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HARMONIC LINEARIZATION OF AUTOMATIC CONTROL SYSTEMS, CONTROLLED BY FINITE AUTOMATON UNDER THE IMPACT OF PARAMETRIC PERTURBATIONS

Analytical dependences, which allow to perform harmonic linearization of systems, controlled by a finite automaton, under the impact of parametric perturbations, which lead to change, over time, of the parameters of nonlinear elements are determined.

Keywords: *harmonic linearization, nonlinear non-stationary systems, parametric perturbations.*

Problem set - up

Increased requirements for quality, accuracy and reliability of modern systems of automatic control of complex objects, parameters of which vary in the process of operation, led to the need for the development and application of new management principles, including the use of logical control laws. Therefore, in such systems logic control devices are widely used [1]. They are applied in control systems of aircraft, industrial processes, etc. Systems of this class retain the advantages of relay systems, namely: high performance, simple structure, high power gain [2]. Logic control at the right time changes the level and sign of the control action that can significantly reduce the amplitude of oscillations. In the systems of the second class (with pulsed-relay logic control), depending on the magnitude of deviation, pulse or relay modes appear. Pulse mode occurs with the growth of the absolute value of deviation, when signs of deviation and rate of change coincide [1, 2]. The behavior of this class of systems is analyzed by known methods [1, 2], which can not be used when their parameters change. However, in real conditions of such systems operation their parameters often vary under the influence of parametric perturbations. Therefore the problem of parameters change accounting that would allow to use known methods of analysis of this class of systems behaviour emerges.

Analysis of recent research and publications

To investigate the above-mentioned classes of systems in engineering practice the method of harmonic linearization [3] is applied due to its simplicity and efficiency. Condition of its application is the availability of filtering properties of the given linear part of nonlinear system. This method can be used in the study of systems with non-filtering linear part at a two-frequency and multifrequency input signal of nonlinearity [4]. The results obtained allow us to study the system at constant parameters of their nonlinear elements, while a significant number of systems contain nonlinear elements, built with the help of electronic circuits, whose parameters vary under the influence of external uncontrollable parametric perturbations (temperature, humidity). Over time, this leads to the change of nonlinear systems parameters. In the existing publications the problems of harmonic linearization of systems under the change of automatic control systems (ACS), controlled by a finite automaton are not solved.

Purpose of research- to obtain analytical relations, which will allow to perform harmonic linearization of systems, controlled by a finite automaton, exposed to parametric perturbations, which eventually lead to the change of nonlinear elements parameters. Thus, the basic assumptions of the classical method of harmonic linearization of the filtering properties of the linear part are proved.

Main part

We solve the problem of determination the coefficients of harmonic linearization coefficients for automatic system controlled by a finite automaton, functional diagram of which is shown in Fig. 1 [5].

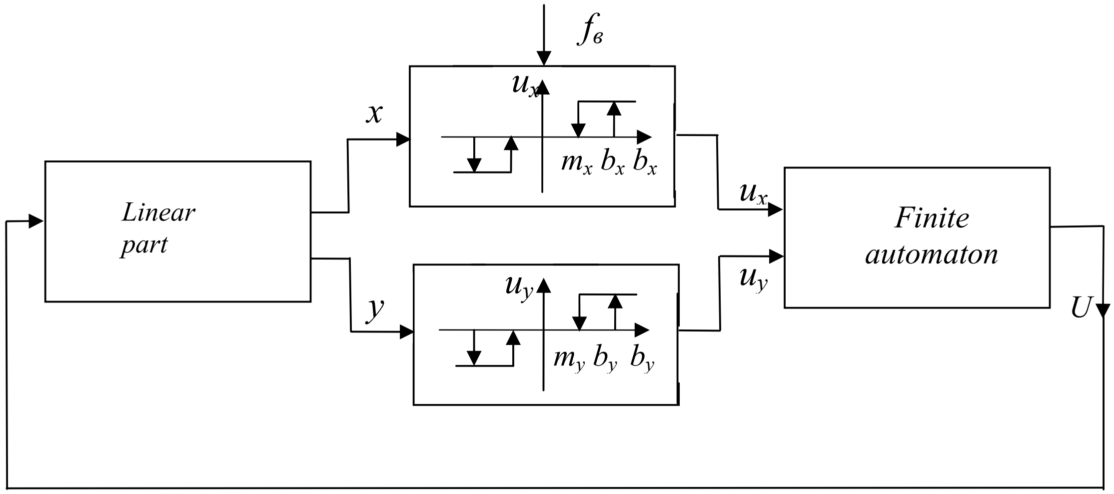

Fig.1. Functional diagram of ACS, controlled by a finite automaton

If hysteresis is available in relay elements, output signals U_x and U_y are determined not only by the values of control signals X, Y, but also by the sign of their derivatives $\dot{X}, \dot{Y}$. In this case, the linearized equation of logic control device with re4lay converter will have the form [5]:

