding information-power streams in geoinformation-power networks:

Information bonds in GIEN are two-directional (bilateral), whereas power bonds – unidirectional (from electric power station to end and intermediate nodes);

Number of information bonds approximately equals the number of energy bonds (provision of autonomous condition, in accordance with the concept of optic geoinormation-power technologies [8]), except open optic channels, which are vertually of information character.

Information exchange occurs according to the process of two-directional requests queue processing; power exchange occurs according to direction of control command arrival on control units of power distribution.

Based on the above-mentioned, taking into account multichannel MQ model, we can write down the model of creation and distribution of information-power streams in optic geoinformation-power networks.

If n_{inf} – is a number of information channels and n_{pow} – is a numbers of power channels with corresponding requests streams λ_{inf} and λ_{pow} and single intensities μ_{inf} and μ_{pow}, on condition that the number of both information and power channels is far more greater than the number of network nodes i. $\mu_{i_{\inf}} = n_{\inf}\mu$, $\mu_{i_{pow}} = n_{pow}\mu$, on condition that $i > n$. In approximation we assume $\mu_{i_{pow}} = \mu_{i_{\inf}} = \mu_i$. Probability of a single information request (for two-directional information channel with requests density or network loading function $\rho_{\inf} = \frac{2\lambda_{\inf}}{\mu_i}$) may be defined as:

$$p_{i\inf} = \frac{2\lambda_{\inf}{}^{i}}{\mu\cdot 2\mu\cdot\ldots n_{\inf}\mu\cdot(n_{\inf}\mu)^{i-n_{\inf}}}\,p_{0\inf} = \frac{2\rho_{\inf}{}^{i}}{n!\,n^{(i-n_{\inf})}}\,p_{0\inf} = \frac{2\cdot\rho_{\inf}{}^{n_{\inf}}}{n!}\cdot\frac{\rho_{\inf}{}^{i-n_{\inf}}}{n^{i-n_{\inf}}}\,p_{0\inf}\,, \tag{5}$$

where $p_{0\inf}$ – is the probability of downtime of all information channels of GIPN.

Similarly for unidirectional power channel $\rho_{pow} = \frac{\lambda_{pow}}{\mu_i}$

$$p_{i\,pow} = \frac{\lambda_{pow}{}^{i}}{\mu\cdot 2\mu\cdot\ldots n_{pow}\mu\cdot(n_{pow}\mu)^{i-n_{\inf}}}\,p_{0\,pow} = \frac{\rho_{pow}{}^{i}}{n!\,n^{(i-n_{\inf})}}\,p_{0\,pow} = \frac{\rho_{pow}{}^{n_{\inf}}}{n!}\cdot\frac{\rho_{pow}{}^{i-n_{\inf}}}{n^{i-n_{\inf}}}\,p_{0\,pow} \tag{6}$$

The probability of downtime of all information $p_{0\inf}$ and power $p_{0\,pow}$ channels of GIPN:

$$\begin{cases} p_{0\inf} = \dfrac{p_{\inf}{}^{n_{\inf}+1}}{(n_{\inf}-1)!(n_{\inf}-\rho_{\inf})^2}\,p_0 \\ p_{0\,pow} = \dfrac{p_{pow}{}^{n_{pow}+1}}{(n_{pow}-1)!(n_{pow}-\rho_{pow})^2}\,p_0 \end{cases}, \tag{7}$$

where p_0 – is the probability of complete downtime of all GIPN channels, $p_0 = 1 - \left(p_{i\,pow} + p_{i\inf}\right)$, average number of occupied information-power channels k_{Sz}

$$\begin{cases} k_{Sz\inf} = \dfrac{p_{\inf}{}^{n_{\inf}+1}\cdot}{n_{\inf} n_{\inf}!(1-\rho_{inf}/n_{\inf})^2} \\ k_{Sz\,pow} = \dfrac{p_{pow}{}^{n_{pow}+1}\cdot}{n_{pow} n_{pow}!(1-\rho_{pow}/n_{pow})^2} \end{cases}. \tag{8}$$

