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SYSTEM OF GENERALIZED CONVERTER OF INERTIA MOMENT, ITS CHARACTERISTICS AND PROPERTIES

The paper considers the theory of measuring conversion of inertia moment of mechanical and electromechanical systems. The concept of generalized converter of inertia moment is introduced, formulated and justified its characteristics and properties are formulated and justified, mathematical model is presented.

Key words: *moment of inertia, measurement conversion, mathematical model, generalized coordinates, Lagrange's equations.*

1. Introduction

The survey of modern state-of-art of moment of inertia measuring converters and analysis of their theoretical fundamentals [1] shows the absence of common approach both to development of the methods for measuring the moment of inertia, and to creation of their mathematical models. Moreover, due to its structure, scientific theory must be a complete and internally differentiated system of hierarchially generalized, logically compatible concepts, laws and regulations, there exist all prerequisites to state that as the system of generalized concepts the theory of converters of moment of inertia today is still in non completed state- and requires further development. In [1 - 6] generalized mathematical models of moment of inertia converters with one (1 st and 2 nd order) and two (3rd order) degrees of freedom are elaborated and suggested. This allowed to systematize the known systems and methods of moment of inertia conversion and created necessary prerequisites for the development of new methods of conversion with improved metrological characteristics. However, such issues remained unsolved. First, mathematical model of generalized converter of moment of inertia of random order (with n degrees of freedom), which should be the logical culmination of generalization process itself is not developed. Secondly Lagrange equations of second order were put in the base of known mathematical models of moment of inertia converters. However, this was done axiomatically, without needed theoretical justification. Thirdly, created mathematical models were limited only to mechanical systems, neglecting electromechanical systems. These issues are important and their solution is possible due to the introduction of abstract measuring device - generalized converter of moment of inertia (GCMI) [7, 8]. According to the author, it enables to provide general theoretical fundamentals of moment of inertia conversion process, and partly solve the problem of the lack of general theory of moment of inertia converters. During the theoretical study of mechanical and electromechanical systems of GCMI by mathematical simulation using the variation principles of analytical mechanics their mathematical models were obtained, which also have generalization feature. In this connection, it becomes possible to determine the fundamental characteristics and properties of GCMI system, and hence formulate and theoretically substantiate basic general laws inherent in the process of moment of inertia conversion.

2. Generalized moment of inertia converter

Generalized moment of inertia of converter is called an abstract measuring device of arbitrary order (with n degrees of freedom), which implements measuring conversion of the moment of inertia into mathematically related physical quantity (geometric, kinematic or dynamic) which is the most common form of relatively well-known and possible in future converters of inertia moment since inertia and, at certain conditions, is converted in them. The block diagram of generalized moment of inertia converter is shown in Fig. 1. In general, this device is exclusively either mechanical or electromechanical system and consists of two interacting parts (subsystems): the object of − measurement (control) itself, which by its nature is either mechanical or electromechanical system (we call this part the subsystem A). By the condition of the problem the moment of inertia J of subsystem A is unknown and represents the input physical quantity; some additional exclusively mechanical system with predetermined properties and relationships (subsystem A), which in some way is connected with the object of measurement (monitoring), and creates for it either the field of active forces, inducing to motion, or field reactions connections, limiting this motion. Thus, as the subsystem A, the subsystem B determines the state of the converter and influences its equation of motion.

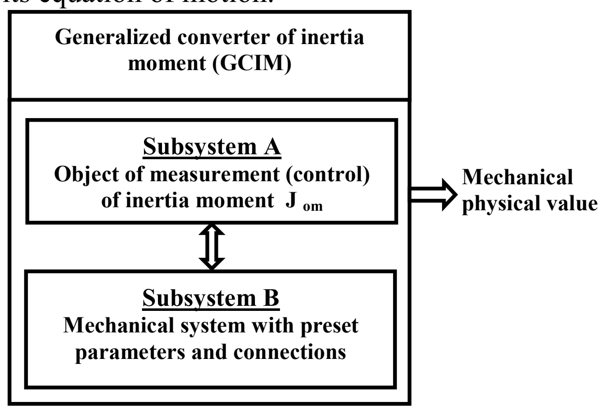

Fig. 1. Block diagram of generalized moment of inertia converter

3. Mathematical model of GCMI mechanical system

Differential equations are obtained taking into account the force interaction between the above-mentioned subsystems. Thus, we consider the motion of each of GCMI subsystems Mathematical model of generalized moment of inertia converter should be the equation of motion of the converter itself as mechanical (electromechanical) system, written in one form or another, since both inertia moment as input value and another mechanical physical value taken as output value, will be present explicitly or implicitly in the system of equations.