If there are elements of the relay hysteresis, output signals and are determined not only by the values of control signals but also a sign of their derivatives in this case, the linearized equation of logic control device with relay converter will have the form [5]:

$$F(x,\dot{x},y,\dot{y}) = \left(\frac{c_{10}}{A_y} + \frac{a_{10}}{A_y\omega}\cdot p\right)y + \left(\frac{b_{01}}{A_x} + \frac{a_{01}}{A_y\omega}\right)\cdot x, \tag{1}$$

where A_x, A_y – are amplitudes of the input signals X and Y, respectively, ω is frequency of input signals, p is Laplace operator.

We define the coefficients c_{10}, a_{10}, b_{01}, a_{01} and, accordingly, the form of the expression (1), provided that the parameters of relay converters (input signal of relay elements and the width of the dead zone) varies linearly. These laws are typical if the system is influenced by such parametric perturbations, as: temperature, humidity, dust. The values of the coefficients are determined by decomposition of logic function describing finite automaton with relay converters into a double Fourier series using the relations [5], which are determined under the conditions that the parameters of relay converters vary linearly:

$$a_{10} = \frac{1}{2\pi^2}\iint_{2\pi} F(x,\dot{x},y,\dot{y})\cos\Psi_y d\Psi_y d\Psi_x = \frac{1}{\pi}\int_0^{2\pi} F(x,\dot{x},y,\dot{y})\cos\Psi_y d\Psi_y, \tag{2}$$

$$a_{01} = \frac{1}{2\pi^2}\iint_{2\pi} F(x,\dot{x},y,\dot{y})\cos\Psi_x d\Psi_y d\Psi_x = \frac{1}{\pi}\int_0^{2\pi} F(x,\dot{x},y,\dot{y})\cos\Psi_x d\Psi_x, \tag{3}$$

$$b_{01} = \frac{1}{2\pi^2}\iint_{2\pi} F(x,\dot{x},y,\dot{y})\sin\Psi_x d\Psi_x d\Psi_y = \frac{1}{\pi}\int_0^{2\pi} F(x,\dot{x},y,\dot{y})\sin\Psi_x d\Psi_x, \tag{4}$$

$$c_{10} = \frac{1}{2\pi^2} \iint\limits_{2\pi} F(x, \dot{x}, y, \dot{y}) \sin \Psi_y d\Psi_y \Psi d\Psi_x = \frac{1}{\pi} \int\limits_{0}^{2\pi} F(x, \dot{x}, y, \dot{y}) \sin \Psi_y d\Psi_y , \qquad (5)$$

where $\Psi_y = \omega t$, $\Psi_x = \omega t + \varphi$, a $\omega -$ frequency of signal Y(t), $\varphi -$ phase shift between the signals

Y(t), X(t).

To solve this problem we give a graphical representation of signals change at the output of a finite automaton, provided that the inputs of the relay links with hysteresis obtain harmonic signals

X(t), Y(t) [5].