Average number of requests is defined as: for information stream $k_{inf} = \rho_{\inf}$,for power stream $k_{pow} = \rho_{pow}$. Time of information requests waiting in the queue

$$t_z = \frac{k_{Sz\inf} + \rho_{\inf}}{\mu n_{\inf}}. \tag{9}$$

If we denote by $p_{inf}(t)$ – the function of information streams distribution of GIPN, and by $p_{pow}(t)$ – the function of power streams distribution, which define p_{inf}, p_{pow} – probabilities of their emerging, then for information and power component the model of formation and distribution of streams may be written as:

$$\begin{cases} \dfrac{dp_{\inf}(t)}{dt} = -\left(\lambda_{i\inf} + \mu_i\right) p_{i\inf}(t) + \lambda_{i-1\inf} p_{i-1\inf}(t) + \mu_{i+1} p_{i+1\inf}(t) \\ \dfrac{dp_{pow}(t)}{dt} = -\left(\lambda_{ipow} + \mu_i\right) p_{i\,pow}(t) + \lambda_{i-1\,pow} p_{i-1\,pow}(t) + \mu_{i+1} p_{i+1\,pow}(t) \end{cases}, \tag{10}$$

where $p_{\inf}(t)$, $p_{pow}(t)$ – are the functions of probabilities distribution of requests emergence information and power streams in time t , from control centre of GIPN.

By means of equation (10) at set values of $\lambda_{0\inf}$, λ_{0pow} and μ it is possible to define dynamics of information and power streams distribution in GIPN on the basis of queueing model.

Applying computer-based modeling in MathCAD environment graphs of changes of single information and power requests probabilities on corresponding downtime probabilities $p_{0\inf}$ and p_{0pow} (Fig.1) on condition that information exchange is bilateral with requests density $\rho_{\inf} = {2\lambda_{\inf}}/{\mu_{\inf}} = 0.4,$ at average stream intensity $\mu_{\inf} = 50$ and number of streams $\lambda_{\inf} = 10$; the number of basic information channels being $n_i = 4$, and power channel is unidirectional with request density $\rho_{pow} = {2\lambda_{pow}}/{\mu_{pow}} = 0.25,$ average intensity the stream $\mu_{pow} = 40$, number of streams $\lambda_{pow} = 5$ and number of power channels $n_i = 4$ have been obtained.

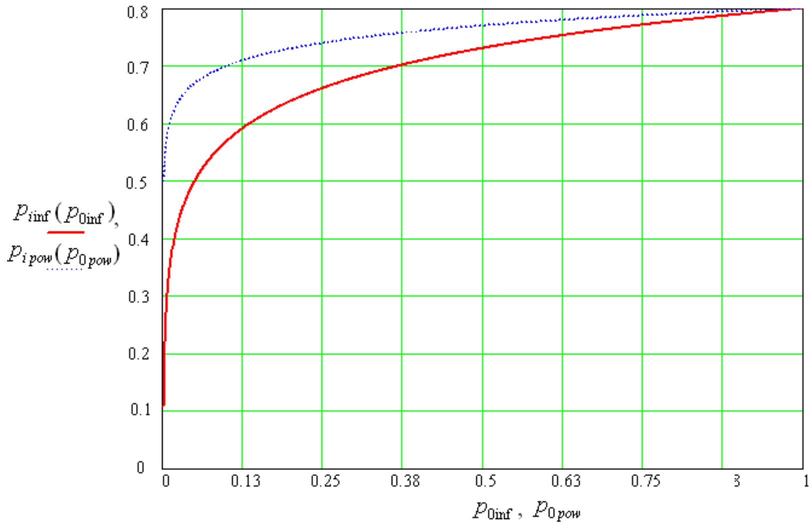

Fig. 1. Graph of changes of single information $p_{i\inf}$ and power $p_{i\,pow}$ requests on probabilities on corresponding downtime probabilities $p_{0\inf}$ and $p_{0\,pow}$

For investigation of changes dynamics of probabilities $p_{i\inf}$ and $p_{i\,pow}$, growth of requests intensity $\lambda_{\inf}$, λ_{pow} can be set by means of graph (Fig.2).