In its turn, the system of equations of motion GCMI is the totality of equations systems of motion of subsystem A and subsystem B, if only these separately

The motion of subsystem A

Let the subsystem A contain N_A material points with masses m_i, where $i = 1, 2, ..., N_A$.

During the movement of this subsystem in the general case to each of i-th material point set of balanced forces is applied [9] (Fig. 2):

$$\vec{F}_i^{(A)} + \vec{R}_i^{(A)} + \vec{P}_i^{(AB)} - m_i \ddot{\vec{r}}_i^{(A)} = 0,$$

where $\vec{F}_i^{(A)}$ is the force, which is a resultant set of active forces applied to an arbitrary i-th material point of subsystem A; $\vec{R}_i^{(A)}$ – force, which would restrict the motion of i-th material point of the subsystem A on condition of the independence of the latter on subsystem B (the resultant of connections reactions of subsystem A); $\vec{P}_i^{(AB)}$ – resultant of active forces and connections reactions which induce to motion or restrict the motion of i-th material point of the subsystem A as a result of force impact by the subsystem B; $-m_i \ddot{\vec{r}}_i^{(A)}$ – D'Alembert inertial force that acts on each i-th material point with mass m_i, where $\vec{r}_i^{(A)}$ – radius vector of i-th material point relatively the preset inertial count system.

According to the principle of d'Alembert-Lagrange [9]:

$$\sum_{i=1}^{N_A}\left(\vec{F}_i^{(A)} + \vec{R}_i^{(A)} + \vec{P}_i^{(AB)} - m_i\ddot{\vec{r}}_i^{(A)}\right)\cdot \delta\vec{r}_i^{(A)} = 0\,. \tag{1}$$

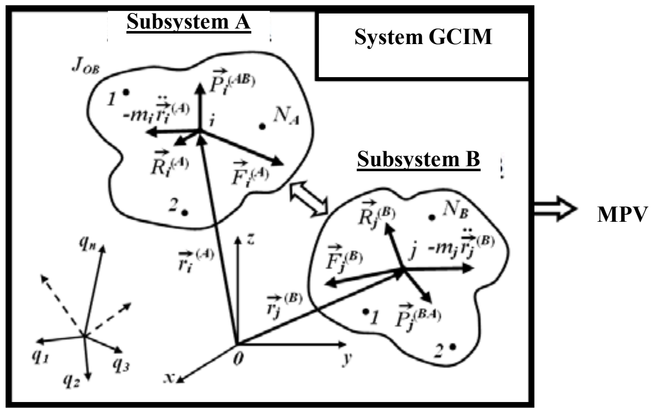

Fig.2 Forces distribution in GCMI system

Detailed mathematical transformations are given in [7].
Referring to them, the general equation of subsystem A motion dynamics (1) can be written in generalized coordinates q_s:

$$\sum_{s=1}^{n}\left(Q_s^{(A)} + \sum_{i=1}^{N_A}\vec{P}_i^{(AB)}\cdot\frac{\partial \vec{r}_i^{(A)}}{\partial q_s} - \frac{d}{dt}\left(\frac{\partial T_A}{\partial \dot{q}_s}\right) + \frac{\partial T_A}{\partial q_s}\right)\cdot \delta q_s = 0$$

or, taking into account the independence of their variations –

$$\frac{d}{dt}\left(\frac{\partial T_A}{\partial \dot{q}_s}\right) - \frac{\partial T_A}{\partial q_s} = Q_s^{(A)} + \sum_{i=1}^{N_A}\vec{P}_i^{(AB)}\cdot\frac{\partial \vec{r}_i^{(A)}}{\partial q_s}, \qquad s = 1,2,\dots,n\,, \tag{2}$$

where T_A – is the kinetic energy of subsystem A; $Q_s^{(A)} = \sum_{i=1}^{N_A}\vec{F}_i^{(A)}\cdot\frac{\partial \vec{r}_i^{(A)}}{\partial q_s}$ – is generalized force, acting on all points of the subsystem A and corresponds to s-th generalized coordinate.