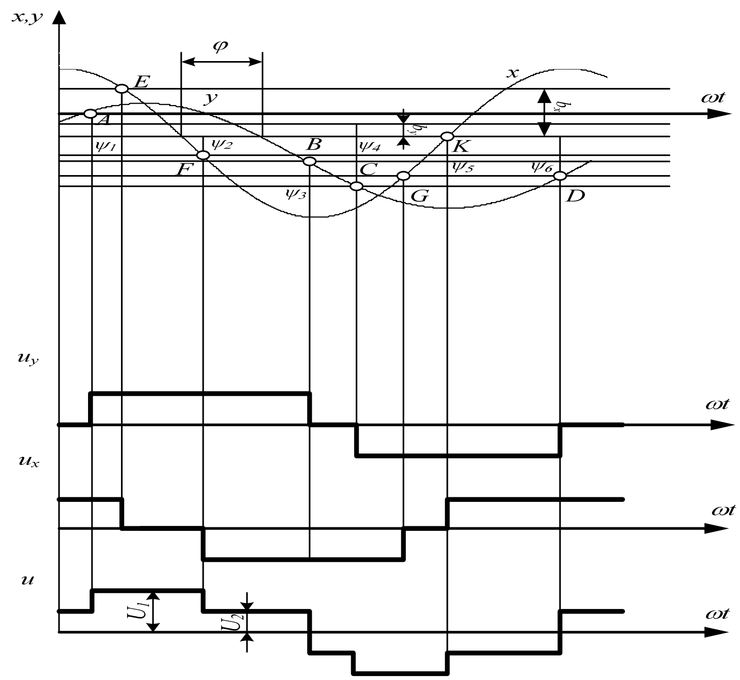

Fig. 2. Signals change in logic control device at self-oscillations

As it is shown in [5], for the preset logic of the logic control device operation, shown in Table 1, and phase shift between input signals, which satisfies the inequality

$0 < \varphi < 90^0$, $b_{01} = c_{10}$ and $a_{01} = a_{10}$. We emphasize that in the Table 1 U_1, $U_2 -$ values of the control signal U .

Table 1

U_x	0	0	0	+1	+1	+1	-1	-1	-1
U_y	0	+1	-1	0	+1	-1	0	+1	-1
U_1	0	1	-1	0	1	0	0	0	-1
U_2	0	0	0	1	0	-1	-1	1	0

As it follows from Fig. 2, automatic switching occurs at points A, B, C, D and E, F, G, K. To determine the coefficients of harmonic linearization we will determine the coordinates of points A, F, B, C, K, D. The coordinates of these points are determined by relations $\psi_1 = \arcsin \frac{b_y}{A_y}$,

$\psi_2 = \pi - \varphi + \arcsin\frac{b_x}{A_x}$, $\quad \psi_3 = \pi - \arcsin\frac{b_y}{A_y}$, $\quad \psi_4 = \pi + \psi_1$, $\quad \psi_5 = 2\pi - \varphi + \arcsin\frac{b_x}{A_x}$,

$\psi_6 = 2\pi - \arcsin\frac{b_y}{A_y}$, respectively.

In [6] it is defined that under the influence of such parametric perturbations as: temperature, dust, humidity-the parameters of output signal of relay link with hysteresis B and width of the dead band C will change linearly

$$B(t) = b_0 \pm bt, b << b_0, \tag{6}$$

$$C(t) = c_0 \pm ct, c << c_0. \tag{7}$$

Using Fig. 2 and (6), (7), the integral (2) is written as:

$$\begin{aligned} a_{10} &= \frac{1}{2\pi^2}\iint_{2\pi} F(x,\dot{x},y,\dot{y})\cos\Psi_y d\Psi_y d\Psi_x = \frac{1}{\pi}\int_0^{2\pi} F(x,\dot{x},y,\dot{y})\cos\Psi_y d\Psi_y = \int_0^{\psi_1}(U_{20} \pm U_2 t)\cos\psi d\psi + \\ &+ \int_{\psi_1}^{\psi_2}(U_{10} \pm U_1 t)\cos\psi d\psi + \int_{\psi_2}^{\psi_3}(U_{20} \pm U_2 t)\cos\psi d\psi + \int_{\psi_3}^{\psi_4}(-U_{20} \pm U_2 t)\cos\psi d\psi + \int_{\psi_4}^{\psi_5}(-U_{10} \pm U_1 t)\cos\psi d\psi + \\ &+ \int_{\psi_5}^{\psi_6}(-U_{20} \pm U_2 t)\cos\psi d\psi + \int_{\psi_6}^{2\pi}(U_{20} \pm U_2 t)\cos\psi d\psi. \end{aligned} \tag{8}$$

If we find separately each of the integrals, which is included in equation (8), we find the final expression for the coefficient a_{10}.