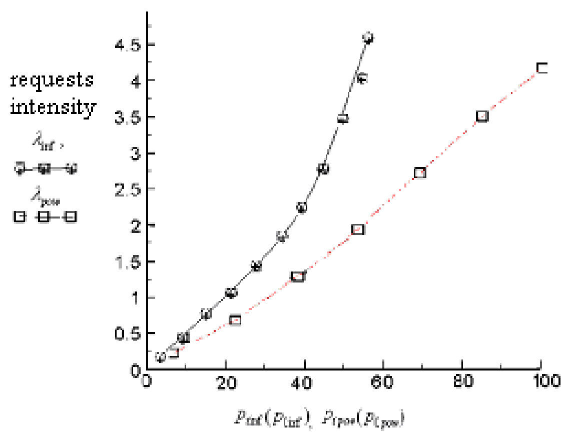

Fig. 2 Graph of requests intensity $\lambda_{\inf}$, λ_{pow} growth

While change of initial conditions of the model, for instance, growth of streams intensity $\mu_{\inf}$ = 70 and number of streams $\lambda_{\inf}$ =20 (for information channels), and μ_{pow} = 60, number of streams λ_{pow} =15 (for power channels) at their simultaneous increase up to n_i = 10, the following dependence of probabilities was obtained (Fig.3).

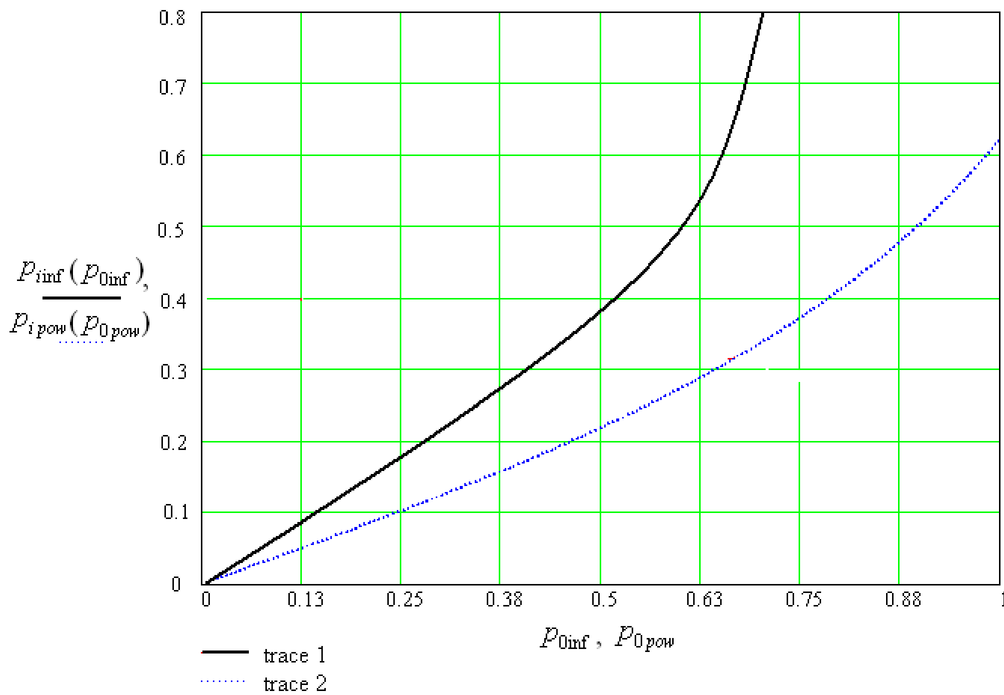

Fig. 3. Change of single information $p_{i\inf}$ and power $p_{i\,pow}$ requests probabilities while increase of model $\mu_{\inf}$, $\lambda_{\inf}$, μ_{pow}, λ_{pow} and n_i parameters