The motion of subsystem B

Equations of subsystem B of GCMI motion we obtain on the basis of the principle of d'Alembert-Lagrange similarly. Then for the subsystemB of GCMI it can be written

$$\frac{d}{dt}\left(\frac{\partial T_B}{\partial \dot{q}_s}\right) - \frac{\partial T_B}{\partial q_s} = Q_s^{(B)} + \sum_{j=1}^{N_B}\vec{P}_j^{(BA)}\cdot\frac{\partial \vec{r}_j^{(B)}}{\partial q_s}, \qquad s = 1,2,\dots,n\,, \tag{3}$$

где T_B – is kinetic energy of the subsystem B; $Q_s^{(B)} = \sum_{j=1}^{N_B}\vec{F}_i^{(B)}\cdot\frac{\partial \vec{r}_j^{(B)}}{\partial q_s}$ – is generalized force of the subsystem B, which corresponds to the *s*-th generalized coordinate.

The equation of motion of GCMI system

Since the differential equations of the subsystems A and B were obtained taking into account force interaction between them, then for obtaining the system of equations of motion GCMI systems of equations (2) and (3) are added coordinatewise, taking into account theorem of the action and counter-action of subsystems A and B in generalized forces. Then

$$\frac{d}{dt}\left[\frac{\partial}{\partial \dot{q}_s}T_A(J_{OB})\right] + \frac{d}{dt}\left(\frac{\partial T_B}{\partial \dot{q}_s}\right) - \left[\frac{\partial}{\partial q_s}T_A(J_{OB}) + \frac{\partial T_B}{\partial q_s}\right] = Q_s^{(A)} + Q_s^{(B)}, \quad s = 1,2,\dots,n\,. \tag{4}$$

The equations system of motion of GCMI (8) is a system of n differential equations of second order relatively generalized coordinates, which is presented in the form of Lagrange equations of the second kind [9]. It is a generalized mathematical model of any theoretically possible converter of moment of inertia. Mathematical model of the mechanical system of GCMI Motion of GCMI system is characterized by some features that simplify the system of equations (4). Their thorough analysis was performed in [7, 8]. Based on the results of this analysis, the system of equations (4) can be represented in the form:

$$\begin{cases} \left(J_{OB} + m_A l^2\right)\ddot{q}_1 = M_A - \dfrac{\partial \Pi}{\partial q_1} - \dfrac{\partial \Phi}{\partial \dot{q}_1}, \\ \dfrac{d}{d\,t}\left(\dfrac{\partial T_B}{\partial \dot{q}_s}\right) - \dfrac{\partial T_B}{\partial q_s} = Q_s - \dfrac{\partial \Pi}{\partial q_s} - \dfrac{\partial \Phi}{\partial \dot{q}_s},\ s = 2,3,...,n, \end{cases} \tag{5}$$

where Π – is potential energy of GCMI system ; Φ – is Rayleigh dissipation function; M_A – is main mechanical torque of subsystem A relatively its rotation axis .

4. Theorems of subsystems A and B of GCMI interaction.

During analytical study of GCMI system motion we can not ignore some important general regularities that are inherent in any currently known, as well as possible in the future converters moment of inertia. We call these regularities as interaction theorems. Let us formulate them and theoretically substantiate

. So, let the subsystem A is in the force field of the subsystem B. This means that, in general, for each i-th of N_A material points of subsystem A on the side of j-th of N_B material points which belong to subsystem B, a set of active forces and connections reactions acts (Fig. 3).

In Fig. 3 force $\vec{F}_{ij}^{(AB)}$ is active , and the force $\vec{R}_{ij}^{(AB)}$ is a reaction of connections, which act on an arbitrary i -th point of the subsystem A from j -th point of subsystem B

$$\vec{P}_{ij}^{(AB)} = \vec{F}_{ij}^{(AB)} + \vec{R}_{ij}^{(AB)}.$$

On the other hand, in general case, each i -th of N_A material points of the subsystem A acts on each j -th of N_B material points which belong to subsystem B, with the force