$$\begin{aligned} a_{10} &= U_{20}\frac{b_y}{A_y} \pm \frac{U_2}{\omega}[\frac{b_y}{A_y}(\arcsin\frac{b_y}{A_y}) + \sqrt{1-(\frac{b_y}{A_y})^2} - 1] + U_{10}(\sin\varphi\sqrt{1-\frac{b_x^2}{A_x^2}} - \cos\varphi\frac{b_x}{A_x}) - U_{10}\frac{b_y}{A_y} \pm \\ &\pm \frac{U_1}{\omega}[(\pi - \varphi + \arcsin\frac{b_x}{A_x})(\sin\varphi\sqrt{1-\frac{b_x^2}{A_x^2}} - \cos\frac{b_x}{A_x} + (\cos\varphi)\sqrt{1-\frac{b_x^2}{A_x^2}} + \frac{b_x}{A_x}\sin\varphi - (\arcsin\frac{b_y}{A_y})(\frac{b_y}{A_y}) - \\ &- \sqrt{1-\frac{b_y^2}{A_y^2}}] - U_{20}\frac{b_y}{A_y} - U_{20}(\sin\varphi\sqrt{1-\frac{b_x^2}{A_x^2}} - \frac{b_x}{A_x}\cos\varphi) \pm \frac{U_2}{\omega}[(\pi - \arcsin\frac{b_y}{A_y})\frac{b_y}{A_y} - \sqrt{1-\frac{b_y^2}{A_y^2}} - \\ &- (\pi - \varphi + \arcsin\frac{b_x}{A_x})(\sin\varphi\sqrt{1-\frac{b_x^2}{A_x^2}} - \frac{b_x}{A_x}\cos\varphi) - (\sqrt{1-\frac{b_x^2}{A_x^2}}\cos\varphi + \frac{b_x}{A_x}\sin\varphi) \pm \frac{U_2}{\omega}[-(\pi + \arcsin\frac{b_y}{A_y})\frac{b_y}{A_y} - \\ &- \sqrt{1-\frac{b_y^2}{A_y^2}} - (\pi - \arcsin\frac{b_y}{A_y})\frac{b_y}{A_y} + \sqrt{1+\frac{b_y^2}{A_y^2}}] + U_{10}(\sin\varphi\sqrt{1-\frac{b_x^2}{A_x^2}} - \frac{b_x}{A_x}\cos\varphi) - U_{10}\frac{b_y}{A_y} \pm \\ &\pm \frac{U_1}{\omega}[(2\pi - \varphi + \arcsin\frac{b_x}{A_x})(\frac{b_x}{A_x}\cos\varphi - \sin\varphi\sqrt{1-\frac{b_x^2}{A_x^2}}) + (\cos\varphi\sqrt{1-\frac{b_x^2}{A_x^2}} + \frac{b_x}{A_x}\sin\varphi) + (\pi + \arcsin\frac{b_y}{A_y})\frac{b_y}{A_y} + \\ &+ \sqrt{1-\frac{b_y^2}{A_y^2}} + U_{20}\frac{b_x}{A_x} - U_{20}(\sqrt{1-\frac{b_x^2}{A_x^2}}\sin\varphi - \frac{b_x}{A_x}\cos\varphi) \pm \frac{U_2}{\omega}[-(2\pi - \arcsin\frac{b_y}{A_y})\frac{b_y}{A_y} + \sqrt{1-\frac{b_y^2}{A_y^2}} - \\ &- (2\pi - \varphi + \arcsin\frac{b_x}{A_x})(\sqrt{1-\frac{b_x^2}{A_x^2}}\sin\varphi - \frac{b_x}{A_x}\cos\varphi) + (\sqrt{1-\frac{b_x^2}{A_x^2}}\cos\varphi + \frac{b_x}{A_x}\sin\varphi)] + U_{20}\frac{b_y}{A_y} \pm \frac{U_2}{\omega}[1 + \\ &+ (2\pi - \arcsin\frac{b_y}{A_y})(\frac{b_y}{A_y}) - \sqrt{1-\frac{b_y^2}{A_y^2}}]. \end{aligned}$$

Similarly, we can determine the coefficient c_{10}.