Taking into account that emergence of information-power streams occurs at the moment of time t and in certain number of channels n_{inf} and n_{ipow} of its totality $N_{inf\text{-}pow}= n_{inf} + n_{ipow}$ and determined character of downtime probabilities of all information $p_{0\inf}$ and power $p_{0\,pow}$ channels (Fig.3), the approximation for equation system (10) can be written in the following manner:

$$\begin{cases} \dfrac{dp_{0\inf}(t)}{dt} = -\lambda_{0\inf} p_{0\inf}(t) + \mu\ p_{1\inf}(t) \\ \dfrac{dp_{0\,pow}(t)}{dt} = -\lambda_{0\,pow} p_{0\,pow}(t) + \mu\ p_{1\,pow}(t) \end{cases}. \tag{11}$$

In Fig 3, greater slope of characteristic for information requests $p_{i\inf}$ (upper curve) probability is observed, it is caused by more rapid growth of requests intensities from information nodes of the networks. The number of information requests since in accordance with the principles of streams distribution information exchange in GIPN is two directional (complete duplex organization of bonds), that provides factor 2 in the formula (5).

Average delay while waiting is defined by the formula:

$$T_c = \sum_{i=1}^{V} \left(\lambda_i \big/ \mu \right) \left[\frac{1}{\mu B_i - \lambda_i} \right], \tag{12}$$

where B_i – transmission rate of single channel of network link.

Constraint while calculation execution is inequality performing $0 \le \lambda_i \big/ \mu < B_i$ [9].

For investigation of average delay T_c growth dynamics the formula [9] is applied:

$$dT_c \big/ d\lambda_i / \mu = B_i \big/ f(B_i - \lambda_i / \mu)^2 \qquad i = 1, n, \tag{13}$$

where $f(B_i - \lambda_i / \mu)$ – is the function of network traffic distribution in time.

Dynamics of average delay growth is positive $dT_c \big/ d\lambda_i / \mu > 0$ at all values of $i = \overline{1,n}$.

For solution of the problem of streams distribution optimization in the network GIPN it is expedient to use distribution along the routes of shortest length l_i. For this purpose it is necessary to find the shortest distance between node – source j and node - destination k and sending the stream in this direction. There exist several efficient algorithms for the shortest distances w_{jh} search, one of them is Floyde algorithm [9]. The essence of this method of streams distribution optimization is connected with the comparison of the length with i- th channel, the value it being [9]:

$$l_i \cdot \Delta\, dT_c \big/ d\lambda_i / \mu = B_i \big/ f(B_i - \lambda_i / \mu)^2 \quad , \tag{14}$$

when the stream in the channel equals value λ_i / μ. Length, calculated in such a manner is a linear rate of growth T_c at infinitely small increase of stream in the channel. Such lengths can be applied for streams searching along the shortest routes. Quality index, used for evaluation of GIPN networks loading dynamics can be averaged functions of information $f_{inf}(t,\lambda)$ and power $f_{pow}(t,\lambda)$ streams distribution (Fig. 4) which characterize the dynamics of GIPN growth in time and are defined by the integer of information – power streams distribution function:

$$f_{\inf}(t,\lambda) = -\int_t^{\Delta t} \lambda_{0\inf} p_{0\inf}(t,\lambda)dt$$
$$f_{pow}(t,\lambda) = -\int_t^{\Delta t} \lambda_{0pow} p_{0pow}(t,\lambda)dt \quad , \tag{15}$$