$$\vec{P}_{ji}^{(BA)} = \vec{F}_{ji}^{(BA)} + \vec{R}_{ji}^{(BA)}.$$

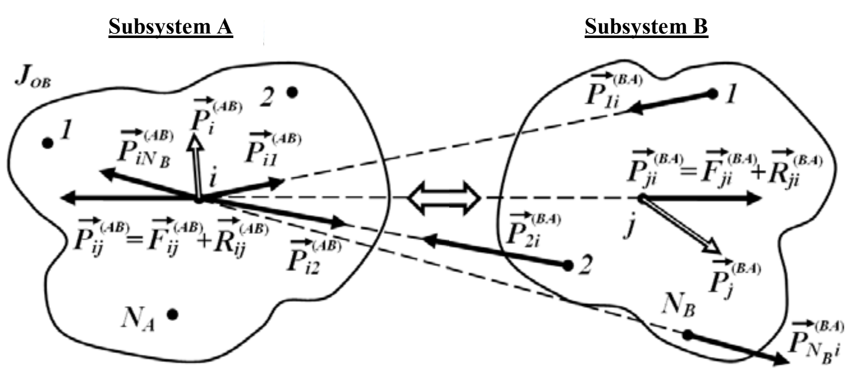

Fig 3. Field of mutual forces of subsystems A and B of GCMI

Accordingly, the law of equality of action and reaction [9], $\vec{P}_{ij}^{(AB)} = -\vec{P}_{ji}^{(BA)}$, that is, during the movement subsystem A to subsystem B force field there is always a couple of forces, scrip $\vec{P}_{ij}^{(AB)}$, $\vec{P}_{ji}^{(BA)}$ such that their

$$\vec{P}_{ij}^{(AB)} + \vec{P}_{ji}^{(BA)} = 0\,. \qquad (6)$$

Equality (6) allows us to formulate and prove the following theorem.

Theorem 1. Of the action and counteraction of subsystems A and B of GCMI

The action always corresponds to the counteraction equal and opposite to it in the direction , i.e.

the action of subsystem A on subsystem BI generates counteraction, equal by value and opposite in direction:

$$\sum_{\mathbf{i=1}}^{\mathbf{N_A}} \vec{\mathbf{P}}_{\mathbf{i}}^{(\mathbf{AB})} + \sum_{\mathbf{j=1}}^{\mathbf{N_B}} \vec{\mathbf{P}}_{\mathbf{j}}^{(\mathbf{BA})} = \mathbf{0}$$

or

$$\vec{\mathbf{P}}^{(\mathbf{AB})} + \vec{\mathbf{P}}^{(\mathbf{BA})} = \mathbf{0}\,. \qquad (7)$$

If the action on subsystem A on the part of subsystem B is the sum $\vec{P}^{(AB)} = \sum_{i=1}^{N_A} \vec{P}_i^{(AB)}$ where the force $\vec{P}_i^{(AB)} = \sum_{j=1}^{N_B} \vec{P}_{ij}^{(AB)}$ -is resultant of all forces applied to i-th point of the subsystem A due to the action of all N_B points of the subsystem B. Then counteract the action of the subsystem A subsystem represents the effect of subsystem A to subsystem B, and, on the other hand, counteraction of the subsystem A to the action of the subsystem B is the action of the subsystem A on subsystem B $\vec{P}^{(BA)} = \sum_{j=1}^{N_B} \vec{P}_j^{(BA)}$ where the force $\vec{P}_j^{(BA)} = \sum_{i=1}^{N_A} \vec{P}_{ji}^{(BA)}$ – is the resultant of all forces applied to the j-th point of the subsystem B as a result of action on the part of all the points that belong to the subsystem A.

For the action on the part of the subsystem A subsystem, taking into account any relation (6) and change the order of summation, we can write:

Forcible action on the part of the subsystem A subsystem B is said to be worth, where the force – the resultant of all forces applied to i-th point of the subsystem A from all points of the subsystem B. Then counteract the action of the subsystem A subsystem B represents the effect of subsystem A to subsystem B, which force - the resultant of all forces applied to the j -th point of the subsystem B as a result of actions on the part of all N_A points that belong to the subsystem A then for the action on subsystem A on the part of subsystem B we can write, taking into account (6) and changing the order of summation:

$$\vec{P}^{(AB)} = \sum_{i=1}^{N_A} \vec{P}_i^{(AB)} = \sum_{i=1}^{N_A}\sum_{j=1}^{N_B} \vec{P}_{ij}^{(AB)} = \sum_{i=1}^{N_A}\sum_{j=1}^{N_B}\left(-\vec{P}_{ji}^{(BA)}\right) = -\sum_{j=1}^{N_B}\sum_{i=1}^{N_A} \vec{P}_{ji}^{(BA)} = -\sum_{j=1}^{N_B} \vec{P}_j^{(BA)} = -\vec{P}^{(BA)}$$

or

$$\vec{P}^{(AB)} + \vec{P}^{(BA)} = 0\,.$$

What must be proved.