$$
\begin{aligned}
c_{10} = & -U_{20}\sqrt{1-\left(\frac{b_y}{A_y}\right)^2} + U_{20} \pm \frac{U_2}{\omega}\left[-\arcsin\left(\frac{b_y}{A_y}\right)\sqrt{1-\left(\frac{b_y}{A_y}\right)^2} + \left(\frac{b_y}{A_y}\right)\right] + \\
& + U_{10}\left(\sqrt{1-\frac{b_x^2}{A_x^2}}\cos\varphi + \frac{b_x}{A_x}\sin\varphi\right) + U_{10}\sqrt{1-\frac{b_y^2}{A_y^2}} \pm \frac{U_1}{\omega}\left[\left(\pi-\varphi+\arcsin\frac{b_x}{A_x}\right)\left(\sqrt{1-\frac{b_x^2}{A_x^2}}\cos\varphi + \frac{b_x}{A_x}\sin\varphi\right.\right. + \\
& + \left(\sqrt{1-\frac{b_x^2}{A_x^2}}\sin\varphi - \frac{b_x}{A_x}\cos\varphi\right) + \left(\arcsin\frac{b_y}{A_y}\sqrt{1-\frac{b_y^2}{A_y^2}}\right) - \frac{b_y}{A_y} + U_{20}\sqrt{1-\frac{b_y^2}{A_y^2}} - U_{20}\left(\cos\varphi\sqrt{1-\frac{b_x^2}{A_x^2}}\right. + \\
& + \left.\frac{b_x}{A_x}\sin\varphi\right) \pm \frac{U_2}{\omega}\left[\left(\pi-\arcsin\frac{b_y}{A_y}\right)\sqrt{1-\frac{b_y^2}{A_y^2}} + \frac{b_y}{A_y} + \left(\pi-\varphi+\arcsin\frac{b_x}{A_x}\right)\left(\sqrt{1-\frac{b_x^2}{A_x^2}}\cos\varphi + \frac{b_x}{A_x}\sin\varphi\right)\right. - \\
& - \left.\left(\sqrt{1-\frac{b_x^2}{A_x^2}}\sin\varphi - \frac{b_x}{A_x}\cos\varphi\right)\right] \pm \frac{U_2}{\omega}\left[-2\frac{b_y}{A_y} + 2\left(\arcsin\frac{b_y}{A_y}\right)\sqrt{1-\frac{b_y^2}{A_y^2}}\right] + U_{10}\left(\sqrt{1-\frac{b_x^2}{A_x^2}}\cos\varphi + \frac{b_x}{A_x}\sin\varphi\right) + \\
& + U_{10}\sqrt{1-\frac{b_y^2}{A_y^2}} \pm \frac{U_{10}}{\omega}\left[-\left(2\pi-\varphi+\arcsin\frac{b_x}{A_x}\right)\left(\sqrt{1-\frac{b_x^2}{A_x^2}}\cos\varphi + \frac{b_x}{A_x}\sin\varphi\right) - \left(\sqrt{1-\frac{b_x^2}{A_x^2}}\sin\varphi - \frac{b_x}{A_x}\cos\varphi\right)\right. - \\
& - \left.\left(\pi+\arcsin\frac{b_y}{A_y}\right)\sqrt{1-\frac{b_y^2}{A_y^2}} - \frac{b_y}{A_y}\right] + U_{20}\sqrt{1-\frac{b_y^2}{A_y^2}} - U_{20}\left(\sqrt{1-\frac{b_x^2}{A_x^2}}\cos\varphi + \frac{b_x}{A_x}\sin\varphi\right) \pm \\
& \pm \frac{U_2}{\omega}\left[-\left(2\pi-\arcsin\frac{b_y}{A_y}\right)\sqrt{1-\frac{b_y^2}{A_y^2}} - \frac{b_y}{A_y} + \left(2\pi-\varphi+\arcsin\frac{b_x}{A_x}\right)\left(\sqrt{1-\frac{b_x^2}{A_x^2}}\cos\varphi + \frac{b_x}{A_x}\sin\varphi\right)\right. + \\
& + \left.\left(\sqrt{1-\frac{b_x^2}{A_x^2}}\sin\varphi - \frac{b_x}{A_x}\cos\varphi\right)\right] - U_{20} + U_{20}\sqrt{1-\frac{b_y^2}{A_y^2}} \pm \frac{U_2}{\omega}\left[(-2\pi) + \left(2\pi-\arcsin\frac{b_y}{A_y}\right)\sqrt{1-\frac{b_y^2}{A_y^2}} - \frac{b_y}{A_y}\right].
\end{aligned}
$$

With the coefficients defined it is possible to obtain the linearized equation of logical control device with relay converter (see equation (1)). Unlike the known values of the above-mentioned coefficients [5], they depend on the amplitude of the input signals A_y A_x, the frequency of the input signals ω, phase shift between input signals X and Y,values of the parameters changes of the output signal and the width of the dead zone of the relay converters.

Conclusion

Coefficients of harmonic linearization of nonlinear part of control systems, which are governed by a finite automaton under the impact of uncontrolled parametric perturbations were defined. The above- mentioned factors are found by decomposition of logic function, describing the finite automaton with relay signal converter, in double Fourier series. By means of defined coefficients we can determine the parameters of self-oscillations of the considered class of systems.

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