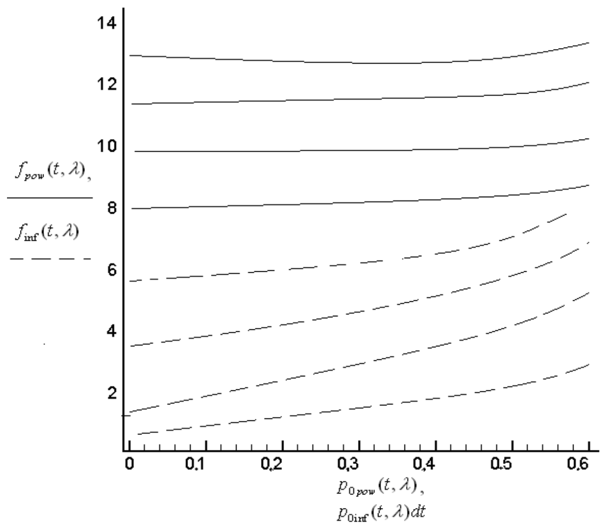

Fig. 4. Graph of information $f_{inf}(t,\lambda)$ and power $f_{pow}(t,\lambda)$ streams distribution function

Analysing the character of curves changes for functions of information $f_{inf}(t,\lambda)$ and power $f_{pow}(t,\lambda)$ streams distribution (Fig. 4) we can observe the dynamics of information streams distribution function growth is greater than $f_{inf}(t,\lambda)$ (family of 4 curves, Fig. 4, below), it is proved by the results obtained in Fig. 3, and testify that intensities of information components of GIPS are more directed and of dynamic character.

Parallel growth of distribution function of energy $f_{pow}(t,\lambda)$ GIPS streams shows the growth of energy load from the side of t erminal devices, which initialize information requests. It is valid since growth of information load and number of channels is impossible without growth of energy load and the number of channels is impossible without growth of energy load. That is why we can state, that the greatest loading. Structures of all known GIPS are hierarchial character i. e. all information – power structures are directed to the centre of control which is the central node of the network.

That is why, we can state that the greatest loading will be in the segment of control centre, since intensities of steams $\lambda_{0\inf}$, λ_{0pow} and $p_{0\inf}(t,\lambda)$, $p_{0pow}(t,\lambda)$ - are functions of probabilities distribution in this segment, which, for conveniences sake, is better to set either by exponential $p_{0\inf}(t,\lambda) = 1 - e^{-\lambda t}$ [7], or by Poisons distribution law point $p_{0\inf}(t,\lambda) = \dfrac{(\lambda t)^i e^{-\lambda t}}{i!}$ [7].Functions of information – power streams distribution will have the greatest values:

$$f_{\inf}(t,\lambda) \to f_{\inf}(t,\lambda)_{\max} \quad f_{pow}(t,\lambda) \to f_{pow}(t,\lambda)_{\max} . \qquad (16)$$

Fig 5 shows the increment of distribution functions of information – power streams.

Fig 5. Graph of distribution function increment of information – power streams.

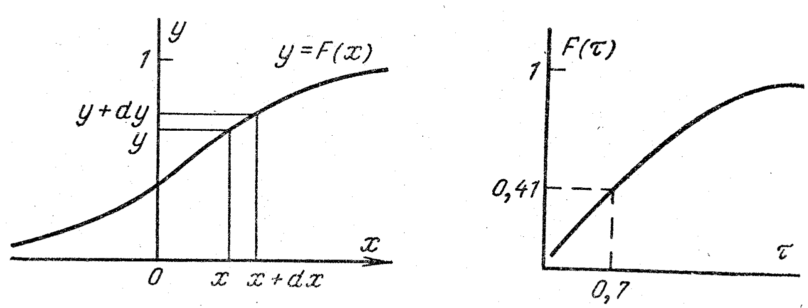

Fig 5. Graph of distribution function increment of information – power streams.

Maxima $f_{\inf}(t,\lambda)_{\max}$, $f_{pow}(t,\lambda)_{\max}$ will characterize the state of load as being critical for GIPS network/ Taking this into account, it is necessary to use in these segments fast – rate trunk lines based on binary semiconductor, which will provide both the highest power and information carrying capacity and high rate of information transmission, which considerably reduces delay time and, correspondingly, decreases the loading of the networks.