Theorem 2. On the equivalence of the interaction of subsystems A and B OPMI

An important consequence of the theorem on the action and reaction subsystems And that is in the fact that motion of subsystems A and B in the external force field is created in the subsystem will be equivalent to the motion of subsystems A and B in the external force field created by a subsystem of A, if only these fields will be an action and reaction.

Corollary (Theorem) follows from the properties of commutativity of summation in equation (7).

Theorem 3. On the action and counteraction of subsystems A and B OPMI in generalized forces

Effects of generalized forces always correspond to equal him and opposition to anti-sign, ie generalized effect on any of the generalized coordinates of a material system to another always produce the same generalized coordinate resistance, equal in magnitude and opposite in sign:

What is and should be proved.

$$\sum_{\mathbf{i=1}}^{\mathbf{N_A}} \vec{\mathbf{P}}_{\mathbf{i}}^{(\mathbf{AB})} \frac{\partial\, \vec{\mathbf{r}}_{\mathbf{i}}^{(\mathbf{A})}}{\partial\, \mathbf{q_s}} + \sum_{\mathbf{j=1}}^{\mathbf{N_B}} \vec{\mathbf{P}}_{\mathbf{j}}^{(\mathbf{BA})} \frac{\partial\, \vec{\mathbf{r}}_{\mathbf{j}}^{(\mathbf{B})}}{\partial\, \mathbf{q_s}} = \mathbf{0}, \quad \mathbf{s = 1, 2, \ldots, n}$$

or

$$\mathbf{D_s^{(AB)} + D_s^{(BA)} = 0}, \quad \mathbf{s = 1, 2, \ldots, n}.$$

Possible work $\delta A^{(AB)}$ of all $\vec{P}_i^{(AB)}$ the forces that are applied to the points of the subsystem A as a result of actions by the subsystem B is equal to:

$$\delta A^{(AB)}\left(\vec{P}_i^{(AB)}\right) = \sum_{i=1}^{N_A} \vec{P}_i^{(AB)}\, \delta\, \vec{r}_i^{(A)}.$$

We rewrite this equality, taking into account that $\delta\, \vec{r}_i^{(A)} = \sum_{s=1}^{n} \frac{\partial\, \vec{r}_i^{(A)}}{\partial\, q_s} \delta\, q_s$ and change the order of summation. Then

$$\delta A^{(AB)}\left(\vec{P}_i^{(AB)}\right) = \sum_{s=1}^{n}\left(\sum_{i=1}^{N_A} \vec{P}_i^{(AB)} \frac{\partial\, \vec{r}_i^{(A)}}{\partial\, q_s}\right)\delta\, q_s. \tag{8}$$

On the other hand, the possible work $\delta\, A^{(BA)}$ of all forces $\vec{P}_j^{(BA)}$ applied to the points in the subsystem B as a result of actions on the part of this subsystem to subsystem A, is:

$$\delta A^{(BA)}\left(\vec{P}_j^{(BA)}\right) = \sum_{j=1}^{N_B} \vec{P}_j^{(BA)}\, \delta\, \vec{r}_j^{(B)} = \sum_{s=1}^{n}\left(\sum_{j=1}^{N_B} \vec{P}_j^{(BA)} \frac{\partial\, \vec{r}_j^{(B)}}{\partial\, q_s}\right)\delta\, q_s. \tag{9}$$

Based on the law of conservation of energy [9]

$$\delta A^{(AB)}\left(\vec{P}_i^{(AB)}\right) = -\delta A^{(BA)}\left(\vec{P}_j^{(BA)}\right). \tag{10}$$

Then, in the aggregate of the relations (8) - (10) it follows:

$$\sum_{s=1}^{n}\left(\sum_{i=1}^{N_A} \vec{P}_i^{(AB)} \frac{\partial\, \vec{r}_i^{(A)}}{\partial\, q_s} + \sum_{j=1}^{N_B} \vec{P}_j^{(BA)} \frac{\partial\, \vec{r}_j^{(B)}}{\partial\, q_s}\right)\delta\, q_s = 0.$$

Since all variations of the generalized coordinates are independent from each other due to the independence of the generalized coordinates themselves, the above- mentioned sum will be zero, unless the factors are equal zero for all variations of $\delta\, q_s$