Conclusions

The problem of definition of streams character in geoinformation – power networks requires the solution not only from the point of view of information component but power component too. The suggested mathematical model allows, applying probabilistic methods to determine such indices of GIPS as loading average time of request queueing, average number of occupied information – power channels, downtime probability. The paper contains mathematical model of information – power streams distribution, based on networks queueing theory, graphical dependencies for basic dynamic characteristics of GIPS are obtained by means of computer simulation in MattCad environment. Application of the given model for design and scaling of geoinformation – power networks is rather perspective.

REFERENCES

1. Оптико–електронна геоінформаційно-енергетична система тотального тестування і оптимального управління науково-освітянськими і бібліотечними ресурсами для створення і розвитку централізованої бази знань / В. П. Кожем’яко, О. Г. Домбровський, І. Д. Івасюк, О. В. Шевченко, С. В. Дусанюк, С. С. Білан, А. В. Кожем’яко // Оптико-електронні інформаційно-енергетичні технології. – 2005. – № 1 (9). – С. 5-11. – ISSN 1681-7893.

2. Кожем’яко В. П. Оптико-електронна геоінформаційно-енергетична система управління інфраструктурою регіону / В. П. Кожем’яко, О. А. Бойко // Оптико-електронні інформаційно-енергетичні технології. – 2007. –№ 1 (13) . – С. 176 – 180. – ISSN 1681-7893.

3. Кожем’яко В. П. Оптико-електронна геоінформаційно-енергетична система біомедичного призначення / В. П. Кожем’яко, С. В. Павлов, О. В. Шевченко, В. В. Дмитрук // Оптико-електронні інформаційно-енергетичні технології. – 2006. № 2 (12). – С. 192 – 196.

4. Кожем’яко В. П. Методи моделювання геоінформаційно–енергетичної системи тотального тестування і оптимального управління науково-освітянськими і бібліотечними ресурсами / В. П. Кожем’яко, С. В. Дусанюк, Л. О. Волонтир // Оптико-електронні інформаційно-енергетичні технології. – 2007. – №1 (13). – С. 169 – 175. – ISSN 1681-7893.

5. Шевченко О. В. Паралельно-ієрархічні методи передачі та обробки інформації у автоматизованій геоінформаційно-енергетичній системі: Дис. канд. техн. наук: 05.13.16 / Шевченко Ольга Вікторівна. – К., 2007. – 168с.

6. Кожем’яко В. П. Паралельно-ієрархічні мережі як структурно-функціональний базис для побудови спеціалізованих моделей образного комп’ютера ; [Монографія] / В. П. Кожем’ яко, Л. І. Тимченко, А. А. Яровий. – Вінниця: УНІВЕРСУМ, 2005. – 160 с.

7. Коршунов Ю. М. Математические основы кибернетики: Учебное пособие для вузов 2-е изд. / Ю.М. Коршунов. – М.: Энергия, 1980. – 424 с.

8. Кожем’яко В. П. Принципи побудови та структурна організація каналів для повністю оптичних геоінформаційно-енергетичних мереж / В. П. Кожем’яко, В. І. Маліновський // Вісник Вінницького політехнічного інституту. – Вінниця, ВНТУ. – 2008. – № 1. – С. 95 – 101.

9. Васильев В. И. Системы связи: Учебное пособие для втузов / В. И. Васильев, А. П. Буркин, В. А. Свириденко. – М.: Высшая школа, 1987. – 280 с.

Kozhemiako Volodymyr – Honored scientist of Ukraine, Academician of the Academy, of Engineering Sciences of Ukraine, Doctor of Sc. (Eng), Professor, Head of the Department of Laser and Optoelectronic Engineering.

Malinovskiy Vadym – Post-graduate, engineer of the Department of Laser and Optoelectronic Engineering,

Vinnytsia National Technical University.