.

$$\sum_{i=1}^{N_A} \vec{P}_i^{(AB)} \frac{\partial\, \vec{r}_i^{(A)}}{\partial\, q_s} + \sum_{j=1}^{N_B} \vec{P}_j^{(BA)} \frac{\partial\, \vec{r}_j^{(B)}}{\partial\, q_s} = 0, \quad s = 1, 2, \ldots, n$$

or

$$D_s^{(AB)} + D_s^{(BA)} = 0, \quad s = 1, 2, \ldots, n.$$

Theorem is proved

5. Mathematical model of electromechanical system of GCMI

If subsystem A is an electromechanical subsystem, its motion as the motion of GCMI, on the whole, is determined by the action and interaction, not only of mechanical forces but also forces of electromagnetic origin [9, 10], which is accompanied by the increase of the number of independent variables which describe the state and the motion of the electromechanical system of GCMI.

In this case, as it was proved in [7], mathematical model of electromechanical system of GCMI will have the form:

$$\begin{cases} \left(J_{OB}+m_A l^2\right)\ddot{q}_1 = M_A - \dfrac{\partial \Pi}{\partial q_1} - \dfrac{\partial \Phi}{\partial \dot{q}_1}, \\ \dfrac{d}{dt}\left(\dfrac{\partial T_B}{\partial \dot{q}_s}\right) - \dfrac{\partial T_B}{\partial q_s} = Q_s - \dfrac{\partial \Pi}{\partial q_s} - \dfrac{\partial \Phi}{\partial \dot{q}_s}, \quad s = 2,3,...,n, \end{cases} \tag{11}$$

where $M_A = M_A^{(м)} + M_A^{(e)}$. The torque of the subsystem A $M_A^{(e)}$ is mathematically related to the generalized electric coordinates $q_{n+1}^{(e)}, q_{n+2}^{(e)}, ..., q_k^{(e)}$ and loop currents $i_{n+1}, i_{n+2}, ..., i_k$ and, in general, depends on the time t.

6. Two general properties of the moment of inertia conversion

Analysis of mathematical models of (GCMI) system (5) and (11) reveals two important properties of any converter of moment of inertia, known or possible. Thus, the structure of systems of differential equations (5) and (11), the possibility of their mathematical solutions allows to make the following assumption, which does not contradict to any of the existing converters of moment of inertia and is represented as the following axiom:

Direct measurement conversion of moment of inertia in physical quantity of non- mechanical origin is either theoretically impossible (for mechanical system of GCMI) or impractical (for electromechanical system of GCMI).

An important consequence of the given axiom is that to convert the moment of inertia, for example, in electric quantity it is necessary to construct measuring channels for moment of inertia with two or more converters, the first of which must be GCMI

From the mathematical models (5) and (11) follows general property of any existing or potential converters of moment of inertia, which we formulate as a theorem.

Theorem 4. On the dynamic mode.

Any transformation of the moment of inertia requires transition process for the object of measurement (monitoring), and is possible only under this condition.

Proof.

Prove the theorem from the opposite. Assume that the transformation of the moment of inertia is possible on condition of the absence of the transition process in the subsystem A of (GCMI)system . This means that the subsystem A should be either at rest $q_1 = const$, or in the state of uniform motion $q_1 = c_1 t + c_2$, where c_1 and c_2 – are constants.

In both cases the acceleration of subsystem A will be absent and the second derivative of the first generalized coordinate in time equals zero

$$\ddot{q}_1 = 0 . \tag{12}$$

Since, except the first equation, moment of inertia J_{OB} does not enter all other systems of equations (5) and (11) either explicitly or implicitly, then equation (12) deprives all possible solutions of systems (5) and (11) dependence on the moment of inertia, as well, and makes impossible its conversion. The assumption has led to a contradiction that is why it is false (the law of non-contradiction).

The theorem is proved.

7. Conclusions

We elaborated theoretical concepts of the process of measuring conversion of moment of inertia of mechanical and electromechanical systems. The concept of generalized converter of moment of inertia (GCMI), which is an abstract, most general form with relatively existing and possible in future converters of moment of inertia is suggested. It allowed to formulate general structural diagram and based on the variation principles of analytical mechanics to develop generalized mathematical model, as well as to reveal and substantiate the important features and properties both of as (GCMI) system , and the process of moment of inertia conversion itself.

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